CORRELATION OF SOLID HOLDUP IN LIQUID-SOLID FLUIDIZED BED

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Key Words: Fluidization, Liquid Fluidized Bed, Slip Velocity, Solid Holdup, Void Function

Solid holdup in vertical fluidized beds for liquid-solid systems was investigated over a wide range of experimental conditions. The solid phases used were sieved glass beads of diameters 66, 118, 243, 465, 900, 2200, 3100 and 4650 μm, sieved porous alumina particles of diameter 3680 μm and sieved ion exchange resin particles of diameter 610 μm. The liquid phase was tap water. The effect of column diameter on solid holdup was negligible when the column diameter was larger than 0.0397 m. A novel model for predicting solid holdup in all flow regimes was developed. Experimental data, including previous results in the literature, were in good agreement with the model.

Introduction

To estimate bed expansion in liquid-solid fluidized systems, we must know the relationship between liquid velocity and solid holdup with good accuracy. When liquid velocity decreases by only 10% in a fluidized bed of coarse particles whose terminal velocity is in Newton's regime, the bed height decreases by more than 50%.

Many correlations for the liquid velocity and solid holdup relationship, so-called void functions, have been presented to date[3, 5-7, 9, 11,12, 14, 18-20]. Richardson and Zaki[14] correlated the void function as

$$U_l \,/\, U_t = (1-\phi_p)^n \tag{1}$$

The exponent, n, was described with four empirical equations assigned to different flow regimes. Each equation was expressed as a function of Reynolds number, Re_t (= d_pU_t/ν), based on terminal velocity of a single particle and the particle to column diameter ratio, d_p/D_T.

Garside and Al-Dibouni[3] and Rowe[15] expressed the exponent as a continuous function of Re_t. At larger Reynolds numbers, however, the relationship between log U_l and log (1 - ϕ_p) deviates from their correlation. Leoffler and Ruth[9], Oliver[13] and Jean and Fan[8] carried out theoretical research on slip velocity for $Re_t^* < 2$. Letan[10] proposed a semi-theoretical equation of slip velocity to be used over the $1.5 < Re_t^* < 2200$ range. Extending the pressure drop equation to porous media, Foscolo *et al.*[2] developed a model that consisted of three equations for laminar, intermediate and turbulent flow regimes. However, most of the previous correlations were applicable only to a narrow range of Reynolds number. Correlations in regions of high solid concentrations were especially unreliable.

In the present study, the relationship between liquid velocity and solid holdup in liquid-solid fluidized beds of spherical particles was systematically investigated. A simple and accurate equation applicable to all flow regimes was developed.

1. Experimental

Fluidized beds 0.0162, 0.0397, 0.070 and 0.150 m in diameter and having heights of 2.0, 2.0, 4.85 and 2.7 m, respectively, were used. Each column was made of acrylic resin pipe and set vertically. The liquid distributor was a packed bed of glass spheres, and a bronze net of 145 mesh was placed at the upper face of the packed layer. The liquid phase was tap water at 293 K. The solid phases were sieved glass beads of diameters 66, 118, 243, 465, 900, 2200, 3100 and 4650 μm (density = 2500 kgm^{-3}), porous alumina particles of diameter 3.86mm (apparent density = 1860kgm^{-3}) and ion exchange resin particles of 0.61mm in diameter (apparent density = 1080kgm^{-3}). Size distributions are shown in **Fig. 1**.

All the experiments were conducted batchwise with respect to solid particles. A prescribed amount of solid particles was fluidized at a set liquid flow and bed height was measured with a maximum error of 2%. Solid holdup was then calculated. The terminal velocity of isolated particles was determined by repeating the falling experiment with more than fifty different particles.

2. Results and Discussion

2.1 Terminal velocity of a particle

When an isolated particle falls in a quiescent liquid, the macroscopic force balance at a steady state is given as

$$(4/3)\, Ga = C_d^*\, Re_t^{*2} \tag{2}$$

where $Ga = d_p^3 g(\rho_p/\rho_l - 1)\,/\,\nu^2$ and $Re_t^* = d_p v^*/\nu$. The

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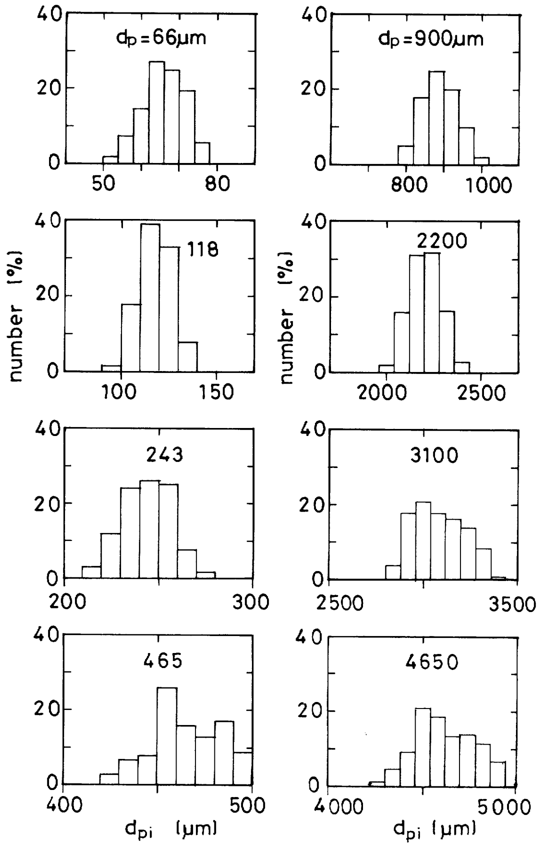

