# MAGNETIC FIELD EFFECTS ON MARANGONI AND NATURAL CONVECTIONS IN A RECTANGULAR OPEN BOAT

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**Effects of magnetic fields on natural and Marangoni convections in a two-dimensional rectangular open boat were studied numerically. Convective velocities in magnetic fields depend on the Marangoni number, the Grashof number, physical properties, direction of the applied magnetic field and strength of magnetic field. For suppression of natural convection, either a horizontal (perpendicular to gravity) or a vertical (parallel to gravity) magnetic fields is effective. For suppression of Marangoni convection, the vertical magnetic field is more effective than the horizontal magnetic field.**

## Introduction

Recent developments in the electronics industry have created greater demand for high quality of bulk single crystals. To grow such single crystals it is important to control convective phenomena in melt during crystal growth.

For melts with high electrical conductivities such as silicon[3] and gallium arsenide[6,17], the application of magnetic fields is useful in controlling the convective phenomena. This technique has been applied to the Czochralski method and put to practical use as the magnetic applied Czochralski (MCZ) method. Hoshi *et al.*[3] studied experimentally the magnetic field effects on the convective phenomena in a silicon melt and the quality of grown crystals. Terashima and Fukuda[17] reported that the application of a magnetic field suppresses thermal fluctuations due to thermal convections in a gallium arsenide melt. Many analytical studies of the effect of magnetic fields on melt convection have also been reported[2,4,5,7-12]. However, many points remain unclear regarding the effects of magnetic field strength and direction on convective velocity due to natural and/or Marangoni convections.

The authors previously studied the natural and Marangoni convections in a two-dimensional rectangular open boat and discussed the effects of natural and Marangoni convections on surface velocity[13]. Applying these results to Czochralski growth of oxide single crystals, correlations between critical crystal rotation rates and various control factors[14] were also discussed. In this study, the effect of a magnetic field on the natural and Marangoni convections in a two-dimensional rectangular open boat was studied numerically. This yielded a quantative evaluation of the effects of the Marangoni number, the Grashof number, melt depth, direction of the magnetic field applied and strength of the magnetic field on convective velocities in the magnetic field.

## 1. Analysis

The theoretical model is shown in **Fig. 1**. A two-dimensional rectangular open boat with a free interface which is heated from one side and cooled from the other is considered. In Fig. 1, (a) shows the case where a vertical magnetic field is applied while (b) shows that where a horizontal magnetic field is applied. The model included the following assumptions (i) steady state, (ii) incompressible and

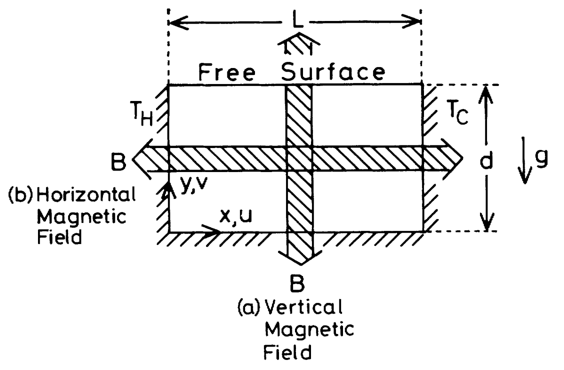


**Fig. 1.** Configuration for analysis
(a) Vertical magnetic field
(b) Horizontal magnetic field

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Newtonian fluid, (iii) free interface and walls that are flat, (iv) bottom wall and the free interface that are adiabatic and electrically insulated, and (v) constant values of all physical properties with the exceptions of interfacial tension in the stress balance equation for the free inteface, and density in the buoyancy force.

