

# Effects of Particle Concentration and Slurry Height on Gas Holdup in a Slurry Bubble Column

Shohei SASAKI, Kengo UCHIDA, Kosuke HAYASHI and  
Akio TOMIYAMA

Graduate School of Engineering, Kobe University, 1-1 Rokkodai Nada, Kobe-shi, Hyogo 657-8501, Japan

**Keywords:** Slurry Bubble Column, Gas Holdup, Initial Slurry Height, Froude Number

Total gas holdups  $\alpha_G$  in a cylindrical slurry bubble column were measured at various values of the superficial gas velocity  $J_G$ , the mean particle volumetric concentration  $C_s$  and the initial slurry height  $H_0$  to investigate their effects on  $\alpha_G$ . The column diameter  $D_H$  and height were 200 mm and 2000 mm, respectively. The gas, liquid and solid phases were air, water and hydrophilic silica particles of 100  $\mu\text{m}$  in mean diameter, respectively. Experimental conditions were  $0.025 \leq J_G \leq 0.40$  m/s,  $0 \leq C_s \leq 0.50$  and  $1.5 \leq H_0^* \leq 5.0$ , where  $H_0^* = H_0/D_H$ . The conclusions obtained are as follows: (1)  $\alpha_G$  decreases with increasing  $H_0$  and becomes independent of  $H_0$  for  $H_0^* > 4$  at low  $J_G$ , whereas it depends on  $H_0$  at higher  $J_G$  even for  $H_0^* > 4$ , (2) the increase in  $C_s$  decreases  $\alpha_G$  up to  $C_s \sim 0.40$ , whereas  $\alpha_G$  becomes independent of  $C_s$  at larger  $C_s$ , and (3)  $\alpha_G$  in the slurry bubble column at various  $C_s$ ,  $H_0$  and  $J_G$  are well correlated in terms of the Froude number  $Fr_H$  using  $H_0$  as a characteristic length.

## Introduction

Slurry bubble column reactors have been utilized in various chemical plants for petrochemical, biochemical and metallurgical engineering applications (Degaleesan *et al.*, 2001). The total gas holdup,  $\alpha_G$ , is one of the key parameters in designing bubble columns, and therefore, many studies on  $\alpha_G$  have been carried out (Koide *et al.*, 1984; Khare and Joshi, 1990; Krishna *et al.*, 1997; Li and Prakash, 1997; Gandhi *et al.*, 1999; Vandu *et al.*, 2004; Mena *et al.*, 2005). These studies have confirmed that  $\alpha_G$  depends on the superficial gas velocity,  $J_G$ , the mean particle volumetric concentration,  $C_s$ , and the initial slurry height,  $H_0$ .

It is well known that  $\alpha_G$  decreases with increasing  $C_s$  of hydrophilic particles (Koide *et al.*, 1984; Li and Prakash, 1997; Gandhi *et al.*, 1999; Mena *et al.*, 2005; Ojima *et al.*, 2014, 2015). Since the bubble frequency in a bubble column decreases with increasing  $C_s$ , it has been speculated that the  $\alpha_G$  reduction is due to the enhancement of bubble coalescence. In our previous studies (Ojima *et al.*, 2014, 2015), the effects of hydrophilic particles on bubble coalescence were quantitatively investigated using a two-dimensional vessel. Measurements of the time,  $t_c$ , elapsed from bubble contact to coalescence made clear that the presence of hydrophilic particles enhances bubble coalescence. The dependency of  $t_c$  on  $C_s$  was similar to that of the bubble frequency in a three-dimensional bubble column. Therefore a coalescence enhancement factor was modeled using the  $t_c$  data and implemented into a multi-fluid model. This model was vali-

dated up to  $J_G = 0.034$  m/s.

In contrast to the studies on the effects of  $J_G$  and  $C_s$ , few studies have been carried out on the effects of  $H_0$ . Koide *et al.* (1984) measured  $\alpha_G$  in a slurry bubble column at various  $H_0^*$ , where  $H_0^*$  is the ratio of  $H_0$  to the hydraulic diameter  $D_H$  of the column. The gas holdup decreases with increasing  $H_0^*$  for  $H_0^* < 7$ , whereas it does not depend on  $H_0^*$  for  $H_0^* \geq 7$ . They proposed a gas holdup correlation for  $H_0^* \geq 7$  to exclude the  $H_0$  effect. Though several  $\alpha_G$  correlations have been proposed for slurry bubble columns (Li and Prakash, 1997; Krishna *et al.*, 1997; Gandhi *et al.*, 1999; Vandu *et al.*, 2004), no discussion on how to account for the  $H_0$  effect in  $\alpha_G$  correlations has been made.

In our previous study (Sasaki *et al.*, 2016), we investigated the effects of  $H_0^*$  on  $\alpha_G$  in air-water bubble columns of  $D_H = 200$  mm for a wide range of  $J_G$  and  $H_0$ , i.e.,  $0.025 \leq J_G \leq 0.40$  m/s and  $1.5 \leq H_0^* \leq 5.0$ . The  $\alpha_G$  data were well correlated in terms of the Froude number  $Fr_H$  defined by using  $H_0$  as the characteristic length. The applicability of  $Fr_H$  in correlating  $\alpha_G$  in slurry bubble columns, however, has not been examined yet.