Fig. 1 Size distributions of glass beads used

drag coefficient, C_d*, is expressed as follows[1]:

$$\textit{Stokes' regime}\,(Re_t^* < 2): C_d^* = 24 / Re_t^* \tag{3}$$

$$\textit{Allen's regime}\,(2 < Re_t^* < 500): \quad C_d^* = 18.5 / Re_t^{*3/5} \tag{4}$$

$$\textit{Newton's regime}\,(Re_t^* > 500): \quad C_d^* = 0.44 \tag{5}$$

By substituting Eq.(3) or (5) into Eq.(2), Re_t* is given as

$$\textit{Stoke's regime}: Re_t^* = Ga / 18 \tag{6}$$

$$\textit{Newton's regime}: Re_t^* = Ga / (Ga / 3.0)^{1/2} \tag{7}$$

Equations (6) and (7) are combined into the following equation that can express Re_t* in all flow regimes including Allen's regime.

$$Re_t^* = \frac{Ga}{\{18^m + (Ga/3.0)^{m/2}\}^{1/m}} \tag{8}$$

where m is the weight factor. When the Galileo number, Ga, in Eq.(2) is eliminated by using Eq.(8), we get .

$$C_d^* = \left[\frac{0.44^{m/2}}{2} + \left\{\frac{0.44^m}{4} + \left(\frac{24}{Re_t^*}\right)^m\right\}^{1/2}\right]^{2/m} \tag{9}$$

Figure 2 shows that experimental data are well correlated when the weight factor, m, is fixed as unity.

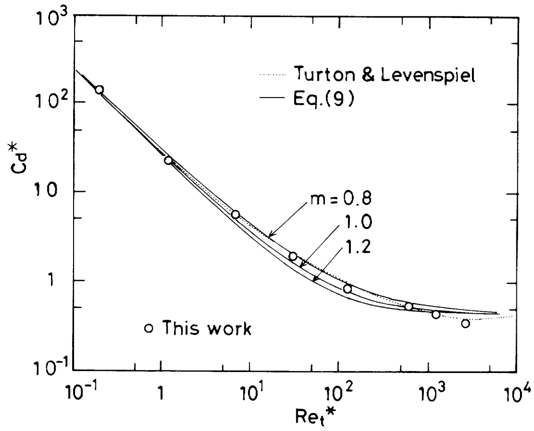

Fig. 2 Determination of m in Eq.(9)

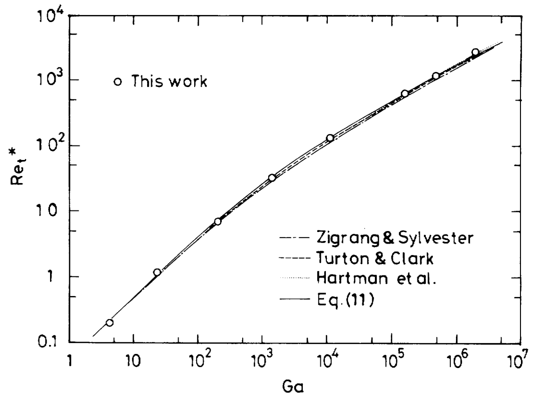

Fig. 3 Relationship between terminal velocity of a single particle and Galileo number

$$C_d^* = \left[\left(\frac{0.44}{4}\right)^{1/2} + \left\{\left(\frac{0.44}{4}\right) + \left(\frac{24}{Re_t^*}\right)\right\}^{1/2}\right]^2 \tag{10}$$

Then Eq.(8) becomes

$$Re_t^* = \frac{Ga}{18 + (Ga/3.0)^{1/2}} \tag{11}$$

Turton and Levenspiel[16] proposed the following equation for C_d* in the range of $Re_t^* < 2\times10^5$.

$$C_d^* = \frac{24}{Re_t^*}\left(1 + 0.173\,Re_t^{*0.657}\right) + \frac{0.413}{1 + 1.63\times10^4\,Re_t^{*-1.09}} \tag{12}$$

Equation (12) also agrees with the experimental data. **Figure 3** shows the suitability of Eq.(11) and that of the available equations listed in **Table 1**.

2.2 Relationship between liquid velocity and solid holdup

Figures 4 and **5** show the plot of log $(1 - \phi_p)$ vs. log Re, obtained in the present study. Data are well represented by straight lines for Reynolds numbers smaller than 30 and curved lines for numbers larger than 100.

