

PREDICTING THE MINIMUM FLUIDIZATION VELOCITY OF MULTICOMPONENT SYSTEMS

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An expression is suggested to appropriately estimate the minimum fluidization velocity of either a perfectly mixed or a completely segregated multicomponent system. Based on experimental data from the literature and our own work it is found that agreement between predictions and experimental results is reasonably good in the context of the inherent limitation of the theory and the accuracy of the data.

Introduction

The study of the fluidization of multicomponent systems, which are most commonly encountered in commercial units, is relatively limited. That is mainly due to the fact that multicomponent systems are extremely complex and, because of that, most authors have simplified the analysis by restricting their investigations to binary mixtures. Thus, although a large number of correlations have been published to predict the U_{mf} of binary systems^{2-4, 6, 10-12, 4)}, very few can be used to estimate the U_{mf} of mixtures with more than two Components^{8, 13)}. In the following, since most published literature on multicomponent systems refers to binary mixtures, the results of some relevant work dealing with the prediction of the U_{mf} of binary systems is briefly noted.

Over the years, several approaches have been used to predict the minimum fluidization velocity of mixtures. One of them implicates introducing an effective particle diameter and density into those correlations which are valid for monodisperse systems. Since there has been no agreement on the values of the effective diameter and density that should be used to describe the mixture, the values predicted from different equations are widely different. Correlations proposed by Goossens *et al.*⁶⁾, Kumar and Sen Gupta⁸⁾ and Noda *et al.*¹⁰⁾ belong to this group.

Other approaches assume that the bed behaves as if it were simply the sum of n single beds in series, n being the number of different species within the bed. Obviously this is an approximation but, surprisingly, it can be used to calculate variations of local voidage with height in reasonable agreement with experimental results even when the mixing of different components is not negligible¹⁾. Besides, Epstein *et al.*⁵⁾ have shown the superi-

ority of this model over the property-averaging models in predicting the expansion characteristics of liquid fluidized beds.

Based on such approach, Chiba *et al.*³⁾ and Obata *et al.*¹¹⁾ obtained by two different methods the same expression to calculate the U_{mf} of binary systems. It should be noticed that while the mathematical derivation by Chiba *et al.* was skillful and complex, the derivation by Obata *et al.* was more simple and was based on a graphical method. In this paper such an expression is first obtained by a procedure very similar to that introduced earlier by Obata *et al.*, and then it is generalized for application to a multicomponent mixture.

Precisely, the purpose of the work was to prove that an equation initially developed for strongly segregating binary mixtures could be used to predict the U_{mf} of either a perfectly mixed or a completely segregated multicomponent system.

1. Theory

Let's consider a binary mixture of species 1 and 2 with different sizes and/or densities. Let us suppose that the weight fractions of the two components are x_1 and x_2 , and that their minimum fluidization velocities are, respectively, U_1 and U_2 ($U_1 < U_2$). If the system is strongly segregating, the variation of pressure drop with superficial velocity for each component and for the binary mixture is shown in **Fig. 1**. According to this figure:

$$\tan \alpha = \frac{x_1}{U_1} \quad (1)$$

$$\tan \beta = \frac{x_{02}}{U_1} = \frac{x_2}{U_2} \quad (2)$$

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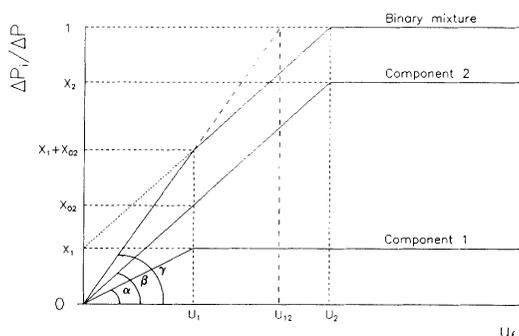


Fig. 1 $(\Delta P_i/\Delta P)$ vs. U_f in a segregating binary fluidized bed

$$\tan \gamma = \frac{x_1 + x_2}{U_{12}} = \frac{1}{U_{12}} = \frac{x_1 + x_{02}}{U_1} \quad (3)$$