In this study, gas holdups in a slurry bubble column were measured at various values of  $J_G$  and  $C_s$  and for  $H_0^* \leq 5.0$  to investigate their effects on  $\alpha_G$ . The applicability of  $Fr_H$  to  $\alpha_G$  in the slurry bubble column was also examined.

## 1. Experimental

### 1.1 Experimental setup

Figure 1 shows the experimental setup. The cylindrical column was made of transparent acrylic resin for flow visualization. The column height was 2000 mm, and  $D_H$  was 200 mm.

The column was initially filled with slurry consisting

Received on November 4, 2015; accepted on May 9, 2016

DOI: 10.1252/jcej.15we280

Correspondence concerning this article should be addressed to A. Tomiyama (E-mail address: tomiyama@mech.kobe-u.ac.jp).

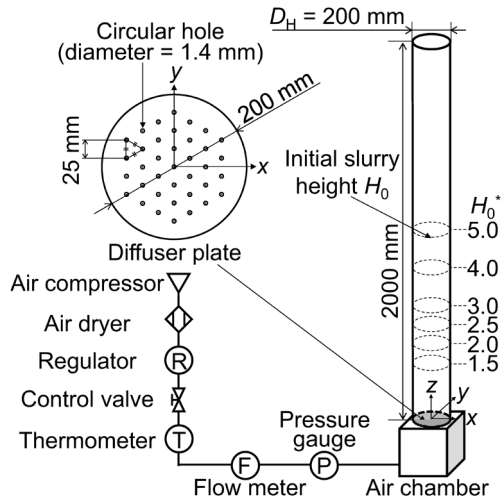


Fig. 1 Experimental setup

of water and fine particles. Water at room temperature ( $19 \pm 1^\circ\text{C}$ ) and atmospheric pressure was used for the liquid phase. Hydrophilic spherical silica particles (CARiACT<sup>®</sup>, Q-10, Fuji Silysia Chemical Ltd.) were used for the solid phase. The mean diameter,  $d_p$ , true density,  $\rho_s$ , apparent density,  $\rho_p$ , and pore volume,  $\theta$ , of the particles were  $100 \mu\text{m}$ ,  $2250 \text{ kg/m}^3$ ,  $1320 \text{ kg/m}^3$  and  $1.3 \times 10^{-3} \text{ m}^3/\text{kg}$ , respectively, where the apparent density was evaluated by taking the volume-weighted average of the true density and the density of water filling its pore volume. The porosity,  $\delta$ , of the particles defined by  $\delta = \rho_s \theta / (\rho_s \theta + 1)$  is 0.75. Since the pore of silica particles could be filled with water, the total particle volume in the slurry was defined as  $V_s + \Theta$ , where  $V_s$  is the total volume of the solid phase in the slurry and  $\Theta$  the total volume of the pore of particles. The mean particle concentration,  $C_s$ , of the slurry was, therefore, defined by  $C_s = (V_s + \Theta) / V_{SL}$ , where  $V_{SL} (= \pi D_H^2 H_0 / 4)$  is the slurry volume. The  $C_s$  can also be expressed as  $C_s = \theta M_s / V_{SL} \delta$ , where  $M_s$  is the total mass of the solid phase. The  $C_s$  ranged from 0 to 0.50. The uncertainty in  $C_s$  at 95% confidence was 0.01.

Air supplied from the compressor (RDG-150C, Anest Iwata Corp.) flowed into the column through the air dryer (SLP-1501EB, Anest Iwata Corp.) and the air chamber. The air diffuser plate of 5 mm in thickness was placed at the bottom of the column. The hole diameter, the number of holes, the hole pitch and the ratio of the total hole area to the area of column cross-section were 1.4 mm, 37, 25 mm and 0.18%, respectively.

The gas volume flow rate was measured using the flowmeters (NVP-I, FLT-H, Nippon Flow Cell Co., Ltd.; AM-1000, full-scale accuracy  $\pm 1.5\%$ , Tokyo Keiso Co. Ltd.). The measured flow rate was converted into the volume flow rate at the middle height of the slurry level by taking into account gas expansion due to the decrease in static pressure. The range of  $J_G$  defined at the latter location was from  $0.025 \pm 0.001$  to  $0.40 \pm 0.01 \text{ m/s}$ , where the uncertainties were evaluated at 95% confidence.