The macroscopic force balance for a particle in slurry

Table 1. Existing correlations for the terminal velocity of a single particle

Investigator	Correlation equation	
Zigrang and Sylvester[21]	$Re_t^* = 1.839Ga^{1/2} + 29.025 - (106.4Ga^{1/2} + 842.45)^{1/2}$	(a-1)
Turton and Clark[17]	$Re_t^* = Ga^{1/3}/\{10.82/Ga^{0.5493}+0.6262/Ga^{0.1373}\}^{1.214}$	(a-2)
Hartman *et al*.[4]	$\log Re_t^* = P(A) + \log R(A)$	(a-3)
	where $P(A) = [(0.0017795A - 0.0573)A + 1.0315]A - 1.26222$	
	$R(A) = 0.99947 + 0.01853\sin(1.848A - 3.14)$	
	$A = \log Ga$	

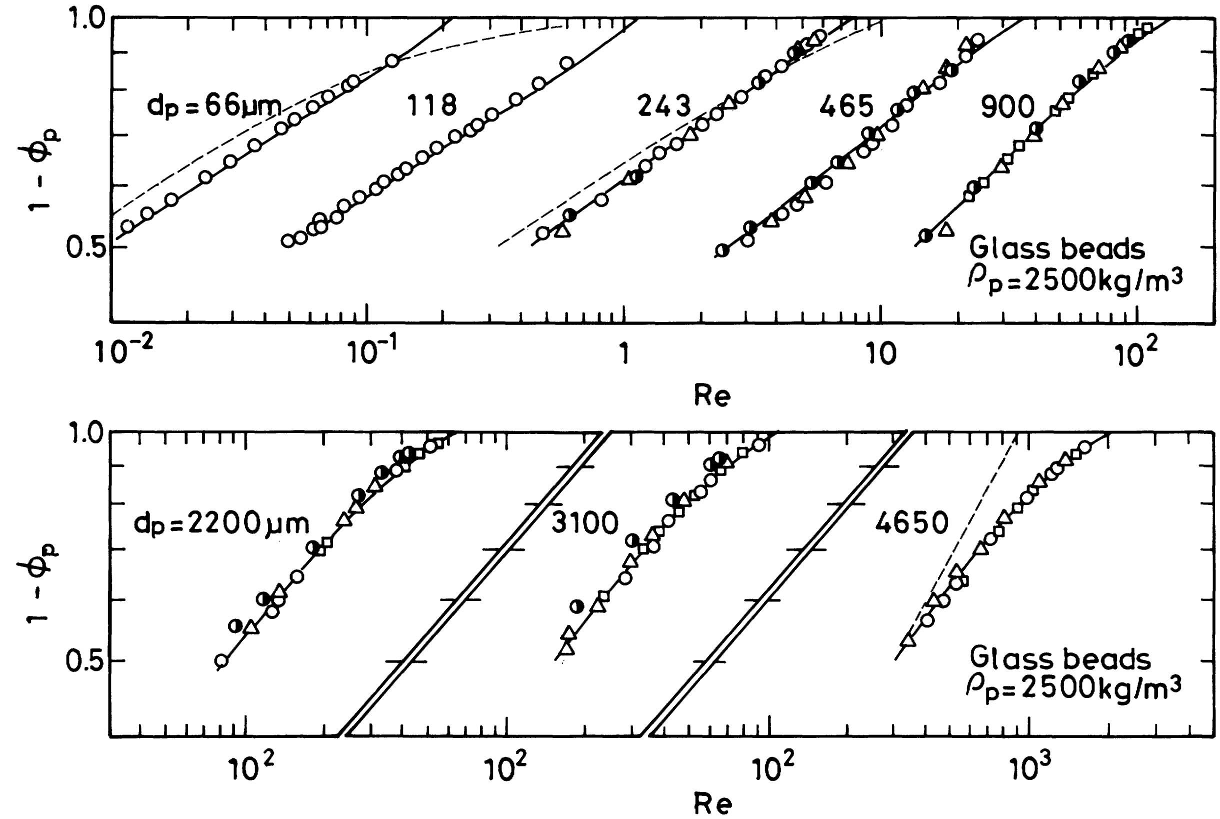

Fig. 4 Experimental and calculated results of liquid velocity-solid holdup relationship
Keys: (◑) $D_T = 0.0162$m, (△) $D_T = 0.0397$m,
(○) $D_T = 0.070$m, (□) $D_T = 0.150$m

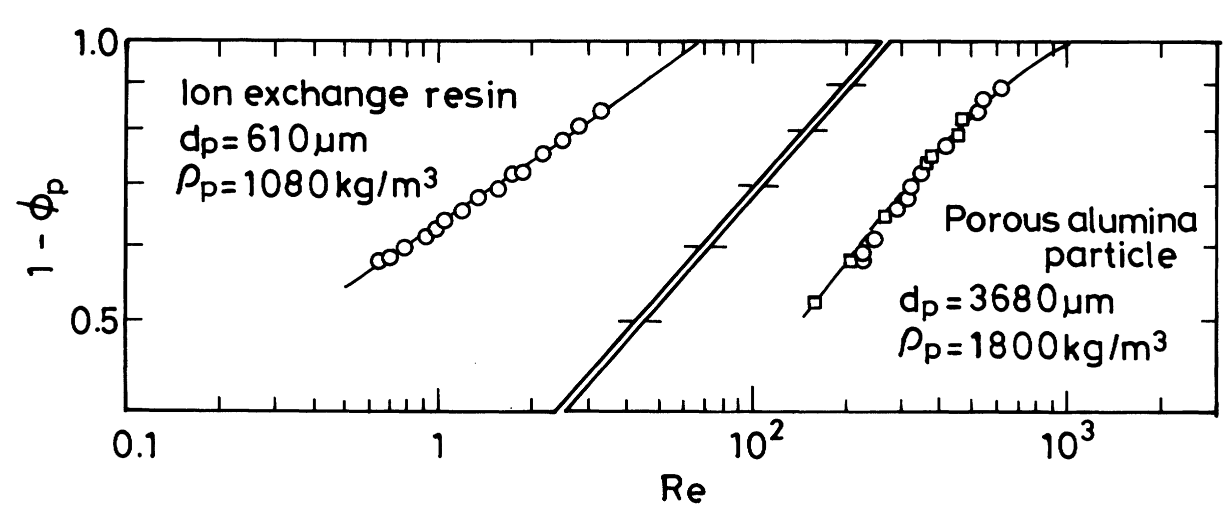

Fig. 5 Experimental and calculated results of liquid velocity-solid holdup relationship
Keys: (○) $D_T = 0.070$m, (□) $D_T = 0.150$m

