A CORRELATION FOR BED VOIDAGE IN THREE-PHASE FLUIDIZED BED*

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Previously published data of three-phase fluidization were correlated to develop a new empirical correlation for predicting bed voidage in gas-liquid-solid fluidized beds. The proposed correlation model, when used in conjunction with any suitable two-phase model for bed voidage, can serve as a correlation for bed voidage in both two- and three-phase fluidized beds. It describes the bed expansion and contraction phenomena observed during fluidization and is valid even as the superficial gas velocity approaches zero. A new criterion for quantifying the bed expansion and contraction phenomena based on this empirical model is derived and is also discussed in this paper.

Introduction

The increased application of two- and three-phase fluidization operations in the chemical processes of industries has led to an increase in studies concerned with fully defining the characteristics of such processes. For the successful design and operation of two- and three-phase fluidized beds, it is important to know the hydrodynamic characteristics of the fluidization process. Gas, liquid and solid phase hold-ups, bed expansion, pressure drop, minimum fluidization velocity, and bubble "wakes" are just some of the many aspects of fluidization processes which have attracted the attention of many researchers. Most of these characteristics, however, have not been fully clarified (especially in the fairly recent and more complex field of three-phase fluidization) which has motivated continued studies aimed at completely defining these systems.

Bed behavior during fluidization, which may be characterized by the bed voidage, has been the subject of extensive research in the past, yet many questions regarding this aspect remain unanswered, particularly in three-phase fluidized bed operations.

Recently, the authors[10] proposed empirical bed voidage correlations in liquid-solid fluidization which explicitly predict bed voidage even as the superficial liquid velocity approaches zero.

Previous correlation equations for estimating phase hold-ups in three-phase fluidization have been summarized and reviewed by Muroyama & Fan[14].

An interesting phenomenon observed during gas-liquid-solid fluidization of fine particles is the bed contraction upon initial introduction of the gas. This has been quantified by the longstanding criterion of El-Temtary & Epstein[8] based on the generalized wake model. (This criterion was recently corrected by Jean & Fan[11].) Some difficulties, however, may be encountered in the evaluation of this criterion due to the inherent need to properly determine the gas and liquid hold-ups.

All previously published empirical correlations[1] for bed voidage in three-phase fluidization cannot describe the bed contraction phenomenon, mainly because the empirical correlations are inapplicable at gas velocities approaching zero.

This study aims to clarify bed expansion (and contraction) behavior during gas-liquid-solid fluidization through the development of a unified empirical correlation that can be used to estimate the bed voidage in both two- and three-phase fluidized beds.

1. Published Gas-Liquid-Solid Fluidization Data

Three-phase fluidization velocity-voidage data of 11 previous studies were gathered from the literature. The solid particles consisted of glass "beads," "balls," "ballotini," "spheres," sand, rockwool shot and iron shot. Air was used as the gas phase and water as the liquid phase in almost all studies. The average particle diameter varied from 0.25×10^{-3} to 8×10^{-3} m, while solid densities ranged from 1430 to 7700 kg/m^3. The Re_t range extended from 10 to about 4100. Details of these experimental systems are summarized in **Table 1**.

The terminal velocity, U_{ti}, of the particle (in an infinite fluid) was estimated using the equation proposed by Hartman *et al.*[9] summarized in **Table 2** as Eqs. (1), (1a), (1b) and (1c).

Wall effects may be significant, especially in small diameter columns, so corrected terminal velocities were calculated to account for such effects. Richardson & Zaki's[17] equation was used in determining the corrected particle terminal velocity, U_t as follows:

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Table 1 Summary of previous experimental systems used in this study