The physical properties of the gas and liquid phases

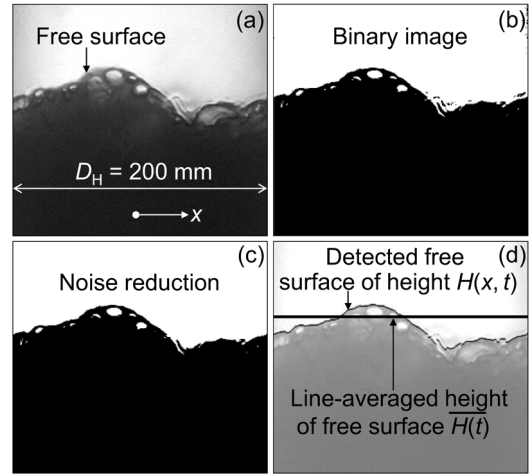


Fig. 2 Image processing for  $\alpha_G$  measurement ( $J_G = 0.050 \text{ m/s}$ ,  $H_0^* = 5.0$ ,  $C_s = 0.20$ )

are as follows: the liquid density was  $998 \text{ kg/m}^3$ , the gas density  $1.2 \text{ kg/m}^3$ , the liquid viscosity,  $\mu_L$ ,  $1.0 \times 10^{-3} \text{ Pa}\cdot\text{s}$ , the gas viscosity  $1.8 \times 10^{-5} \text{ Pa}\cdot\text{s}$ , and the surface tension  $0.072 \text{ N/m}$ . The initial slurry height,  $H_0$ , was varied from 300 to 1000 mm. The ratio,  $H_0^*$ , of  $H_0$  to  $D_H$ , therefore, ranged from 1.5 to 5.0. Since the Stokes number,  $St (= \rho_p d_p^2 J_G / 18 \mu_L D_H)$  (Ojima *et al.*, 2015), was much smaller than unity, e.g.,  $St = 7.3 \times 10^{-4}$  at  $J_G = 0.20 \text{ m/s}$ , and bubbles strongly stirred the slurry, the particle concentration would be uniform in the column.

## 1.2 Measurement method

The gas holdup of the bubble column was measured by processing images of the free surface. The images were taken using a high-speed video camera (Motion Pro X-3, Integrated Device Technology Inc.) with spatial and temporal resolutions of  $0.36 \text{ mm/pixel}$  and  $1/100 \text{ s}$ , respectively. As shown in **Figure 2**, they were transformed into binary images. Then, the instantaneous slurry height,  $H(x, t)$ , at the horizontal position  $x$  and the time  $t$  was detected using a region growing method (Hojjatolaslami and Kittler, 1998) after noise reduction. The mean slurry height at  $t$  was calculated as

$$\overline{H(t)} = \frac{1}{D_H} \int_{-D_H/2}^{D_H/2} H(x, t) dx \quad (1)$$

The total gas holdup was then calculated as

$$\alpha_G = \frac{1}{T} \int_{t_1}^{t_1+T} \frac{\overline{H(t)} - H_0}{H(t)} dt \quad (2)$$

where  $t_1$  is the time, at which the recording was started, and  $T$  the sampling time. Instantaneous and time-averaged  $\overline{H(t)}$  at  $J_G = 0.40 \text{ m/s}$  in the slurry bubble column are shown in **Figure 3**. For the latter,  $T$  was varied from 0 to 30 s to examine the convergence of the time-averaged value. The instantaneous  $\overline{H(t)}$  fluctuates largely, whereas the time-averaged  $\overline{H(t)}$  at  $T = 30 \text{ s}$  converges to within  $\pm 0.5\%$  de-

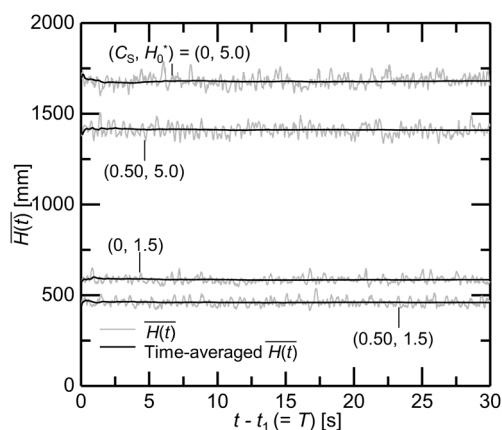


Fig. 3 Instantaneous and time-averaged  $\overline{H(t)}$  at  $J_G = 0.40$  m/s

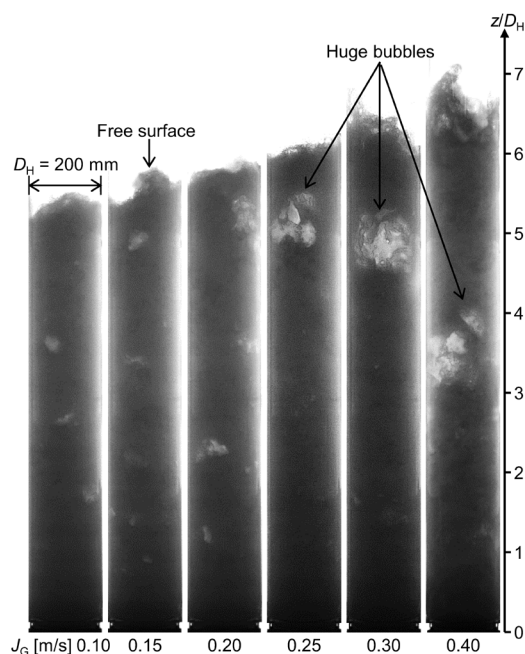


Fig. 4 Bubbly flows at  $C_s = 0.20$  and  $H_0^* = 5.0$

viations. In the following,  $\alpha_G$  data evaluated using  $T = 30$  s in Eq. (2) are presented. The relative standard errors in  $\alpha_G$  were evaluated by repeating the measurement 10 times under the conditions of  $C_s = 0$  and  $0.50$ ,  $J_G = 0.025$  and  $0.40$  m/s, and  $H_0^* = 1.5$  and  $5.0$ . The errors in  $\alpha_G$  were within  $\pm 1\%$ .