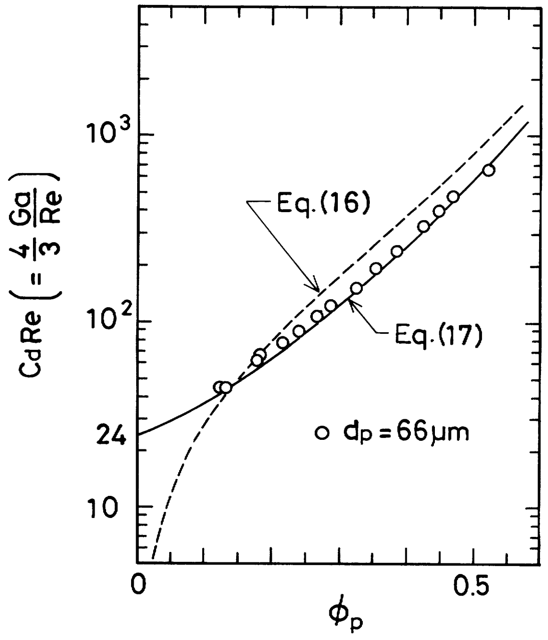

Fig. 6 Effect of ϕ_p on C_d for small Reynolds number

at a steady state condition is expressed as

$$(4/3)\,Ga = C_d Re^2 \tag{13}$$

where C_d is a function of ϕ_p and Re, and Re is Reynolds number defined as $d_p U_l / \nu$. The pressure drop in a fixed bed for Re smaller than 0.2 is given by the Blake-Kozeny equation[1].

$$-\frac{dP}{dz} = 150\frac{\mu U_l}{d_p^2}\frac{\phi_p^2}{(1-\phi_p)^3} \tag{14}$$

and is balanced with the gravity force acting on particles in a fluidized bed.

$$-\frac{dP}{dz} = (\rho_p - \rho_l)\,\phi_p g \tag{15}$$

From Eqs.(13), (14) and (15), we obtain the drag coefficient for the range of small Reynolds number.

$$C_d = \frac{4}{3Re}\cdot\frac{150\phi_p}{(1-\phi_p)^3} \tag{16}$$

Figure 6 shows the relationship between $C_d Re$ (= (4/3) Ga / Re) calculated from Eq.(13) and ϕ_p, indicating that Eq.(16) cannot describe the data for glass beads of diameter 66 μm. This may be caused mainly by the fact that particles are not statically suspended in the present fluidization system and that the value of 150 in Eq.(16) is representatively selected for particles regardless of their shapes. It is also desirable that for the limiting case of isolated particles, i.e. for very small values of ϕ_p, $C_d Re$ takes the value of 24 as suggested by Eq.(3). However, Eq.(16) does not agree with this assumption. Thus, to get a good correlation equation, the following equation is adopted in place of Eq.(16).

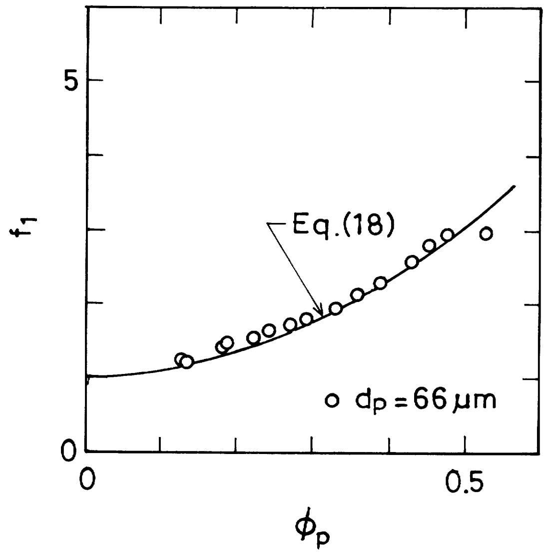

Fig. 7 Correlation of f_l

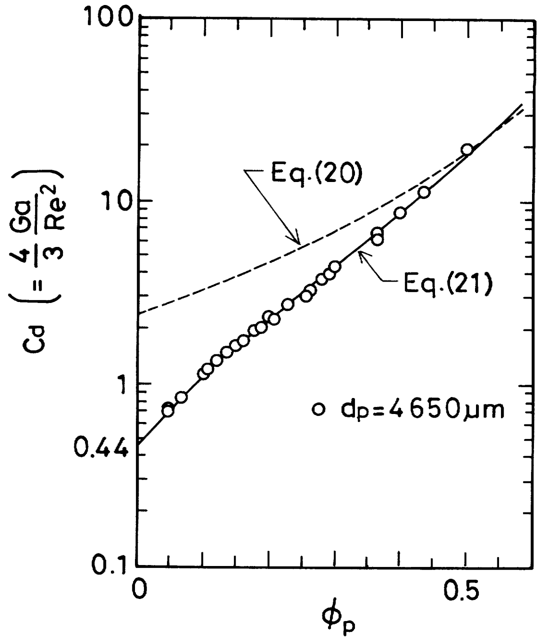

Fig. 8 Effect of ϕ_p on C_d for large Reynolds number

$$C_d = \frac{24}{Re}\frac{f_l}{(1-\phi_p)^3} \tag{17}$$

where f_l is an empirical function of ϕ_p. f_l is experimentally determined from Eqs.(13) and (17) by using the data for glass beads of diameter 66 μm, and is plotted in **Fig. 7.** Experimental results are well expressed by the following equation.