The basic equations are continuity, Navier–Stokes and energy equations. The Navier–Stokes equation in vector form is written as

$$\frac{\partial \boldsymbol{v}}{\partial t}+(\boldsymbol{v}\cdot\nabla)\boldsymbol{v}=-\frac{1}{\rho}\nabla \boldsymbol{p}+\nu\nabla^2\boldsymbol{v}+\boldsymbol{F}_g+\boldsymbol{F}_L \tag{1}$$

$\boldsymbol{F}_g$ is the buoyancy force and $\boldsymbol{F}_L$ is the Lorentz force, which is written as

$$\boldsymbol{F}_L=\boldsymbol{J}\times\boldsymbol{B} \tag{2}$$

That is,

$$F_{Lx}=\sigma_e(E_y+wB_x-uB_z)B_z-\sigma_e(E_z+uB_y-vB_x)B_x \tag{3}$$

$$F_{Ly}=\sigma_e(E_z+uB_y-vB_x)B_x-\sigma_e(E_x+vB_z-wB_y)B_z \tag{4}$$

Magnetic fields generated by induced current are negligibly small compared with the applied magnetic field[7]. Therefore, when vertical magnetic fields are applied,

$$\boldsymbol{B}=(0, B, 0) \tag{5}$$

and when horizontal magnetic fields are applied,

$$\boldsymbol{B}=(B, 0, 0) \tag{6}$$

Because the walls and the free surface are insulated, $E_z=0$[15] and because a two-dimensional rectangular boat is used, $w=0$. Therefore, the Lorentz forces are expressed as follows.

When vertical magnetic fields are applied,

$$F_{Lx}=-\sigma_e uB^2\,,\qquad F_{Ly}=0 \tag{7}$$

When horizontal magnetic fields are applied,

$$F_{Lx}=0\,,\qquad F_{Ly}=-\sigma_e vB^2 \tag{8}$$

The final basic equations to be solved in this study are as follows:

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{9}$$

Navier–Stokes equation

for the case of vertical magnetic field:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma_e B^2 u}{\rho} \tag{10}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-g\beta\Delta T \tag{11}$$

for the case of horizontal magnetic field:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{12}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\sigma_e B^2 u}{\rho}-g\beta\Delta T \tag{13}$$

Energy equation

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{14}$$

The boundary conditions are described as follows:

(a) along the free surface; $0\leqq x\leqq L$, $y=d$.

$$\mu(\partial u/\partial y)=-(\partial\sigma/\partial T)\cdot(\partial T/\partial x) \tag{15}$$

$$v=0\,,\qquad \partial T/\partial y=0 \tag{16}$$

(b) along the bottom wall; $0\leqq x\leqq L$, $y=0$.

$$u=v=0\,,\qquad \partial T/\partial y=0 \tag{17}$$

(c) along the hot wall; $x=0$, $0\leqq y\leqq d$

$$u=v=0\,,\qquad T=T_H \tag{18}$$

(d) along the cold wall; $x=L$, $0\leqq y\leqq d$

$$u=v=0\,,\qquad T=T_C \tag{19}$$

A numerical analysis according to the finite difference method, the same as in our previous paper,[13] are applied to the basic equations and boundary conditions. The value of the Prandtl number is $10^{-2}$. This value is equal to the value of semiconductor melts such as silicon and gallium arsenide.

## 2. Results and Discussion

**Figures 2** and **3** show respectively the effect of vertical and horizontal magnetic fields on the velocity distributions and temperature distributions. The velocity distributions in melts without magnetic fields are shown in **Fig. 4**. Column (A) shows the case where only natural convection exists; column (B) shows the results when there is coexistence of natural and Marangoni convections; column (C) shows the case where only Marangoni convection exists. In Figs. 2 and 3, the strength of the applied magnetic field is fixed. When a vertical magnetic field is applied, the value of the Hartmann number is independent of