The flow was observed by using a high-speed video camera (Fastcam SA-X2, Photron Ltd.). Four fluorescent lights were used for back illumination. The shutter speed and the frame rate were  $1/20000$  and  $500$  fps, respectively. The sampling time was  $20$  s. The spatial resolution of the flow visualization was  $0.49$  mm/pixel.

## 2. Results and Discussion

### 2.1 Flow structure and gas holdup

Typical images of bubbles in the slurry at  $H_0^* = 5.0$  and  $C_s = 0.20$  are shown in Figure 4. Though the presence of

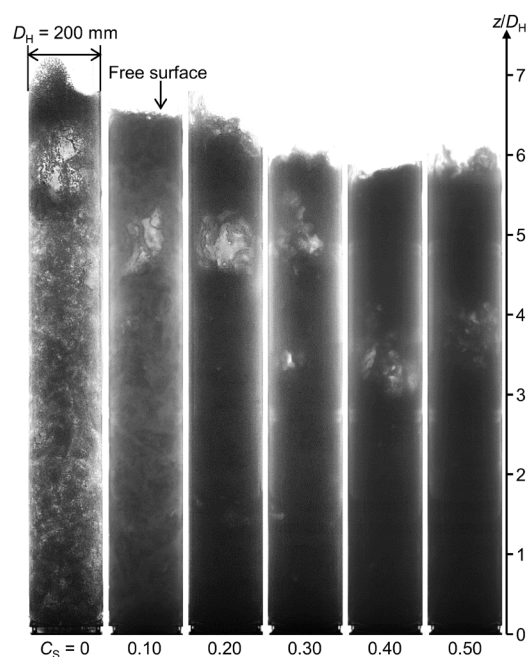


Fig. 5 Bubbly flows at  $J_G = 0.30$  m/s and  $H_0^* = 5.0$

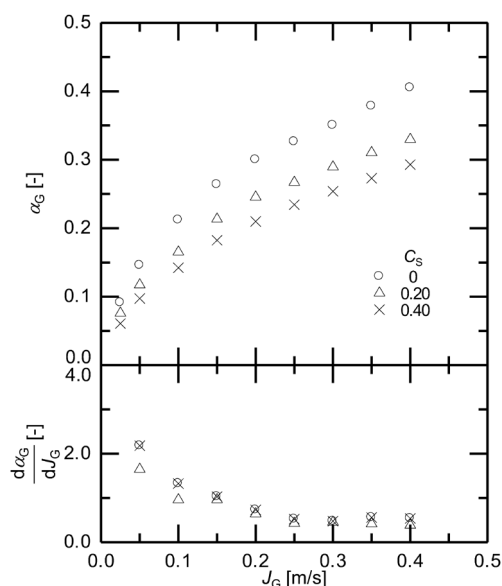


Fig. 6 Total gas holdup  $\alpha_G$  and gradient  $d\alpha_G/dJ_G$  at  $H_0^* = 5.0$

particles strongly deteriorates the transparency of slurry, high-speed video images confirmed that the flows were heterogeneous. For  $J_G > 0.20$  m/s, huge bubbles were formed at  $z/D_H \sim 2$ , where  $z$  is the vertical distance from the bottom of the column. The huge bubbles strongly agitated the flow over the whole column cross section. This trend was similar to that at  $C_s = 0$  in Sasaki *et al.* (2016). The characteristics of flow structures rarely depended on  $C_s$ .

Figure 5 shows flows at  $H_0^* = 5.0$  and  $J_G = 0.30$  m/s. The increase in  $C_s$  decreases the height of the free surface, in other words, the total gas holdup. The heights for  $C_s \geq 0.40$  are, however, almost the same.

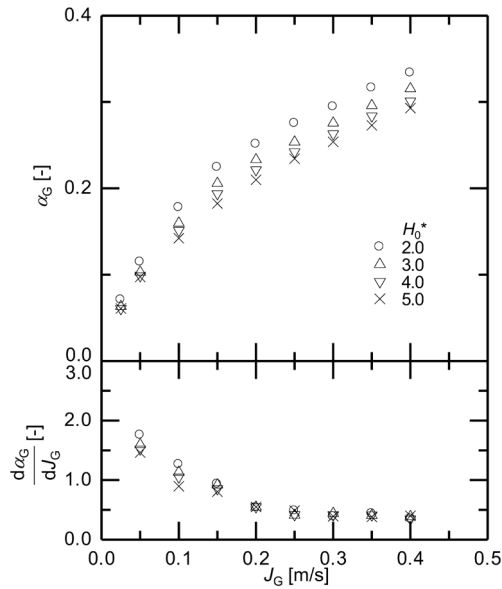


Fig. 7  $\alpha_G$  and  $d\alpha_G/dJ_G$  at  $C_s = 0.40$

Gas holdups at  $H_0^* = 5.0$  are shown in **Figure 6**, where the  $\alpha_G$  data at  $C_s = 0$  were quoted from Sasaki *et al.* (2016). The  $\alpha_G$  monotonously increases as  $J_G$  increases. This trend is the same as that of the so-called pure heterogeneous flow (Ruzicka *et al.*, 2003). The increase in  $C_s$  decreases  $\alpha_G$ . This  $\alpha_G$  reduction is due to the bubble coalescence enhancement by hydrophilic particles (Ojima *et al.*, 2014, 2015).