$$f_l = 1 + 8\phi_p^2 \tag{18}$$

Thus, from combining Eq.(13) and Eq.(17) with Eq.(18), Re is given as

Table 2. Existing correlations for the liquid velocity-solid holdup relationship in liquid-solid fluidized bed

Investigator	Correlation equation	Range of applicability	
Richardson and Zaki[14]	$U_l/U_t = (1-\phi_p)^n$ where $n = 4.65 + 19.5 d_p/D_T$ $n = (4.35 + 17.5 d_p/D_T) Re_t^{-0.3}$ $n = (4.45 + 18.0 d_p/D_T) Re_t^{-0.1}$ $n = 4.45 Re_t^{-0.1}$ $n = 2.39$	$Re_t < 0.2$ $0.2 < Re_t < 1$ $1 < Re_t < 200$ $200 < Re_t < 500$ $Re_t > 500$	(b-1)
Wen and Yu[19]	$(1-\phi_p)^{4.7} Ga = 18 Re + 2.7 Re^{1.687}$	$0.01 < Re_t^* < 10^4$	(b-2)
Wen and Fan[18]	$\frac{Ga}{13.9 Re^{1.4}} \; Ga^{0.0463(\phi-1)} = \phi^{3.026}(1 + 0.00113\phi^{4.025})$ $\frac{3Ga}{Re^2} \; Ga^{0.0384(\phi-1)} = \phi^{3.819}(1 + 0.00150\phi^{3.102})$ where $\phi = (1 - 1.21\phi_p^{2/3})^{-1}$	$18 < Ga < 10^5$ $Ga > 10^5$	(b-3)
Letan[10]	$\frac{U_l}{U_t} = \frac{(1 + 0.15 Re_t^{0.687})(1-\phi_p)^{3.5}}{1 + 0.15(Re_t U_l/U_t)^{0.687}(1-\phi_p)^{1.72}}$	$1.5 < Re_t < 2200$	(b-4)
Garside and Al-Dibouni[3]	$U_l/U_t = (1-\phi_p)^n$ where $(5.1 - n)/(n - 2.7) = 0.1 Re_t$		(b-5)
Foscolo *et al.*[2]	$\frac{U_l}{v^*} = \frac{(1-\phi_p)^4}{4\phi_p + (1-\phi_p)^3}$ $\frac{U_l}{v^*} = \frac{\{0.077 Re_t^*(1 + 0.0194 Re_t^*)(1-\phi_p)^{4.8} + 1\}^{0.5} - 1}{0.0388 Re_t^*}$ $\frac{U_l}{v^*} = \left\{\frac{(1-\phi_p)^4}{3.55\phi_p + (1-\phi_p)^3}\right\}^{0.5}$	$Re_t^* < 0.2$ $0.2 < Re_t^* < 500$ $Re_t^* > 500$	(b-6)
Hirata and Bulos[6]	$\phi_p = \phi_{pk} - \phi_{pk}(1-\phi_p)^A \exp(B\phi_p)$ where $A = 2.2n + 8 d_p/D_T$ $B = 2.1n$ n is the exponent of Rowe[15].		(b-7)

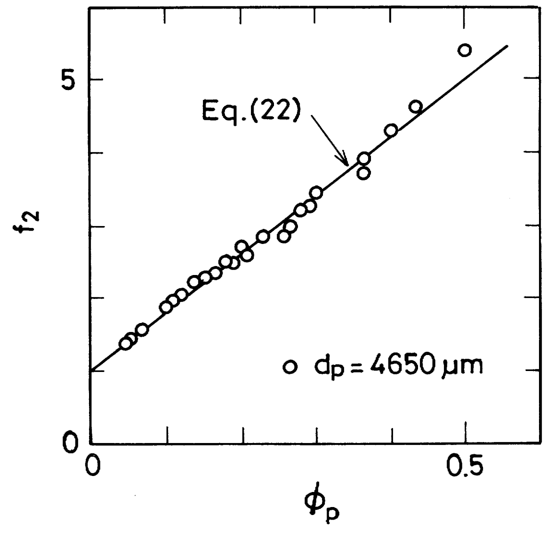

Fig. 9 Correlation of f_2

$$Re = Ga \Big/ \left\{ 18 \frac{1 + 8\phi_p^2}{(1-\phi_p)^3} \right\} \tag{19}$$

On the other hand, for Re larger than 500, the Burke-Plummer equation[1] leads to