CODE	SYMBOL in FIGS. 1 and 3	SOLID	GAS	LIQUID	BED D m	SOLID d_p m	SOLID ρ_s kg/m^3	LIQUID ρ kg/m^3	LIQUID μ kg/m/s	U_t m/s	Re_t -
Ostergaard[15]	■	Glass ballotini	Air	Water	0.0760	0.00050	2830	999	0.00117	0.8671	371.0
Ostergaard & Theisen[16]	◧	Glass ballotini	Air	Water	0.1016	0.00028	2960	997	0.00089	0.0465	14.5
		Glass ballotini	Air	Water	0.1016	0.00058	2940	997	0.00087	0.1047	69.4
		Glass ballotini	Air	Water	0.1016	0.00120	2700	997	0.00089	0.1866	250.3
		Glass ballotini	Air	Water	0.1016	0.00220	2500	997	0.00089	0.2796	687.5
		Glass ballotini	Air	Water	0.0508	0.00200	2880	997	0.00089	0.2849	636.9
Efremov & Vakrushev[6]	◇	Glass beads	Air	Water	0.1000	0.00061	2460	999	0.00122	0.0803	40.0
		Glass beads	Air	Water	0.1000	0.00215	2460	999	0.00122	0.2534	444.8
Michelsen & Ostergaard[13]	▲	Glass beads	Air	Water	0.1524	0.00125	2670	(997)	(0.00089)	0.1929	269.6
		Glass beads	Air	Water	0.1524	0.00295	2450	(997)	(0.00089)	0.3410	1124.3
		Glass beads	Air	Water	0.1524	0.00593	2630	(997)	(0.00089)	0.5178	3432.1
Dakshinamurty, *et al.*[4]	□	Rockwool shot	Air	Water	0.0560	0.00130	2700	995	0.00080	0.2006	324.3
		Rockwool shot	Air	Kerosene	0.0560	0.00130	2700	800	0.00170	0.1900	116.2
		Sand	Air	Water	0.0560	0.00106	2700	995	0.00080	0.1694	223.3
		Sand	Air	Water	0.0560	0.00224	2710	995	0.00080	0.2985	829.6
		Glass beads	Air	Water	0.0560	0.00335	2400	995	0.00080	0.3354	1396.6
		Glass balls	Air	Kerosene	0.0560	0.00335	2400	800	0.00170	0.3577	563.6
		Glass beads	Air	Water	0.0560	0.00684	2400	995	0.00080	0.4237	3606.9
		Glass balls	Air	Kerosene	0.0560	0.00684	2400	800	0.00170	0.5033	1621.0
		Glass beads	Air	Water	0.0560	0.00489	2260	995	0.00080	0.3705	2253.1
		Glass balls	Air	Kerosene	0.0560	0.00489	2260	800	0.00170	0.4204	967.4
		Iron shot	Air	Water	0.0560	0.00300	7707	995	0.00080	0.7233	2698.8
		Iron shot	Air	Kerosene	0.0560	0.00300	7707	800	0.00170	0.7782	1098.6
Bhatia[2]	○	Glass beads	Air	Water	0.0508	0.00027	2938	999	0.00109	0.0397	9.9
		Glass beads	Air	Water	0.0508	0.00046	2935	999	0.00112	0.0729	29.6
		Glass beads	Air	Water	0.0508	0.00108	2949	999	0.00105	0.1732	177.7
		Lead shot	Air	Water	0.0508	0.00218	11030	999	0.00114	0.7494	1430.5
Bruce & Revel-Chion[3]	△	Glass spheres	Air	Water	0.0463	0.00200	2750	995	0.00078	0.2774	704.0
		Glass spheres	Air	Water	0.0463	0.00400	2750	995	0.00078	0.3943	2001.9
		Glass spheres	Air	Water	0.0463	0.00600	2450	995	0.00078	0.3988	3036.5
		Glass spheres	Air	Water	0.0463	0.00800	2360	995	0.00078	0.3979	4039.7
El-Temtamy[7]	◆	Glass beads	Air	Water	0.0500	0.00045	2599	998	0.00100	0.0658	29.4
		Glass beads	Air	Water	0.0500	0.00096	2930	998	0.00100	0.1572	150.0
		Glass beads	Air	Water	0.0500	0.00200	2936	998	0.00100	0.2829	562.2
		Glass beads	Air	Water	0.0500	0.00300	2926	998	0.00100	0.3626	1080.8
Dhanuka & Stepanek[5]	▽	Glass spheres	Air	Water	0.0500	0.00198	2960	(997)	(0.00089)	0.2896	640.8
		Glass spheres	Air	Water	0.0500	0.00408	2960	(997)	(0.00089)	0.4248	1937.3
		Glass spheres	Air	Water	0.0500	0.00586	2960	(997)	(0.00089)	0.4707	3082.7
Sinha *et al.*[19]	▼	Glass spheres	N_2	Kerosene	0.0762	0.00200	1450	(800)	(0.00200)	0.1347	107.8
		Glass spheres	N_2	Kerosene	0.0762	0.00130	1450	(800)	(0.00200)	0.0894	46.5
		Glass spheres	N_2	Heptane	0.0762	0.00130	1450	(680)	(0.00040)	0.1668	368.6
Lee & Lasa[12]	●	Glass beads	Air	Water	0.2000	0.00025	(2700)	(997)	(0.00089)	0.0361	10.1