**Figures 6 and 7** also show  $d\alpha_G/dJ_G$ . The  $d\alpha_G/dJ_G$  decreases with increasing  $J_G$  and depends on  $H_0^*$  for  $J_G \leq 0.20$  m/s, whereas the dependency of  $d\alpha_G/dJ_G$  on  $J_G$  and  $H_0^*$  is weak at larger  $J_G$ . The critical value of  $J_G$  corresponds to that for the formation of huge bubbles. Though all the flows in the present experiments are classified into the heterogeneous regime, the flow structure can be further divided into two regimes, i.e., flows without huge bubbles and those with huge bubbles. Hereafter, we refer to the ranges for the former and latter as Regime 1 and Regime 2, respectively (Sasaki *et al.*, 2016).

**Figure 7** shows gas holdups at  $C_s = 0.40$ . The increase in  $H_0^*$  decreases  $\alpha_G$  both in Regimes 1 and 2. The lower  $\alpha_G$  at high  $H_0^*$  means that the mean bubble diameter in the column was larger compared to that at low  $H_0^*$ .

The total gas holdups are re-plotted against  $H_0^*$  in **Figure 8** to clearly show the effects of  $H_0$  on  $\alpha_G$ . At a constant  $J_G$ ,  $\alpha_G$  decreases with increasing  $H_0^*$ . Koide *et al.* (1984) measured  $\alpha_G$  for  $C_s \leq 0.04$ . They concluded that the  $H_0$  effect is negligible for  $H_0^* > 7$ . In the present data,  $\alpha_G$  becomes independent of  $H_0$  for  $H_0^* > 4$  at low  $J_G$ , i.e.  $J_G = 0.025$  and  $0.050$  m/s. The effect, however, appears at higher  $J_G$  even for  $H_0^* > 4$ .

The  $C_s$  effect on  $\alpha_G$  at  $H_0^* = 1.5$  and  $5.0$  is shown in **Figure 9**. Due to the particle-induced enhancement of bubble coalescence,  $\alpha_G$  decreases with increasing  $C_s$ . At  $H_0^* = 5.0$ ,  $\alpha_G$  for  $C_s \geq 0.40$  are independent of  $C_s$ . On the other hand,  $\alpha_G$  slightly decrease with increasing  $C_s$  even for  $C_s \geq 0.40$  when  $H_0^* = 1.5$ , especially at high  $J_G$ . The reason for this

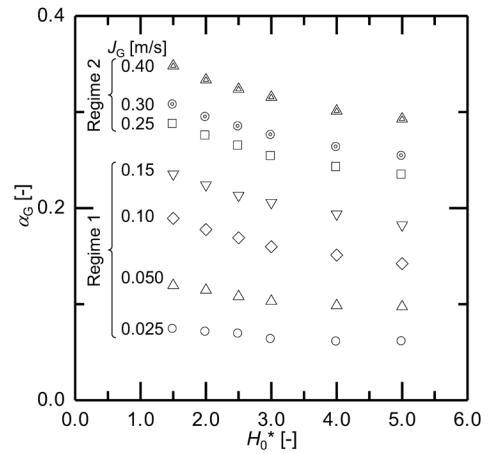


Fig. 8 Effects of  $H_0$  on  $\alpha_G$  at  $C_s = 0.40$

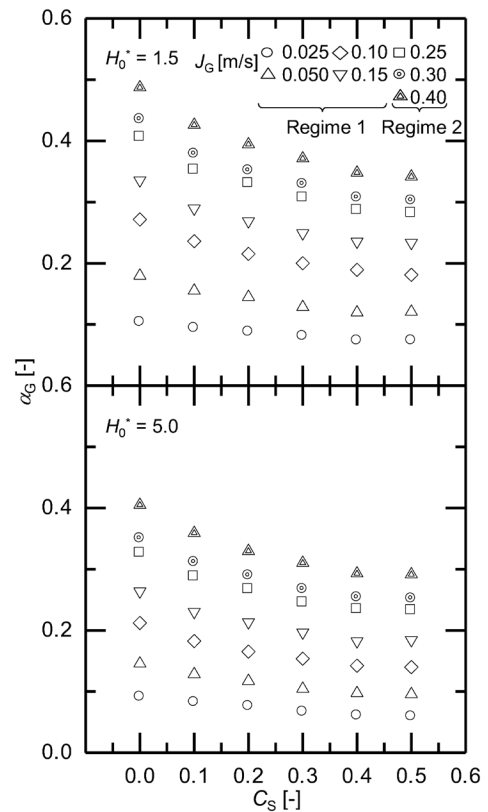


Fig. 9 Effects of  $C_s$  on  $\alpha_G$  at  $H_0^* = 1.5$  and  $5.0$

difference can be understood as follows: since huge bubbles were formed at  $z/D_H \sim 2$ , bubble coalescence was dominant for  $z/D_H \leq 2$ . On the other hand, the bubble coalescence rate was smaller for  $z/D_H \geq 2$  compared to the former region. The particle effect on bubble coalescence enhancement might, therefore, be of importance in the former. The ratio of the former region to the whole flow region decreases as  $H_0^*$  increases. Hence, the increase in  $C_s$  reduces  $\alpha_G$  even at the high  $C_s$  when  $H_0^* = 1.5$ .