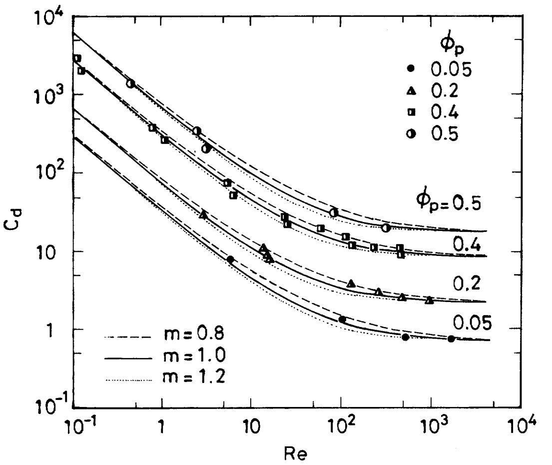

Fig. 10 Determination of m in Eq.(25)

$$C_d = \frac{4}{3} \frac{1.75}{(1-\phi_p)^3} \tag{20}$$

Figure 8 shows experimental results of C_d (= (4 / 3) Ga / Re^2) calculated from Eq.(13) by using the data of glass beads of diameter 4650 μm. In this figure, Eq.(20) is compared with the experimental results. Eq.(20) cannot describe the data well. It is also desirable that C_d for large Reynolds numbers approaches 0.44 near ϕ_p = 0 as suggested by Eq.(5). The following equation is presumed

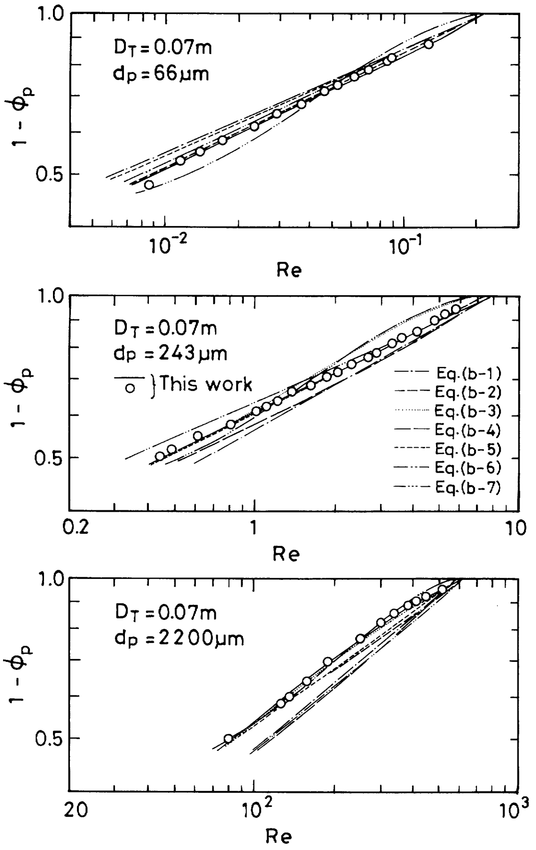

Fig. 11 Liquid velocity-solid holdup relationship calculated from Eq. (26) and those in the literature. Equation number in this figure is given in Table2

to correlate C_d, as well as the case for small Reynolds numbers.

$$C_d = 0.44 \frac{f_2}{(1-\phi_p)^3} \tag{21}$$

where f_2 is an empirical function of ϕ_p. As shown in **Fig. 9**, f_2 is represented by

$$f_2 = 1 + 8\phi_p \tag{22}$$

Then, for large Reynolds numbers, we get

$$Re = Ga \Big/ \left\{ \frac{Ga}{3.0} \frac{1+8\phi_p}{(1-\phi_p)^3} \right\}^{1/2} \tag{23}$$

In order to obtain an equation to include the intermediate range of Re, the following equation is assumed by combining Eqs.(19) and (23), based on the same argument used in deriving Eq.(8) for the terminal velocity of a single particle.

$$Re = \frac{Ga}{\left[\left\{ \frac{18(1+8\phi_p^2)}{(1-\phi_p)^3} \right\}^m + \left\{ \frac{Ga(1+8\phi_p)}{3.0(1-\phi_p)^3} \right\}^{m/2} \right]^{1/m}} \tag{24}$$

where m is the weight factor. Equation (24) is rewritten as an explicit function of C_d by using Eq.(13).

$$C_d = \left[\frac{1}{2} \left\{ \frac{0.44(1+8\phi_p)}{(1-\phi_p)^3} \right\}^{m/2} + \left[\frac{1}{4} \left\{ \frac{0.44(1+8\phi_p)}{(1-\phi_p)^3} \right\}^m + \left\{ \frac{24(1+8\phi_p^2)}{Re(1-\phi_p)^3} \right\}^m \right]^{1/2} \right]^{2/m} \tag{25}$$

Equation (25) becomes Eq.(17) or (21) in the case of small or large Re, respectively. **Figure 10** shows that experimental data are well correlated by Eq.(25) with m = 1. Eq.(24) then becomes

$$Re = \frac{Ga}{\frac{18(1+8\phi_p^2)}{(1-\phi_p)^3} + \left[\frac{Ga(1+8\phi_p)}{3.0(1-\phi_p)^3} \right]^{1/2}} \tag{26}$$

As shown in Figs. 4 and 5, the solid lines calculated from Eq.(26) agree satisfactorily with the data over a wide range of Reynolds numbers for column diameters larger than 0.0397m and particle to column diameter ratios less than 0.12. When the Galileo number is large, the first term in the denominator of Eqs.(11) and (26) is neglected. The two equations are then combined into a simplified form.

$$Re = Re_t^* \frac{(1-\phi_p)^3}{1+8\phi_p} \tag{27}$$

Equation (27) is identical with the equation derived by Hidaka *et al.*[5]

If Eqs.(16) and (20) are adopted, the relationship between Ga and Re gives Ergun's equation.[3]

$$(4/3)\,Ga = \left(\frac{4}{3} \frac{150\phi_p}{Re(1-\phi_p)^3} + \frac{4}{3} \frac{1.75}{(1-\phi_p)^3} \right) Re^2 \tag{28}$$

The broken lines in Fig. 4 are calculated from Eq.(28), the deviation being quite large in the lean region of solid holdup.

