EVALUATION OF COMMON SCALING-UP RULES FOR A STIRRED VESSEL FROM THE VIEWPOINT OF ENERGY SPECTRUM FUNCTION

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Introduction

A major problem in industrial processes is successful scale-up of equipment under turbulent conditions from small to large size. Though various methods of scale-up have been proposed[1,2,4,9-12] on the premise that all factors about the equipment geometry are similar between small and large equipment, no practical method based on theoretical considerations has been established. The main reason is the difficulty in satisfying the basically required condition for turbulent flow field which governs phenomena as the object of attention.

The energy spectrum function (ESF) is one of the representative factors which express the mechanism of turbulent flow field. The purpose of this paper is to evaluate common rules for scaling-up from the viewpoint of the ESF.

1. ESF Form and Scaling-up

Concerning the ESF, a simple expression of one-dimensional ESF for wide wavenumber ranges was proposed, based on an information entropy conception[6,7] (see Appendix).

$$\frac{EK_s}{u^2}=\frac{1}{\sum_{j=1}^{m} 2^{j-1}}\sum_{j=1}^{m}\left\{6^{j-1}\exp\left(-3^{j-1}\frac{k}{K_s}\right)\right\} \quad (1)$$

It was confirmed that each ESF curve for m eddy-groups appears to be a satisfactory approximation of the measured one in a flow: pipe flow, jet flow, downstream of a grid and so on[7]. **Figure 1** shows an example of application of Eq. (1) to measured ESF values at a center position far from the tip of the impeller in a discharge flow in a stirred vessel (six-blade disk-turbine impeller diameter/vessel diameter ratio is 102 mm/312 mm). The ESF curve for the number of eddy-groups $m=2$ can express the measured ESF values satisfactorily. It is also confirmed that Eq. (1) can approximately express the measured ESF values by Sato *et al.*[13] in a stirred vessel (different geometric style of stirred vessel from that described above) by determining the appropriate number m. In addition, in the case of fully developed turbulent pipe flows there was a clear relationship between the pipe inner diameter and the number of eddy-groups m[7]; a new subharmonic larger eddy-group appears when the pipe inner diameter is at least tripled. Using these considerations, the principle for discussion about common rules for scaling-up can be written as the following.

1. Scale-up is completely successful if the ESF curve for the large equipment is the same as that for the small one. This is established when the scaling-up ratio is less than three.
2. When it is impossible to achieve the same ESF curves before and after scaling-up, scale-up is successful enough if the respective ESF curves of the large and small equipment in the wavenumber range which is significant for the phenomena overlap sufficiently.

2. Evaluation of Rules for Scaling-up

Some representative rules for scaling-up of a stirred

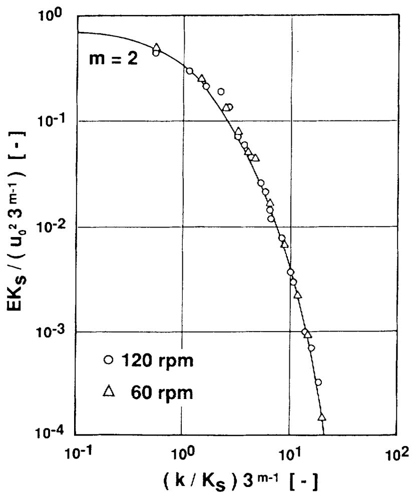

Fig. 1. ESF curve for an impeller discharge flow in a stirred vessel

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vessel are taken up as the object of discussion: N=constant, $ND^{2/3}$=constant, ND=constant and ND^2=constant. With these rules, the impeller revolutional speed N can be rewritten by the mean square turbulent velocity u^2 in Eq. (1). This is because, it is experimentally clear[3] that impeller tip speed ND is in proportion to $(u^2)^{1/2}$.

ESF curves, when scaling-up is done by holding the respective parameter constant, are drawn as shown in Fig. 2. In the figures, each ESF curve is relative to that for the case of $m=1$ and the curve for $m=i$ is the result when the size of the large vessel is 3^{i-1} times that in the case of $m=1$.

N=constant (u^2D^{-2}=const.):

No curve crosses any other.

$ND^{2/3}$=constant ($u^2D^{-2/3}$=const.):

This rule has been most widely discussed[5] and used.

As shown in **Fig. 2(a)**, in the wavenumber range higher than that to which the Kolmogoroff spectrum law($E \propto k^{-5/3}$) can apply, all curves seem to overlap, though they do not overlap one another in the lower wavenumber range. This tendency is more complete in the case of $m \geq 2$. These results show that there is a clear physical reliability in this scaling-up rule from the viewpoint of ESF if the turbulence in the higher wavenumber range plays a significant role, that is, if such a microscale mixing is significant in the process.

ND=constant (u^2=const.):

Every curve has only one cross-point with any other curve.