Ojima *et al.* (2014, 2015) measured the film drainage time,  $t_c$ , i.e., the time elapsed from bubble contact to coales-

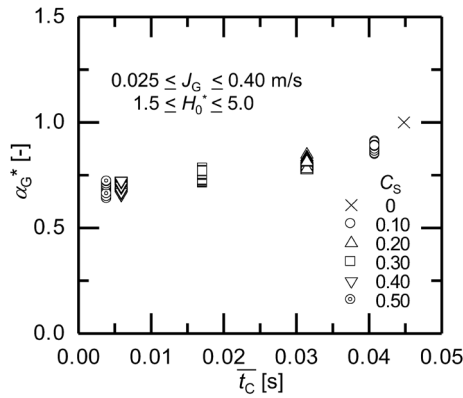


Fig. 10 Normalized gas holdup vs. film drainage time

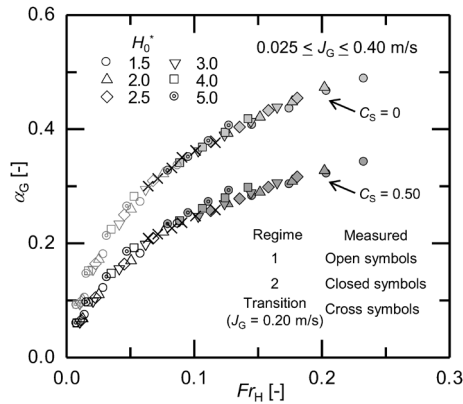


Fig. 11  $\alpha_G$  plotted against  $Fr_H$

cence, to investigate the effects of hydrophilic silica particles on bubble coalescence in a quasi two-dimensional vessel. The  $t_c$  decreased with increasing  $C_s$  for  $C_s < 0.45$ . Hence, bubble coalescence is enhanced with increasing  $C_s$ . For  $C_s \geq 0.45$ , further bubble-coalescence enhancement was not observed even with increasing  $C_s$ . Figure 10 shows a comparison between  $t_c$  in the 2D vessel and the following normalized gas holdup,  $\alpha_G^*$ , in the cylindrical bubble column:

$$\alpha_G^* = \frac{\alpha_G(H_0^*, J_G, C_s)}{\alpha_G(H_0^*, J_G, 0)} \quad (3)$$

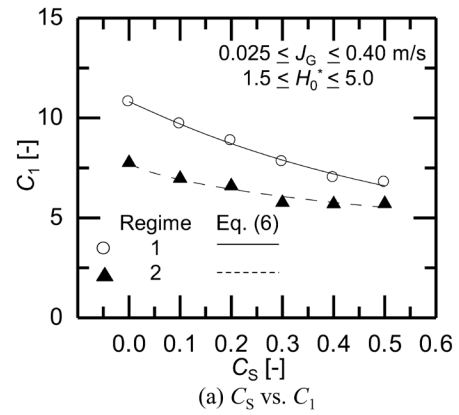
The strong correlation between  $\alpha_G^*$  and  $t_c$  implies that the  $\alpha_G$  reduction with increasing  $C_s$  in the bubble column is caused by the bubble coalescence enhancement due to the presence of particles.

## 2.2 Applicability of Froude Number to $\alpha_G$ correlation

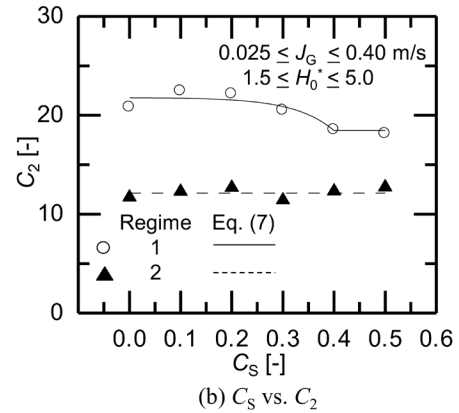
In our previous study (Sasaki *et al.*, 2016), we confirmed that the gas holdups in air-water bubble columns for  $1.5 \leq H_0^* \leq 5.0$  is well correlated in terms of the following Froude number:

$$Fr_H = \frac{J_G}{\sqrt{gH_0}} \quad (4)$$

where  $g$  is the acceleration of gravity. Figure 11 shows  $\alpha_G$  plotted against  $Fr_H$ . The  $\alpha_G$  increases with increasing  $Fr_H$ .