( ) assumed values (data not available in literature)

Table 2 Hartman *et al.*'s[9] equation for predicting the free-fall velocity in an infinite medium

$\log Re_t = P(A) + \log R(A)$	(1)
where	
$P(A) = [(0.0017795A - 0.0573)A + 1.0315]A - 1.26222$	(1a)
$R(A) = 0.99947 + 0.01853 \sin(1.848A - 3.14)$	(1b)
$A = \log Ar$	(1c)

$$\log U_t = \log U_{ti} - (d_p / D) \quad (2)$$

The terminal velocity determined from Eq. (2) was used in calculating Re_t, the Reynolds number at terminal velocity conditions, and in all subsequent references to particle terminal velocity throughout this study.

2. Development of the Model

An empirical correlation which satisfies the following conditions is desired:

a. as $U_G \to 0$, $\varepsilon_3 \to \varepsilon_2$

b. both bed expansion and contraction behavior can be described

In order to satisfy these conditions, the following expression of ε_3.

$\varepsilon_3 = (\varepsilon_2) \times$ ("expansion factor")

was proposed. Where ε_3 is the bed voidage of three-phase fluidized bed and ε_2 is the two-phase (U_G = 0) bed voidage. The "expansion factor" must approach a value of 1 as U_G approaches zero and must vary from values less than 1 to values greater than 1 to be able to account for both bed expansion or contraction. The proposed form of the

Table 3 Results of regression and optimization

Constant	*a*	*b*	*c*	*d*	*e*	*f*
value	34	1	0.060	-0.325	0.024	0.013
t-value	-	-	9.63	26.08	20.68	3.50
critical t-value (99% confidence level): 2.326						

"expansion factor" is

$$f(U_G)+\{1-f(U_G)\}(K)$$

where $f(U_G) \rightarrow 1$ as $U_G \rightarrow 0$, $f(U_G) \rightarrow 0$ as U_G increases, and K is a function of other operating variables. A suitable model for $f(U_G)$ was found to be

$$f(U_G)=\exp(-a\,U_G/U_t) \tag{3}$$

with constant coefficient a. The ratio U_G/U_t was suggested in order to make this independent variable dimensionless. K, on the other hand, is assumed to be a function of different operating variables as follows:

$$K=b(U_L/U_t)^c(\varepsilon_2)^d(d_p/D)^e\{(\rho_S-\rho_L)/\rho_L\}^f \tag{4}$$

with constants b, c, d, e and f. ε_2 is used in Eq. (4) because bed expansion or contraction is greatly affected by bed voitage before gas comes into the bed.