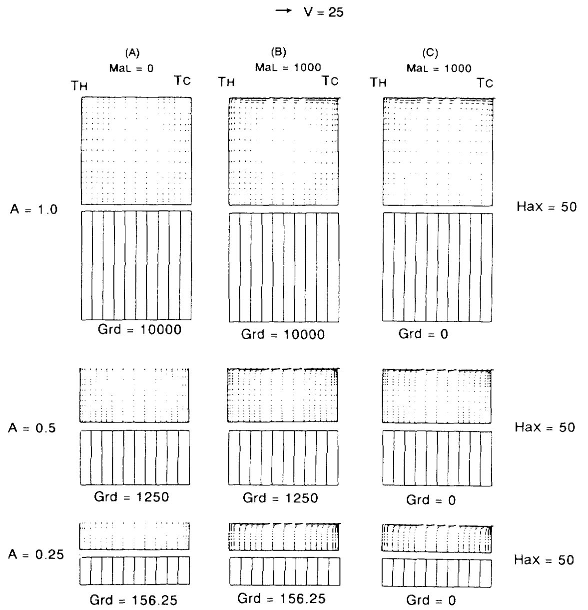


**Fig. 2.** Effect of vertical magnetic field on velocity distributions and temperature distributions (Arrows show velocity vector at each point in the melt.)

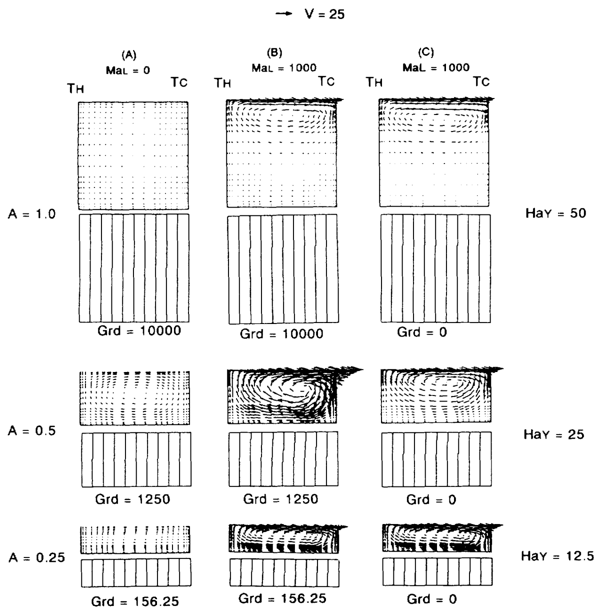


**Fig. 3.** Effect of horizontal magnetic field on velocity distributions and temperature distributions (Arrows show velocity vector at each point in the melt.)

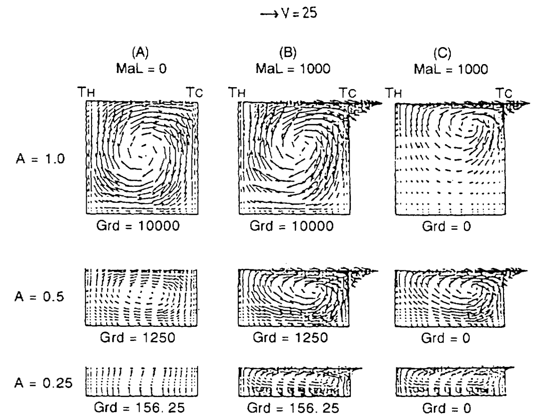


**Fig. 4.** The velocity distributions in melts without magnetic fields (Arrows show velocity vector at each point in the melt.)[13]

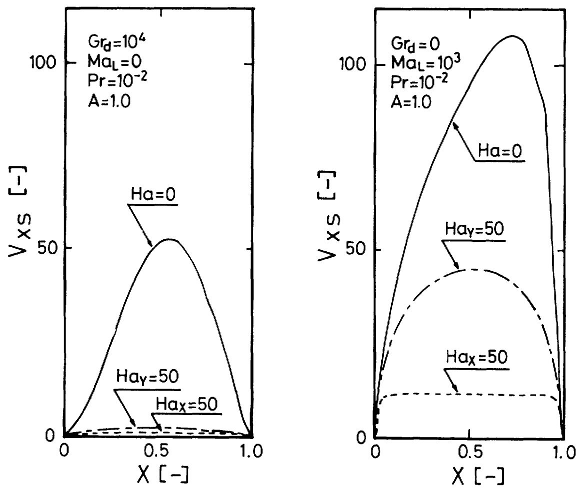


**Fig. 5.** Effect of magnetic fields on surface velocities

melt depth. When a horizontal magnetic field is applied, the Hartmann number decreases with decreasing melt depth because the value of the melt depth is used for the characteristics length in $Ha_y$. **Figure 5** shows the magnetic field effect on the surface velocity. From Figs. 2–5, the following points can be noticed.