(a)  $C_s$  vs.  $C_1$



(b)  $C_s$  vs.  $C_2$

Fig. 12 Interpolation functions of  $C_1$  and  $C_2$  in Eq. (5)

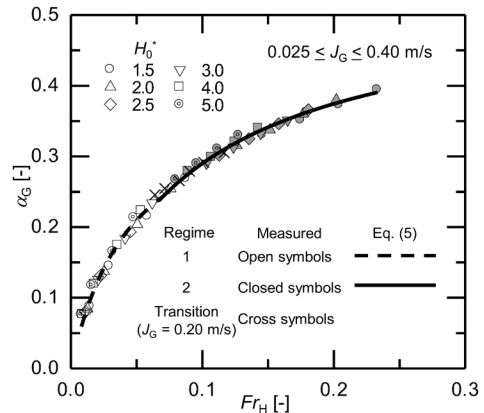


Fig. 13  $\alpha_G$  in the slurry bubble column correlated in terms of  $Fr_H$  at  $C_s = 0.20$  ( $C_1$  and  $C_2$  are calculated using Eqs. (6) and (7))

The Froude number well correlates not only  $\alpha_G$  in the air-water bubble column but also those in the slurry bubble column.

The following functional form was also used to express the  $\alpha_G$  data in Sasaki *et al.* (2016):

$$\alpha_G^R = \frac{C_1^R Fr_H^R}{1 + C_2^R Fr_H^R} \quad (R = R_1 \text{ or } R_2) \quad (5)$$

where  $C_1$  and  $C_2$  are regime-dependent coefficients and the superscripts,  $R_1$  and  $R_2$ , denote Regimes 1 and 2, respectively. Values of  $C_1$  and  $C_2$  determined by fitting Eq. (5) to



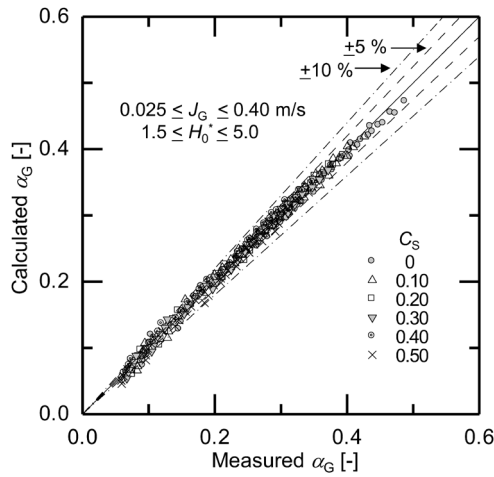


Fig. 14 Comparison between Eq. (5) and  $\alpha_G$  data

the data are shown in **Figure 12**. In both regimes,  $C_1$  decreases with increasing  $C_S$ , whereas it becomes constant for  $C_S \geq 0.40$ . This reflects the fact that the dependency of  $\alpha_G$  on  $C_S$  is very weak for this  $C_S$  range. The  $C_2$  in Regime 1 also decreases as  $C_S$  increases. On the other hand,  $C_2$  in Regime 2 is constant. Interpolation functions of  $C_1$  and  $C_2$  in terms of  $C_S$  were obtained as

$$C_1 = \begin{cases} \frac{8.12}{0.75 + C_S^{1.07}} & \text{in Regime 1} \\ \frac{11.4}{1.47 + C_S^{0.73}} & \text{in Regime 2} \end{cases} \quad (6)$$

$$C_2 = \begin{cases} \max[21.8 - 0.018e^{13C_S}, 18.5] & \text{in Regime 1} \\ 12.2 & \text{in Regime 2} \end{cases} \quad (7)$$

**Figures 13 and 14** show comparisons of the  $\alpha_G$  data at  $C_S = 0.20$  with Eq. (5) and those of all the data with Eq. (5), respectively. Equation (5), which includes only two fitting parameters  $C_1$  and  $C_2$ , can well correlate all the data.

## Conclusion

Total gas holdups,  $\alpha_G$ , in a cylindrical slurry bubble column were measured by using a high-speed video camera and an image processing method to investigate the effects of the initial slurry height,  $H_0$ , the superficial gas velocity,  $J_G$ , and the mean particle volumetric concentration,  $C_S$ , on  $\alpha_G$ . The column diameter,  $D_H$ , was 200 mm. Air, water and hydrophilic spherical silica particles were used for the gas, liquid and solid phases, respectively. The mean particle diameter was 100  $\mu\text{m}$ . The  $C_S$  varied from 0 to 0.50. The  $J_G$  ranged from 0.025 to 0.40 m/s. The ratio,  $H_0^* = H_0/D_H$ , of initial slurry height to diameter was from 1.5 to 5.0. The conclusions obtained under the present experimental conditions are as follows.

1. The gas holdup decreases with increasing  $H_0$  and becomes independent of  $H_0$  for  $H_0^* > 4$  at low  $J_G$ . The height effect, however, appears at higher  $J_G$  even for

$H_0^* > 4$ .

2. The gas holdup decreases with increasing  $C_S$  up to  $C_S \sim 0.40$ , whereas it becomes independent of  $C_S$  at larger  $C_S$ .
3. The gas holdups at various initial slurry heights are well correlated in terms of the Froude number defined by using the superficial gas velocity and the initial slurry height as the characteristic velocity and length, respectively.

## Acknowledgement

This work has been supported by the Japan Society for the Promotion of Science (JSPS) (Grants-in-Aid for Scientific Research (B), No. 15H03920).