From the above relations it is deduced that the U_{mf} of a binary mixture may be expressed as:

$$\frac{1}{U_M} = \frac{1}{U_{12}} = \frac{x_1}{U_1} + \frac{x_2}{U_2} \quad (4)$$

As pointed out above, the graphical method we have used is very similar to that of Obata *et al.* (1982). The only differences are 1) that they inferred U_M from a $\Delta P - U_f$ plot, instead of the $(\Delta P_i/\Delta P) - U_f$ plot we used; and 2) that their geometrical deduction of U_M is based on the fact that $\gamma = \alpha + \beta$ (α , β and γ being segments representing, respectively, the pressure drop due to each component and the binary mixture at $U_f = U_1$). Our geometrical deduction considers $\tan \gamma = \tan \alpha + \tan \beta$ (α , β and γ being angles whose tangents are: $\tan \alpha = (\Delta P_1)_x/U_x$, $\tan \beta = (\Delta P_2)_x/U_x$ and $\tan \gamma = (\Delta P_1 + \Delta P_2)_x/U_x$. U_x represents any velocity from zero to U_1 . $(\Delta P_1)_x$, $(\Delta P_2)_x$ and $(\Delta P_1 + \Delta P_2)_x$ are, respectively, the pressure drops due to each component and the binary mixture at $U_f = U_x$).

Similarly, the U_{mf} of a ternary mixture can be obtained by considering the variation of pressure drop with superficial gas velocity for the segregating ternary mixture. Thus, according to Fig. 2 we may write:

$$\tan \alpha' = \frac{x_1}{U_1} \quad (5)$$

$$\tan \beta' = \frac{x_{02}}{U_1} = \frac{x_2}{U_2} \quad (6)$$

$$\tan \gamma' = \frac{x_{03}}{U_1} = \frac{x_3}{U_3} \quad (7)$$

$$\tan \delta' = \frac{x_1 + x_2 + x_3}{U_{123}} = \frac{1}{U_{123}} = \frac{x_1 + x_{02} + x_{03}}{U_1} \quad (8)$$

and, therefore, the U_{mf} of a ternary mixture may be expressed as:

$$\frac{1}{U_M} = \frac{1}{U_{123}} = \frac{x_1}{U_1} + \frac{x_2}{U_2} + \frac{x_3}{U_3} \quad (9)$$

where x_i and U_i ($i = 1, 2, 3$) are the weight fractions and the minimum fluidization velocities of each component of the ternary mixture. (Equation (9) is similar to the expression that one would have obtained by considering that the three-component mixture is a binary one, in

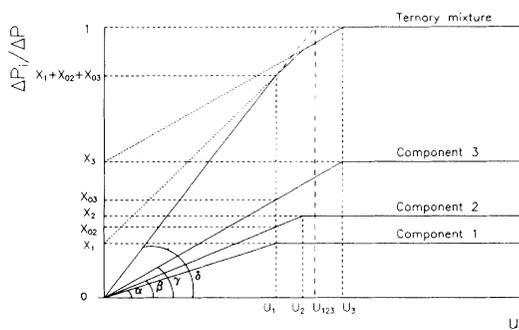


Fig. 2 $(\Delta P_i/\Delta P)$ vs. U_f in a segregating ternary fluidized bed

which the hypothetical two components have minimum fluidization velocities U_{12} , the U_{mf} of the mixture of component 1 and 2, and U_3 the U_{mf} of pure component 3).

Results indicated above may be extended to an n -component mixture and, therefore, their U_{mf} expressed as¹¹⁾:

$$\frac{1}{U_M} = \frac{1}{U_{12\dots n}} = \sum_{i=1}^n \frac{x_i}{U_i} \quad (10)$$

2. Experimental

All experiments were conducted in a perspex column 9 cm in diameter and 90 cm in length fitted to a perforated-plate distributor. A manometer was employed to measure the pressure drop across the bed. Air metered by a calibrated rotameter was used as a fluidizing gas. One size fraction of plastic granules ($\rho_p = 980 \text{ kg/m}^3$; $d_p = 2900 \mu\text{m}$) and six size fractions of glass beads ($\rho_g = 2700 \text{ kg/m}^3$; $d_p = 1090, 920, 775, 655, 550$ and $460 \mu\text{m}$) were used as the fluidized solids. The minimum fluidization velocities of the plastic and glass particles were, respectively, 0.86, 0.62, 0.53, 0.42, 0.36, 0.26 and 0.19 m/s. Each component in the mixture was sieved for a time long enough (100 g of material was sieved during 15 minutes) to give a narrow size distribution.