The model may then be rewritten as

$$\frac{\varepsilon_3/\varepsilon_2-\exp(-aU_G/U_t)}{1-\exp(-aU_G/U_t)}=b\left(\frac{U_L}{U_t}\right)^c(\varepsilon_2)^d\left(\frac{d_p}{D}\right)^e\left(\frac{\rho_S-\rho_L}{\rho_L}\right)^f \tag{5}$$

The evaluation of the constants in the model Eq. (5) requires non-linear correlation techniques. Thus, to arrive at the best values of the coefficients, an assumed value is assigned to the coefficient a, and then multiple regression methods are applied after taking the logarithm of both sides of the model to convert it into the general multiple regression form. After obtaining the best fit values of the other constants, the sum of the squares of the errors (SSE) is computed for this set of constants. Then, a new value is assigned to a and the prodcess is repeated until the optimum set of values is obtained (based on the computed SSE for each set).

The final results of this optimization scheme are summarized in **Table 3**. The t-value of each parameter over 2.326 is necessary for the equation to achieve 99% confidence level.

Based on the 1535 experimental data points collected from 210 fluidization experimental runs, the following constant coefficients of Eq. (5) were obtained.

$a = 34$, $b = 1$, $c = 0.060$, $d = -0.325$, $e = 0.024$ and $f = 0.013$

Then Eq. (5) can be arranged as Eq. (6).

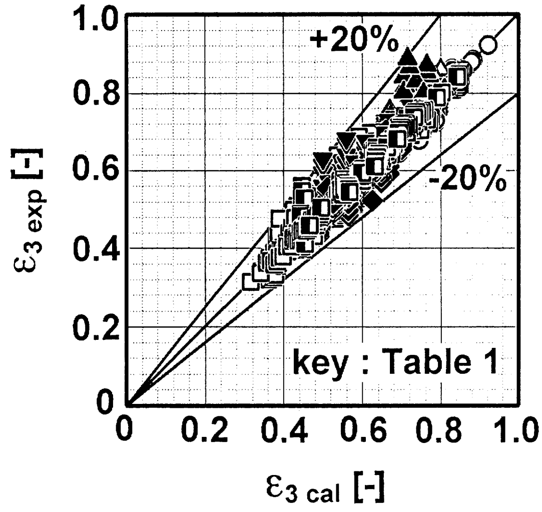

Fig. 1 Proposed three-phase correlation vs. experimental data

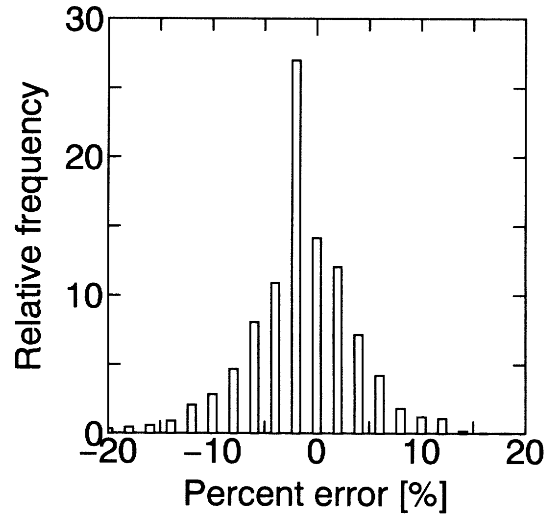

Fig. 2 Error frequency distribution of the proposed three-phase correlation

Table 4 Recent correlation of the authors[10)] for predicting ε_2

$\varepsilon_2 = \varepsilon_{PK} + (1-\varepsilon_{PK})\varepsilon_{RZ}^{A}\exp(B(1-\varepsilon_{RZ}))$	(7)
$A = 2.2n + 8d_p/D$	(7a)
$B = 2.1\,n$	(7b)
$\varepsilon_{RZ} = (U_L/U_t)^{1/n}$	(Richarson & Zaki)[17)]
$n = \frac{2.35(2+0.175\,Re_t^{0.75})}{(1+0.175Re_t^{0.75})}$	(Rowe)[18)]

$$\varepsilon_3=\varepsilon_2\left[\exp\left(\frac{-34U_G}{U_t}\right)+\left\{1-\exp\left(\frac{-34U_G}{U_t}\right)\right\}\times\left(\frac{U_L}{U_t}\right)^{0.060}(\varepsilon_2)^{-0.325}\left(\frac{d_p}{D}\right)^{0.024}\left(\frac{\rho_S-\rho_L}{\rho_L}\right)^{0.013}\right] \tag{6}$$