## Nomenclature

$C_S$	= particle volumetric concentration	[—]
$C_1, C_2$	= coefficients in Eq. (5)	[—]
$D_H$	= hydraulic diameter	[m]
$d_p$	= mean particle diameter	[m]
$Fr_H$	= Froude number ( $Fr_H = J_G / \sqrt{gH_0}$ )	[—]
$g$	= acceleration of gravity	[m/s <sup>2</sup> ]
$H_0$	= initial slurry (or liquid) height	[m]
$H_0^*$	= dimensionless slurry height ( $H_0^* = H_0/D_H$ )	[—]
$H$	= instantaneous slurry height	[m]
$J_G$	= superficial gas velocity	[m/s]
$M_S$	= total mass of solid phase	[kg]
$St$	= Stokes number ( $St = \rho_p d_p^2 J_G / 18 \mu_L D_H$ )	[—]
$t_C$	= film drainage time	[s]
$t$	= time	[s]
$t_1$	= time, at which video recording is started	[s]
$T$	= sampling time	[s]
$V_S$	= total volume of solid phase in slurry	[m <sup>3</sup> ]
$V_{SL}$	= slurry volume ( $V_{SL} = \pi D_H^2 H_0 / 4$ )	[m <sup>3</sup> ]
$x$	= horizontal coordinate	[m]
$z$	= vertical coordinate	[m]
$\alpha_G$	= total gas holdup	[—]
$\alpha_G^*$	= normalized gas holdup	[—]
$\delta$	= porosity of particles ( $\delta = \rho_s \theta / (1 + \rho_s \theta)$ )	[—]
$\theta$	= pore volume of particles	[m <sup>3</sup> /kg]
$\Theta$	= total volume of pore of particles	[m <sup>3</sup> ]
$\mu_L$	= liquid viscosity	[Pa·s]
$\rho_s$	= true particle density	[kg/m <sup>3</sup> ]
$\rho_p$	= apparent particle density	[kg/m <sup>3</sup> ]

## Literature Cited

- Degaleesan, S., M. Dudukovic and Y. Pan; "Experimental Study of Gas-Induced Liquid-Flow Structures in Bubble Columns," *AIChE J.*, **47**, 1913–1931 (2001)
- Gandhi, B., A. Prakash and M. A. Bergougnou; "Hydrodynamic Behavior of Slurry Bubble Column at High Solids Concentrations," *Powder Technol.*, **103**, 80–94 (1999)
- Hojjatolleslami, S. A. and J. Kittler; "Region Growing: A New Approach," *IEEE Trans. Image Process.*, **7**, 1079–1084 (1998)
- Khare, A. S. and J. B. Joshi; "Effect of Fine Particles on Gas Hold-Up in Three-Phase Sparged Reactors," *Chem. Eng. J.*, **44**, 11–25 (1990)
- Koide, K., A. Takazawa, M. Komura and H. Matsunaga; "Gas Holdup and Volumetric Liquid-Phase Mass Transfer Coefficient in Solid-

- Suspended Bubble Columns,” *J. Chem. Eng. Japan*, **17**, 459–466 (1984)
- Krishna, R., J. W. A. de Swart, J. Ellenberger, G. B. Martina and C. Maretti; “Gas Holdup in Slurry Bubble Columns: Effect of Column Diameter and Slurry Concentrations,” *AIChE J.*, **43**, 311–316 (1997)
- Li, H. and A. Prakash; “Heat Transfer and Hydrodynamics in a Three-Phase Slurry Bubble Column,” *Ind. Eng. Chem. Res.*, **36**, 4688–4694 (1997)
- Mena, P. C., M. C. Ruzicka, F. A. Rocha, J. A. Teixeira and J. Drahoš; “Effect of Solids on Homogeneous–Heterogeneous Flow Regime Transition in Bubble Columns,” *Chem. Eng. Sci.*, **60**, 6013–6026 (2005)
- Ojima, S., K. Hayashi and A. Tomiyama; “Effects of Hydrophilic Particles on Bubbly Flow in Slurry Bubble Column,” *Int. J. Multiph. Flow*, **58**, 154–167 (2014)
- Ojima, S., S. Sasaki, K. Hayashi and A. Tomiyama; “Effects of Particle Diameter on Bubble Coalescence in a Slurry Bubble Column,” *J. Chem. Eng. Japan*, **48**, 181–189 (2015)
- Ruzicka, M. C., J. Drahoš, P. C. Mena and J. A. Teixeira; “Effect of viscosity on homogeneous-heterogeneous flow regime transition in bubble columns,” *Chem. Eng. J.*, **96**, 15–22 (2003)
- Sasaki, S., K. Hayashi and A. Tomiyama; “Effects of Liquid Height on Gas Holdup in Air-Water Bubble Column,” *Exp. Therm. Fluid Sci.*, **72**, 67–74 (2016)
- Vandu, C. O., K. Koop and R. Krishna; “Volumetric Mass Transfer Coefficient in a Slurry Bubble Column Operating in the Heterogeneous Flow Regime,” *Chem. Eng. Sci.*, **59**, 5417–5423 (2004)