ND^2=constant (u^2D^2=const.):

No curve crosses any other, and they are parallel to each other in the lower wavenumber range.

$ND^{1.5}$=constant (u^2D=const.):

This rule is outside of the common rules. This condition means that impeller discharge energy per unit discharge area and unit circulation time are constant. As shown in **Fig. 2(b)**, though no curve crosses any other, they are very close to one another in the lower wavenumber range. These results show that this rule will be valuable for scaling-up only if macroscale mixing is important in the process.

As mentioned above, for scaling up the most important point is to make sure which constant parameter is most significant for the phenomena as the object of attention. The discussion above is applicable not only to a stirred vessel but also to other equipment.

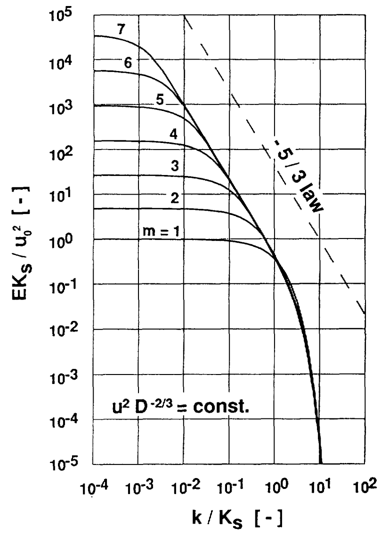

Fig. 2(a). ESF curves in the case of $ND^{2/3}$=constant

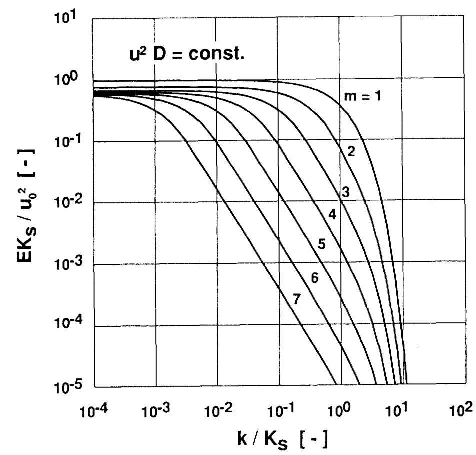

Fig. 2(b). ESF curves in the case of $ND^{3/2}$=constant

Appendix[7]

The expression of Eq. (1) was derived according to the following considerations.

First, by assuming that

1. turbulent flow field consists of m eddy-groups: a basic eddy-group and its sequential subharmonic m-1 eddy-groups,
2. eddy-group j takes the ESF curve which associates the maximum amount of information entropy under the condition that an average wavenumber K_j exists, and
3. weight factor of the turbulent kinematic energy of eddy-group j to u^2 is $P_j(\sum_{j=1}^{m} P_j=1)$, then the one-dimensional ESF curve is derived as follows.

$$E=\sum_{j=1}^{m}\frac{u^2P_j}{K_j}\exp\left(-\frac{k}{K_j}\right) \tag{A-1}$$

Second, it is supposed that the following simple relationships exist among the average wavenumbers and weight factors of each eddy-group.

$$\frac{K_j}{K_{j-1}}=\alpha, \quad \frac{P_j}{P_{j-1}}=\beta \tag{A-2}$$

Eq. (A-1) is rewritten as follows by using α and β in Eq. (A-2).

$$E=\frac{u^2}{K_s\sum_{j=1}^{m}\beta^{j-1}}\sum_{j=1}^{m}\left\{\left(\frac{\beta}{\alpha}\right)^{j-1}\exp\left(-\frac{1}{\alpha^{j-1}}\frac{k}{K_s}\right)\right\} \tag{A-3}$$

where $K_s=K_1$ (average wavenumber of basic eddy-group). By comparing the ESF curves expressed by Eq. (A-3) under many value combinations of α and β, the most suitable value combination of them is determined as

$$\alpha = 1/3\,,\quad \beta = 2 \tag{A-4}$$

In this case of value combination of $\alpha = 1/3$ and $\beta = 2$, Eq. (A-3) can be transformed into Eq. (1).

Nomenclature

D	= representative length of equipment	[m]
E	= one-dimensional energy spectrum	[m^3/s^2]
K_j	= average wavenumber of eddy-group j	[1/m]
K_s	= average wavenumber in the case of $m=1$	[1/m]
k	= wavenumber	[1/m]
m	= number of eddy-group	[—]
N	= impeller speed of revolution	[1/s]
P_j	= weight factor of turbulent kinematic energy of eddy-group j	[—]
u^2	= turbulent kinematic energy	[m^2/s^2]
u_0^2	= turbulent kinematic energy for the case of $m=1$	[m^2/s^2]
α	= ratio of average wavenumber	[—]
β	= ratio of weight factor of turbulent kinematic energy	[—]

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