$\varepsilon_{3cal.}$ values predicted by Eq. (6) using $\varepsilon_{2exp.}$ are plotted against the actual $\varepsilon_{3exp.}$ data from each experimental system considered in this study in **Fig. 1**. The frequency distribution of the error, on the other hand, is shown in **Fig. 2**. The standard deviation and average absolute error of this

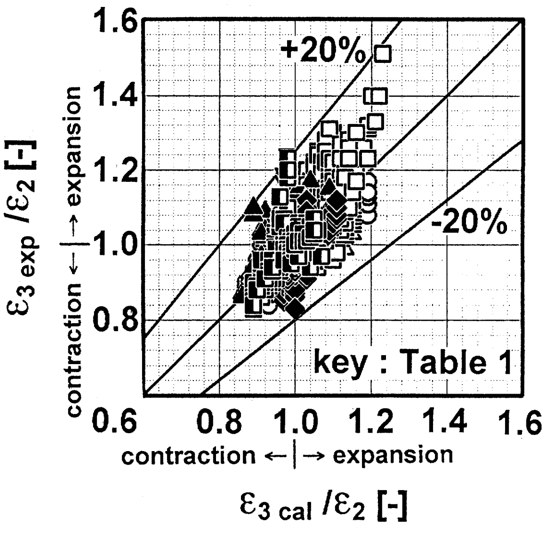

Fig. 3 Expansion/contraction of proposed three-phase correlation vs. experimental data

model were computed to be 5.0% and 3.5%, respectively.

Table 4 shows the recent correlation of the authors[10] for predicting bed voidage in liquid-solid fluidization. Using Eqs. (7), (7a) and (7b) to estimate ε_2 in Eq. (6), a correlation for bed voidage in both two- and three-phase fluidized beds is obtained.

3. Bed Expansion and Contraction Criterion

Based on the developed correlation, a criterion for bed expansion and contraction may be derived. The bed expansion or contraction behavior upon introduction of gas may be stated as follows:

when $d\,\varepsilon_3/dU_G > 0$ as $U_G \rightarrow 0$: bed expansion occurs

when $d\,\varepsilon_3/dU_G < 0$ as $U_G \rightarrow 0$: bed contraction occurs

and using the developed correlation (Eq. (6)), $d\varepsilon_3/dU_G$ may be expressed as

$$\frac{d\varepsilon_3}{dU_G}=\varepsilon_2\left\{-\frac{a}{U_t}\exp\left(\frac{-aU_G}{U_t}\right)+K\frac{a}{U_t}\exp\left(\frac{-aU_G}{U_t}\right)\right\}$$

Furthermore,

as $U_G \rightarrow 0$, $d\varepsilon_3/dU_G \rightarrow \varepsilon_2\,(a/U_t)\,(K-1)$

The slope, $d\varepsilon_3/dU_G$ is therefore positive when $K > 1$ and negative when $K < 1$, which forms the basis for the following proposed criterion for predicting bed expansion and/or contraction in three-phase fluidized beds:

if $K > 1$: bed expansion takes place

$K < 1$: bed contraction takes place

where K has been earlier defined as

$$K=\left(\frac{U_L}{U_t}\right)^{0.060}(\varepsilon_2)^{-0.325}\left(\frac{d_p}{D}\right)^{0.024}\left(\frac{\rho_S-\rho_L}{\rho_L}\right)^{0.013} \quad (8)$$

This criterion was tested against all the experimental systems considered in this study and it was verified that the criterion successfully predicted bed contraction or expansion behavior in 170 of the 210 total runs (81%).