The experimental procedure in all experiments was basically the same. A weighted quantity of each component of the mixture was poured into the fluidization column. Prior to recording any data the charge was vigorously fluidized with air at a velocity either twice the U_{mf} of the heavier/larger component or the maximum velocity at which entrainment was not observed. Thereafter the bed was allowed to settle by gradually stopping the air supply during a period of 20 minutes. This slow defluidization procedure was designed to maximize the chances of segregation. During the defluidization period the fluid velocity and pressure drop were recorded. U_{mf} was taken as the velocity at which the two extrapolated straight lines (representing the fixed and fluidized states) of the curve $\Delta P_i/\Delta P$ vs. U_f intersect. This definition of the U_{mf} of mixed-particle systems, which is equivalent to that of a pure component, has been widely accepted^{2-4, 11-14)}.

Table 1. Comparison of experimental and predicted results for binary mixtures (Data from present work)

Species	U_{mf1}/U_{mf2}	x_p (%)	U_M (m/s)		E* (%)
			exp.	cal.	
Plastic + Glass (1090 μm)	1.4	6.4	0.65	0.63	-3.1
			14.2	0.66	-1.5
			27.9	0.67	0.0
			47.5	0.76	-6.6
Plastic + Glass (920 μm)	1.6	Idem	0.55	0.54	-1.8
			0.57	0.56	-1.8
			0.59	0.59	0.0
			0.64	0.65	1.6
Plastic + Glass (775 μm)	2.1	Idem	0.70	0.72	2.9
			0.41	0.43	4.8
			0.47	0.45	-4.3
			0.50	0.49	-2.0
Plastic + Glass (655 μm)	2.4	Idem	0.55	0.55	0.0
			0.67	0.65	-2.9
			0.37	0.37	0.0
			0.39	0.39	0.0
Plastic + Glass (550 μm)	3.3	Idem	0.44	0.43	-2.2
			0.53	0.50	-5.6
			0.60	0.60	0.0
			0.28	0.27	-3.5
Plastic + Glass (460 μm)	4.5	Idem	0.33	0.29	-12.1
			0.33	0.32	-3.0
			0.39	0.39	0.0
			0.47	0.50	6.3
Plastic + Glass (460 μm)	4.5	Idem	0.19	0.20	5.2
			0.22	0.21	-4.5
			0.23	0.24	4.3
			0.37	0.30	-18.9
Plastic + Glass (460 μm)	4.5	Idem	0.38	0.41	7.8
			0.38	0.41	7.8

*: $E = 100 [U_M(\text{cal}) - U_M(\text{exp})]/U_M(\text{exp})$

3. Results and Discussion

In this section, using our own data and other authors' results, the validity of Eqs. (4) and (10) in estimating the U_{mf} of mixtures is discussed. Equation (10) for multicomponent systems and Eq. (4) for binary mixtures, the simplest multicomponent system, are evaluated by the experimental data. Likewise, the ability of Eq. (4) to predict the effect of adding small proportions of fines to a monocomponent bed is shown.