3. Comparison with Previous Works

The applicability of the equations listed in **Table 2** was reviewed. As shown in **Fig. 11**, all equations are approximately in agreement with the data for glass beads of d_p = 66 μm (Re_t* = 0.22). With beads of d_p = 243 μm, the correlations of Garside and Al-Dibouni[3] (Eq.(b-5)) and Wen and Yu[19] (Eq.(b-2)) are satisfactory, but the others are not. With d_p = 2200 μm, all correlations except those of Wen and Fan[18] (Eq.(b-3)) and Hirata and Bulos[6] (Eq.(b-7)) do not represent the curvature of data for ϕ_p < 0.2 well. They also overestimate the solid concentration. In the case where d_p = 2200 μm and Re = 300, the solid concentration calculated from the equation of Richardson and Zaki[14] (Eq.(b-1)) is twice the experimental value.

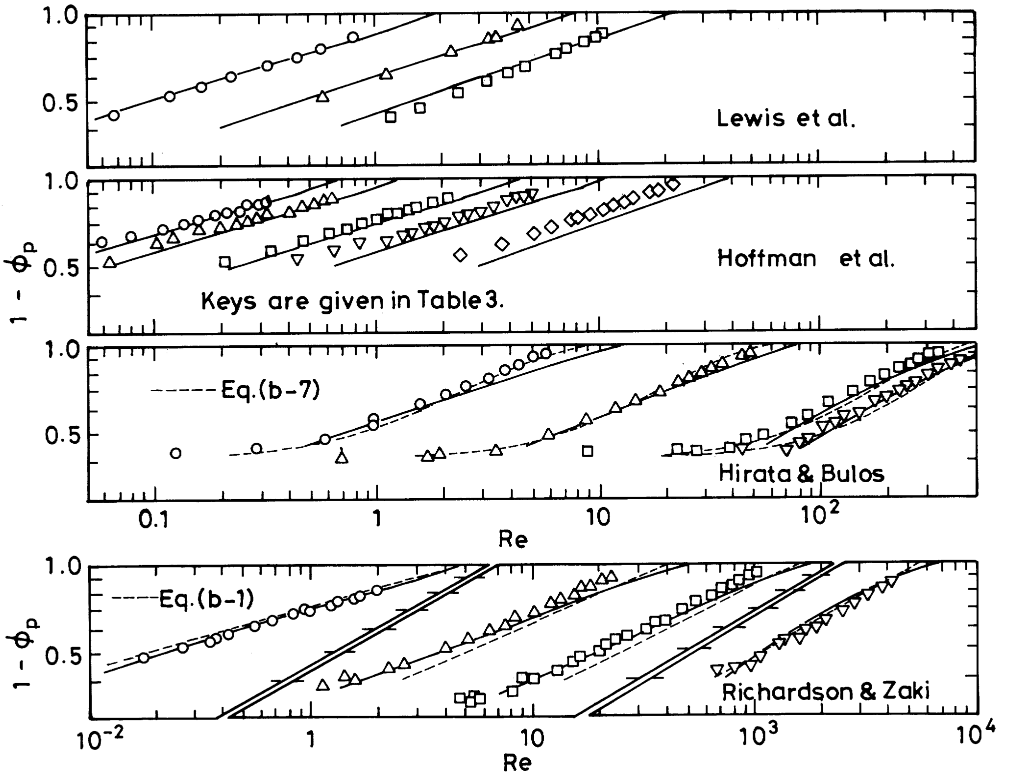

Fig. 12 Applicability test of Eq.(26) to previously published data. Solid lines are calculated results from Eq.(26). Equation number and keys in this figure are given in Tables 2 and 3, respectively