The proposed criterion suggests that initial bed contraction is favored in systems with small particle diameters and low particle density. Bed contraction is also favored as the liquid velocity increases due to the subsequent increase in ε_2. These deductions are consistent with previous observations on the bed expansion and contraction phenomena published in the literature.

The extent of bed expansion or contraction predicted by Eq. (6) for each of the experimental data is plotted against that based on actual voidgae data from each experimental system considered in this study in **Fig. 3.** When $\varepsilon_{3cal.}/\varepsilon_{2exp.} > 1$ or $\varepsilon_{3exp.}/\varepsilon_{2exp.} > 1$, bed expansion is predicted or occur. When $\varepsilon_{3cal.}/\varepsilon_{2exp.} < 1$ or $\varepsilon_{3exp.}/\varepsilon_{2exp.} < 1$, bed contraction is predicted or occur. This plot can be divided into four areas (a, b, c and d) as follows. (a) Both predicted results and experimental data are expansion. (d) Both predicted results and experimental data are contraction. (b and c) One is expansion and the other is contraction. The predicted ratio of bed expansion or contraction by Eq. (6) is 83% except $\varepsilon_{3cal.} = 1$ and $\varepsilon_{3exp.} = 1$

a. $\varepsilon_{3cal.}/\varepsilon_{2exp.} > 1$, $\varepsilon_{3exp.}/\varepsilon_{2exp.} > 1$: 619 plots
b. $\varepsilon_{3cal.}/\varepsilon_{2exp.} > 1$, $\varepsilon_{3exp.}/\varepsilon_{2exp.} < 1$: 101 plots
c. $\varepsilon_{3cal.}/\varepsilon_{2exp.} < 1$, $\varepsilon_{3exp.}/\varepsilon_{2exp.} > 1$: 106 plots
d. $\varepsilon_{3cal.}/\varepsilon_{2exp.} < 1$, $\varepsilon_{3exp.}/\varepsilon_{2exp.} < 1$: 362 plots

Conclusions

An empirical correlation for predicting bed voidage in three-phase fluidized beds has been developed as shown in Eq. (6).

In contrast to the previously published empirical models, this new correlation is valid even as the superficial gas velocity approaches zero. Furthermore, it is capable of describing the initial bed expansion and contraction phenomena observed in three-phase fluidized bed operations.

Furthermore, a new criterion for predicting bed expansion and contraction in three-phase fluidized beds upon initial introduction of the gas has been derived as follows:

$(U_L/U_t)^{0.060}\,(\varepsilon_2)^{-0.325}\,(d_p/D)^{0.024}\,\{(\rho_S-\rho_L)/\rho_L\}^{0.013}$
> 1: expansion
< 1: contraction

The quantitative implications of this criterion are consistent with previously published observations regarding initial bed expansion and contraction.

Nomenclature

Ar	=	$d_p^3 g\,\rho_L(\rho_S-\rho_L)/\mu^2$ = Archimedes number	[-]
D	=	Bed diameter	[m]
d_p	=	Particle diameter	[m]
g	=	Acceleration due to gravity	[m/s^2]
n	=	Richardson-Zaki's Eq. exponent estimated by Rowe's Eq.	[-]
Re_t	=	$d_p U_t \rho_L/\mu$ = Particle Reynolds no. at terminal velocity conditions	[-]
U_G	=	Superficial gas velocity	[m/s]
U_L	=	Superficial liquid velocity	[m/s]

U_t = Terminal velocity of a single particle corrected for wall effect [m/s]
U_{ti} = Terminal velocity of a single particle in an infinite fluid [m/s]
ε_{PK} = Static or "packed" voidage ($U_L = 0$) [-]
ε_{RZ} = Bed voidage from Richardson & Zaki's Eq. [-]
ε_2 = Two-phase bed voidage [-]
ε_3 = Three-phase bed voidage [-]
μ = Liquid viscosity [kg/m/s]
ρ = Density [kg/m^3]

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cal. = calculated with Eq. (6)
exp. = experimental data
L = liquid
S = solid

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