For binary mixtures, the comparison between our experimental data and those predicted by Eq. (4) is shown in **Table 1**. It can be seen that the agreement is very good, having an absolute percentage error smaller than 10 % in almost all cases (28 out of 30). The good fitting was not unexpected since the experiments were designed to reach such agreement. First, the experimental systems were chosen so that the densities ratio (ρ_g/ρ_p) was large enough to assure that strong bed segregation would occur in conditions close to those of minimum fluidization. Second, the defluidization procedure was slow enough to allow particles to rearrange themselves according to their natural trends, i.e., to allow heavier components to sink while lighter ones rise. These two facts, the chosen experimental systems and the oper-

Table 2. Comparison of experimental and predicted results for binary mixtures (Data from Thonglimp *et al.*, 1984)

Mixture	Species	U_{mf1}/U_{mf2}	Number of points with $E^* < 15\%$	Total number of points**
Type 1	different sized glass	1.1	5	5
		2.0	5	5
		3.1	5	5
		4.0	4	5
		4.8	4	5
		7.0	3	5
Type 2	glass + alumina	8.0	0	5
		1.2	5	5
		1.6	5	5
		1.9	2	5
		2.0	5	5
		3.3	4	5
Type 2	glass + steel	3.6	5	5
		4.0	2	5
		4.3	2	5
		1.1	0	5
		1.3	5	5
		1.4	5	5
Type 2	glass + steel	1.5	5	5
		1.6	4	5
		2.1	5	5
		2.2	2	5
		3.0	5	5
		3.4	3	5
Type 2	glass + steel	7.8	5	5
		7.8	5	5

*: $E = [U_M(\text{cal}) - U_M(\text{exp})]/U_M(\text{exp})$

** : For each U_{mf} ratio components were mixed in 5 different proportions

ation procedure, permitted both the hypothesis of completely segregated layers to be accomplished and the fitting of experimental data to Eq. (4) to be excellent.

The validity of the expression to predict experimental data obtained under conditions where the mixing of the different components was not negligible was evaluated using the experimental data reported by Otero and Corella¹¹, Yang and Keairns¹⁰, Thonglimp *et al.*¹⁴ and Koniuta and Taha⁷. In this case it was observed that the fitting to Eq. (4) was slightly worse. Nevertheless, the absolute percentage error was less than 15 % in most cases, which for binary (or multicomponent) mixtures is quite acceptable¹⁴.

Results from Thonglimp *et al.*¹⁴ are shown in **Table 2**. The mixtures analyzed differed in size (Type 1) as well as in size and density (Type 2). In the experimental range they studied, for Type 1 mixtures it was shown that the agreement between experimental and predicted data is not affected by the U_{mf} ratio and, therefore, neither by size ratio. In the case of Type 2 mixtures, however, the density ratio does affect the agreement and best fitting was obtained for large density ratios, as expected. (The absolute percentage error was less than 15 % in 26 out of 35 experimental points when glass-glass systems were analyzed, in 30 out of 40 in the cases of glass-alumina ($\rho_g/\rho_{al} = 1.6$) systems and in 39 out of 50 for glass-steel ($\rho_s/\rho_g = 2.8$) mixtures). Experimental data reported by Otero and Corella¹², Yang and

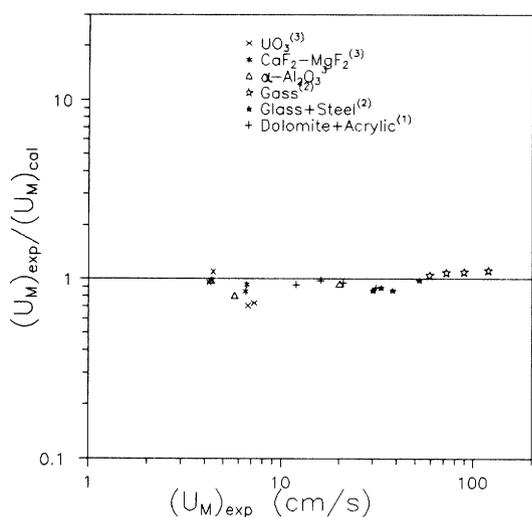


Fig. 3 Comparison of experimental and predicted results for binary mixtures. ⁽¹⁾: Data from Yang and Keairns, 1982; ⁽²⁾: Data from Koniuta and Taha, 1990; ⁽³⁾: Data from Otero and Corella, 1971)