(a) Convective velocity in the melt depends on the strength of the magnetic field, direction of the magnetic field, melt depth and physical properties of the melt.

(b) A vertical magnetic field is effective in suppressing convection induced by natural and/or Marangoni convection in the range of $A \leqq 1$ in which numerical analysis was carried out.

(c) A horizontal magnetic field is effective in suppressing convection induced by natural convection but is not very effective against Marangoni convection in the region of $A \leqq 1$.

**Figure 6** shows in greater detail the effect of the direction of magnetic field on the melt when natural convection is dominant. When a vertical magnetic field is applied under natural convection-dominant conditions, convection perpendicular to the direction of gravity is suppressed. A horizontal magnetic field, on the other hand, suppresses convection in the direction of gravity. When dissolution of crucible materials occurs, e.g. in silicon crystal growth using a quartz crucible, the vertical magnetic field may induce a high concentration of impurities from the crucible in the melt.

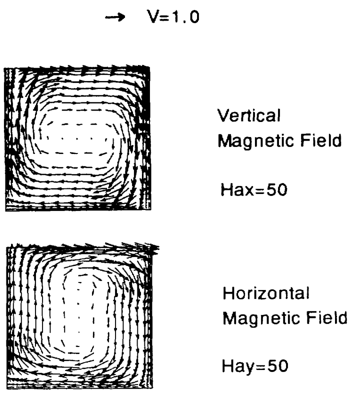


**Fig. 6.** Effect of the direction of magnetic field on velocity distribution under natural convection-dominant condition ($Gr_d = 10^4$, $Ma_L = 0$, $Pr = 10^{-2}$)

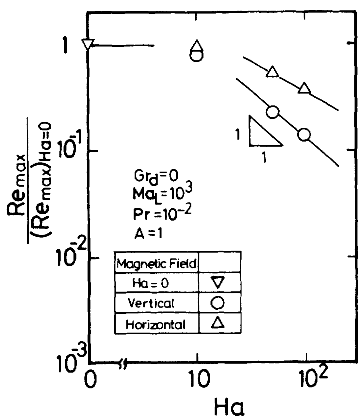


**Fig. 7.** Relationship between the maximum Reynolds number and Hartmann number under Marangoni convection-dominant condition

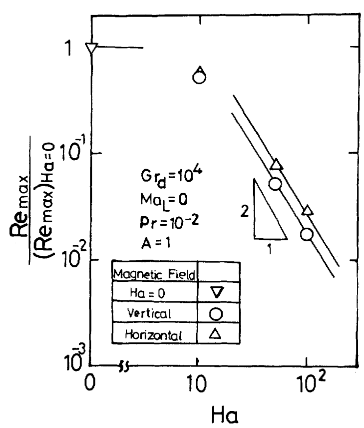


**Fig. 8.** Relationship between the maximum Reynolds number and Hartmann number under natural convection-dominant condition

**Figures 7** and **8** show respectively the effect of applied magnetic field strength on maximum velocity in the boat under Marangoni- and natural convection-dominant regimes. The following relationships can be obtained in the region of $Ha \gg o[1]$.

(a) For suppression of Marangoni convection, a vertical magnetic field is more effective than a horizontal one. When a vertical magnetic field is applied under Marangoni convection-dominant conditions,

$$Re = o[Ma_L/Ha_x] \tag{20}$$

and

$$u/u_{Ha=0} = Ma_L^{1/3}/Ha_x \tag{21}$$

(b) When a horizontal magnetic field is applied under natural convection-dominant conditions,

$$Re = o[Gr_d/Ha_y^2] \tag{22}$$

and

$$v/v_{Ha=0} = Gr_d^{1/2}/Ha_y^2 \tag{23}$$

This relationship agrees with the result obtained by Munakata and Tanasawa[12].