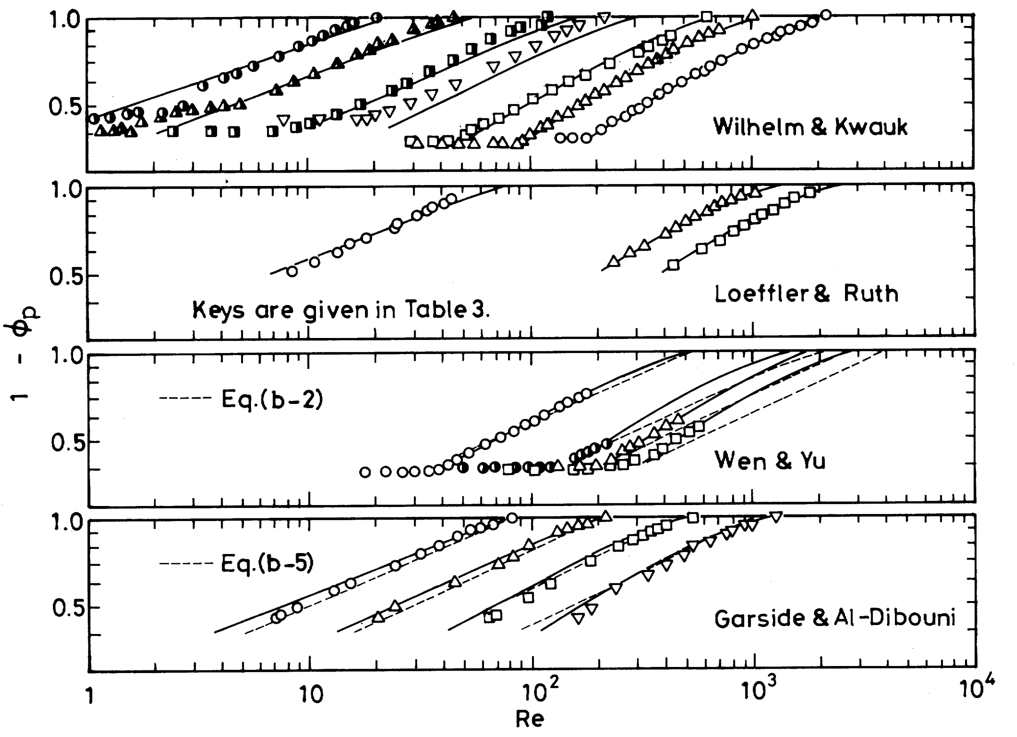

Fig. 13 Applicability test of Eq.(26) to previously published data. Solid lines are calculated results from Eq.(26). Equation number and keys in this figure are given in Tables 2 and 3, respectively

Table 3. Previously published data used for applicability test of Eq.(26).

Investigator	Particle	Particle diameter d_p [μm]	Particle density ρ_p[kg/m^3]	Column diameter D_T [m]	Key
Wilhelm & Kwauk[20]	Lead Shot	1283	10790	0.0754	□
	Soconey Beads	4420	1603	0.0754	△
	Glass Beads	5207	2351	0.0754	○
	Sea Sand	373	2639	0.0754	◑
		556	2639	0.0754	◭
		998	2639	0.0754	◨
	Crushed Rock	1415	2636	0.0754	▽
Lewis *et al.*[11]	Glass Beads	155	2451	0.0635	○
		284	2435	0.0635	△
		452	2515	0.0635	□
Richardson & Zaki[14]	Divinyl Benzene Beads	253	1060	0.0620	○
	Glass Beads	1100	2745	0.0381	△
		2600	2745	0.0381	□
	Ball Bearing	6350	7740	0.0620	▽
Loeffler & Ruth[9]	Glass Beads	659	2630	0.0260	○
		3360	2572	0.0551	△
		5060	2514	0.0551	□
Hoffman *et al.*[7]	Glass Beads	97	2435	0.0254	○
		121	2467	0.0254	△
		191	2465	0.0254	□
		269	2486	0.0254	▽
		464	2525	0.0254	◇
Wen & Yu[19]	Glass Beads	2032	2450	0.1016	○
		5004	2460	0.1016	△
		6350	2360	0.1016	□
	Steel Balls	2380	7840	0.1016	◑
Garside & Al-Dibouni[3]	Glass Beads	655	2600	0.052	○
		1100	2600	0.052	△
		2600	2600	0.052	□
		3070	2600	0.052	▽
Hirata & Bulos[6]	Glass Beads	267	2470	0.05	○
		631	2458	0.05	△
		1876	2487	0.05	□
		2357	2485	0.05	▽

Liquid used in above experiments was water.

Figures 12 and **13** show the comparison between the prediction by Eq.(26) and previously published data listed in **Table 3**. Equation (26) is also in good agreement with all the data for spherical particles over a wide range of *Re*, except those of one or two particle diameter obtained by Hoffman *et al.*[7] and Hirata and Bulos[6]. In the case of non-spherical particles, there are small deviations from the data with sea sand and crushed rock reported by Wilhelm and Kwauk[20].

Conclusion

A new equation for the liquid velocity-solid holdup relationship in liquid-solid fluidized beds was developed. In spite of its simpleness, Eq.(26) predicted well experimental results including literature data in all flow regimes for the cases of column diameters larger than 0.0397m and particle to column diameter ratios less than 0.12.

Nomenclature

C_d	=	drag coefficient defined by Eq.(13)	[-]
C_d^*	=	drag coefficient defined by Eq.(2)	[-]
D_T	=	column diameter	[m]
d_p	=	particle diameter	[m]
g	=	gravitational acceleration	[m s^{-2}]
Ga	=	Galileo number defined by $d_p^3 g(\rho_p/\rho_l - 1)/v^2$	[-]
m	=	exponent in Eqs.(8) and (25)	[-]
n	=	exponent in Eq.(1)	[-]
P	=	static pressure	[Pa]
Re	=	Reynolds number defined by $d_p U_l/v$	[-]
Re_t	=	Reynolds number defined by $d_p U_t/v$	[-]
Re_t^*	=	Reynolds number defined by $d_p v^*/v$	[-]
U_l	=	superficial liquid velocity	[m s^{-1}]
U_t	=	terminal velocity of a single particle extrapolated to $\phi_p = 0$	[m s^{-1}]
v^*	=	terminal velocity of a single particle	[m s^{-1}]
z	=	distance from bottom of column	[m]
ϕ_p	=	solid holdup defined as volume fraction of particulate phase in bed	[-]
ϕ_{PK}	=	volume fraction of particulate phase in static bed	[-]
ρ_l	=	density of liquid	[kg m^{-3}]
ρ_p	=	density of solid particles	[kg m^{-3}]
μ	=	viscosity of liquid	[Pa s]
v	=	kinematic viscosity of liquid	[m^2 s^{-1}]

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