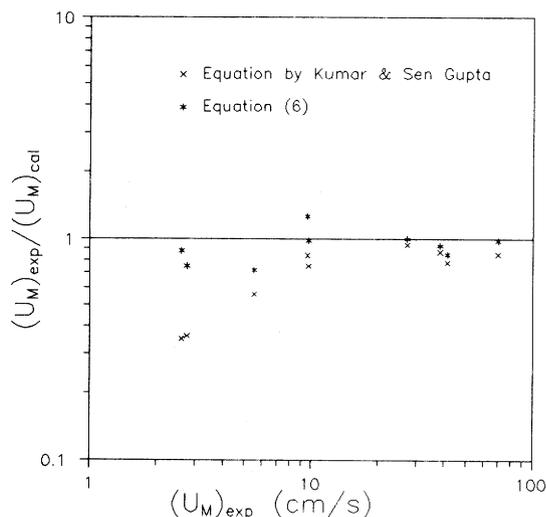


Fig. 4 Minimum fluidization velocities of multicomponent mixtures. Comparison of correlations

Keairns¹⁶⁾ and Koniuta and Taha⁷⁾ are plotted in **Fig. 3**. They showed the same feature as those by Thonglimp *et al.*¹⁴⁾; i.e., while size ratio does not affect agreement between experimental and predicted results, density ratio does affect agreement. (The absolute percentage error was less than 15 % in 10 out of 14 experimental points when systems of similar density were analyzed, and in all cases for dolomite-acrylic ($\rho_d/\rho_s = 2.5$) and glass-steel ($\rho_s/\rho_g = 2.8$) mixtures).

In relation to multicomponent systems, it should be noted that most of the literature correlations⁸⁾ for calculating their U_{mf} are based on the assumption that the bed is completely mixed and homogeneous¹⁵⁾. In contrast, Eq. (10) of this paper was derived on the assumption that the bed was completely segregated. The comparison between predictions from these two different approaches is shown in **Fig. 4**. It can be seen that Kumar and Sen

Table 3. Comparison of experimental and predicted U_M data for multicomponent mixtures

Mixture	Species	d_p (μm)	x_j (%)	U_{mf} (m/s)	U_M (cm/s)		E^* (%)
					exp.	calc.	
A ¹	Glass + Plastic	2900	70.5	86.0			
		1090	5.9	62.0	69.00	70.10	1
		920	10.3	53.0	(62.0)		(13)
		775	13.3	42.0			
B ¹	Glass + Plastic	2900	49.5	86.0			
		655	20.2	36.0	38.10	40.90	7
		550	20.2	26.0	(35.5)		(15)
		460	10.1	19.0			
C ¹	Glass	1090	20.0	62.0			
		920	35.0	53.0	41.20	48.70	18
		755	45.0	42.0	(39.1)		(13)
D ¹	Glass	655	40.0	36.0			
		550	40.0	26.0	27.10	27.00	0
		460	20.0	19.0	(25.0)		(8)
E ²	CaF ₂ -MgF ₂	338	25.0	15.3			
		274	50.0	10.0	9.70	9.90	2
		224	25.0	7.2			
F ²	α -Al ₂ O ₃	274	33.4	15.6			
		224	33.3	7.6	9.60	7.59	-26
		179	33.3	5.0			
G ²	UO ₃	548	33.4	49.0			
		224	33.3	12.9	5.50	7.66	28
		98	33.3	3.4			
H ³	Glass	569	16.7	27.04			
		452	16.7	18.54			
		284	25.0	8.04	2.59	2.94	12
		155	16.6	2.54			
		102	25.0	1.14			
I ³	Glass	569	20.0	27.04			
		284	20.0	8.04	2.74	3.67	25
		155	60.0	2.54			

*: $E = 100 [U_M(\text{cal}) - U_M(\text{exp})]/U_M(\text{exp})$

¹: Data from present work

²: Data by Otero and Corella, 1971

³: Data by Lewis *et al.*, 1949

⁴: As recalculated by Rowe and Nienow, 1975

Gupta's correlation always overpredicts experimental results. Equation (10) also tends to overestimate U_M but their predictions are significantly more satisfactory than those of the equation by Kumar and Sen Gupta⁸⁾. Expression by Rowe and Nienow¹³⁾ (which expresses the U_{mf} of the mixture as a function of the U_{mf} of the single components, their relative proportions, the voidage of a pure component and the voidage of the mixture) could not be compared because voidage data were not reported for the experimental systems taken from the literature.