Here, $Ma^{1/3}/Ha$ and $Gr^{1/2}/Ha^2$ are dimensionless numbers defined as follows:

$$\frac{Ma^{1/3}}{Ha} \equiv \frac{\text{inertial force due to Marangoni convection}}{\text{suppressive force due to magnetic field}}$$

and

$$\frac{Gr^{1/2}}{Ha^2} \equiv \frac{\text{inertial force due to natural convection}}{\text{suppressive force due to magnetic field}}$$

From Eqs. (20)–(23), and from Figs. 7 and 8, the following can be noted:

(a) Under a vertical magnetic field, the Reynolds number based on the velocity induced by Marangoni convection is inversely proportional to the Hartmann number in the region where $Ha_x > o[1]$.

(b) Under either a vertical or a horizontal magnetic field the Reynolds number based on the velocity induced by natural convection is inversely proportional to the square of the Hartmann number in the region where $Ha_y > o[1]$.

## Conclusion

The effect of magnetic fields on Marangoni and natural convections were studied theoretically. The following conclusions were obtained:

(a) Convective velocities under magnetic fields depend on the Marangoni number, Grashof number, physical properties, melt depth, direction of applied magnetic field and strength of magnetic field.

(b) The Reynolds number based on the velocity induced by Marangoni convection is inversely proportional to the Hartmann number based on the strength of a vertical magnetic field.

(c) The Reynolds number based on the velocity induced by natural convection is inversely proportional to the square of the Hartmann number based on the strength of either a vertical or a horizontal magnetic field.

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**Nomenclature**

| | | |
|---|---|---|
| $A$ | = aspect ratio, equal to $d/L$ | [—] |
| $B$ | = magnetic flux density | [T] |
| $d$ | = melt depth | [m] |
| $E$ | = electric field strength | [V/m] |
| $g$ | = gravitational acceleration | [m/s²] |
| $Gr_d$ | = Grashof number, equal to $g\beta\Delta Td^3/v^2$ | [—] |
| $Ha$ | = Hartmann number | [—] |
| $Ha_x$ | = Hartmann number based on applied vertical magnetic field, equal to $(\sigma_e B^2 L^2/\mu)^{1/2}$ | [—] |
| $Ha_y$ | = Hartmann number based on applied horizontal magnetic field, equal to $(\sigma_e B_2 d^2/\mu)^{1/2}$ | [—] |
| $J$ | = current density | [A/m²] |
| $L$ | = length of free interface | [m] |
| $Ma_L$ | = Marangoni number, equal to $\lvert\partial\sigma/\partial T\rvert\Delta TL/(v\cdot\mu)$ | [—] |
| $o[\ ]$ | = order of magnitude | [—] |
| $P$ | = pressure | [N/m²] |
| $Pr$ | = Prandtl number, equal to $v/\alpha$ | [—] |
| $Re$ | = Reynolds number, equal to $uL/v$ | [—] |
| $T$ | = temperature | [K] |
| $u$ | = x direction velocity | [m/s] |
| $v$ | = y direction velocity | [m/s] |
| $w$ | = z direction velocity | [m/s] |
| $\alpha$ | = thermal diffusivity | [m²/s] |
| $\beta$ | = thermal expansion coefficient | [1/K] |
| $\Delta T$ | = temperature difference, equal to $T_H - T_C$ | [K] |
| $\mu$ | = viscosity | [Pa·s] |
| $v$ | = kinematic viscosity | [m²/s] |
| $\rho$ | = density | [kg/m³] |
| $\sigma$ | = interfacial tension | [N/m] |
| $\sigma_e$ | = electrical conductivity | [1/(Ω·m)] |

⟨Subscripts⟩

| | |
|---|---|
| $C$ | = cold |
| $d$ | = based on melt depth |
| $H$ | = hot |
| $Ha=0$ | = without applied magnetic field |
| $L$ | = based on free interface |
| max | = maximum |
| $s$ | = free interface |
| $x$ | = x direction |
| $y$ | = y direction |
| $z$ | = z direction |

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