A further analysis of Eq. (10) is shown in **Table 3**. It can be observed that, for the U_{mf} data obtained in this work, the agreement between theory and experiments is quite good and is independent of the type of system analyzed (namely, mixtures differing in size or mixtures differing in both size and density). This should be imputed to the fact that the data were obtained from a slow defluidization procedure which ensured that the bed corresponded as closely as possible to a segregating state.

The argument was less satisfactory for results reported by other authors^{9, 12}. In principle the differences could be attributed to the possibility that these data may have not been obtained from a slow defluidization procedure and, therefore, that the bed conditions could have been far from those under which the theoretical expression for U_{mf} was derived. The argument, however, becomes less convincing in light of the fact that the U_{mf} of mixtures obtained in this work from a fast procedure (bracketed numbers in Table 3) did not show high deviations from the predicted value. Most probably, the reason for the disagreement is that for small values of U_{mf} minor experimental errors in the determination of U_{mf} lead to high absolute percentage errors. Therefore, in light of the results analyzed, one may state that the use of Eq. (10) for the estimation of U_{mf} of multicomponent mixtures is justified.

Finally, it is worth noting that a great reduction in U_{mf} occurs when small quantities of fines are added to a monocomponent bed of coarse particles. However, a brisk increase is rare when coarse particles are poured into a bed of finer ones. Although the effect has been implicitly studied above, it will be shown here that Eq. (4) can predict these experimental findings.

Let's suppose a hypothetical binary mixture with components 1 and 2 having $U_1 = 1$ and $U_2 = 5U_1$. It can be calculated from Eq. (4) that when 10 % of a component with $U_{mf} = U_1$ is added to a monocomponent bed of $U_{mf} = U_2$, the decrease in U_{mf} (relative to the pure component 2) triples the increase in U_{mf} (relative to the pure component 1) if 10 % of coarse particles is incorporated into a bed of finer ones. For real binary mixtures the effect is shown in Table 1, where experimental results are compared against predictions of Eq. (4).

Conclusion

Taking all results together it may be said that, though the ideal bed classification may not be exactly realized in practice, it is a good approximation for predicting the U_{mf} of binary and multicomponent mixtures as well as the effect of adding small proportions of fines to a monocomponent mixture.

The degree of agreement between theory and experiments has been found to be variable, ranging from very good when segregation occurred during measurement to acceptable in cases where such segregation (because of the system or because of the defluidization procedure) was not probable. This is true even for multicomponent mixtures of similar density and slightly different particle size when use of Eq. (10) beyond extends the original limiting conditions.

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Nomenclature

d_p	= particle diameter	[m]
E	= absolute percentage error	[-]
U_f	= superficial fluidization velocity	[m/s]
U_{mf}	= minimum fluidization velocity	[m/s]
U_M	= minimum fluidization velocity of a mixture	[m/s]
U_i	= minimum fluidization velocity of component i of a mixture	[m/s]
U_{12}	= U_{mf} of the mixture of components 1 and 2	[m/s]
U_{123}	= U_{mf} of the mixture of components 1, 2 and 3	[m/s]
$U_{1..n}$	= U_{mf} of the mixture of components 1, 2 ... n	[m/s]
x_i	= weight fraction of component i of a mixture	[-]
x_{oi}	= weight fraction corresponding to the abscissa U_1 of the $(\Delta P/\Delta P) - U_f$ curve of component i in Figures 2 and 3	[-]
ΔP	= total pressure drop	[N/m ²]
ΔP_i	= pressure drop due to component i	[N/m ²]
ρ	= particle density	[kg/m ³]

<Subscripts>

a	= acrylic
al	= alumina
d	= dolomite
g	= glass
j	= jetsam component (heavier and/or larger component)
p	= plastic
s	= steel

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