

# COMPRESSION CHARACTERISTICS OF EXCESS ACTIVATED SLUDGES TREATED BY FREEZING-AND-THAWING PROCESS

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Excess activated sludge is very difficult to be filtered, but its solid-liquid separation characteristics can be improved by a freezing-and-thawing process. In this study, centrifugal and the gravitational settling experiments were done by using unfrozen and frozen activated sludges.

An equation for the average solid compressive pressure ( $(p_s)_{av}$ ) in the sediment zone was derived, based on the general equation for the local compression characteristics of a solid-liquid mixture. Also, the relationship between  $(p_s)_{av}$  and the average porosity ( $(\epsilon)_{av}$ ) in the sediment zone ( $1 - (\epsilon)_{av} = E(p_s)_{av}^\beta$ ;  $E$  and  $\beta = \text{constants}$ ) was confirmed by centrifugal and the gravitational settling experiments.

The order of magnitude of  $(\epsilon)_{av}$  was unfrozen sample < fast-frozen sample < slow-frozen sample. This order was consistent with that of the average specific filtration resistance obtained from the constant-pressure filtration experiments reported in a previous paper.

## Introduction

It is essential to the design of the solid-liquid separation processes for sludgy materials such as excess activated sludge to find the relationship between the solid compressive pressure and the porosity of a particulate layer. If the material is incompressible or moderately compressible, such as inorganic sludge (zinc oxide, ferric oxide, Mitsukuri-Gairome clay, etc.), the conventional compression-permeability cell method<sup>3,8)</sup>, the constant-pressure expression method<sup>10)</sup>, the centrifugal settling method<sup>1,11,12)</sup>, the gravitational sedimentation method<sup>13,14)</sup>, and the filtration method<sup>9)</sup> have been proposed and used to obtain the desired compression characteristics. In the case of treating an excess activated sludge that is highly compressible, however, the relationship between the solid compressive pressure and the porosity of the sediment has not yet been determined. Nor has it been examined whether the above methods can be applied to activated sludge or not.

It is difficult to filter excess activated sludge, but treatment by freezing and thawing makes the settling velocity of its suspended solids rapid<sup>2,4,5)</sup>. The authors<sup>6,7)</sup> have reported previously, based on gravitational and centrifugal sedimentation experiments, that by freezing and thawing of excess activated sludge the gravitational sedimentation characteristics of the sludge are improved remarkably and the

values of density and dry solid fraction of floc and suspended solid become larger.

In this study the relationship between average solid compressive pressure and average wet-basis porosity of a solid-liquid mixture is examined over a wide solid compressive pressure range, by the centrifugal and the gravitational settling experiments using unfrozen and frozen excess activated sludges produced at a municipal wastewater treatment plant. A new equation is presented for an average solid compressive pressure in the entire sediment corresponding to the average porosity based on conventional local compression characteristics of a solid-liquid mixture. Further, the effect of the freezing and thawing process on the compression characteristics of excess activated sludge is studied.

## 1. Theory

### 1.1 Basic consideration

The suspended solids of activated sludge hold much water (bound water), inside and at the surface, which cannot be removed easily by mechanical separation techniques. The bound water, however, may be removed by drying, by which the shape of the suspended solids is deformed. Therefore, the properties of wet suspended solids must be different from those of dry ones.

As mentioned above, in dealing with this kind of sludge, which contains hydrophilic suspended solids of unfixed shape, it is very important to know the physical characteristics of the wet solid that is in

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water. Consequently, the equipment for solid-liquid separation must be designed according to the wet-basis porosity rather than the dry-basis porosity which has been used conventionally.

In this paper the wet-basis porosity  $\varepsilon$  of sediment is defined as the volume fraction of free water (= (all water in sediment) – (bound water)) which is removed easily by filtration, etc.

$$\varepsilon = 1 - \frac{\text{volume of wet sludge solid}}{\text{volume of sediment}} = \frac{\text{volume of free water}}{\text{volume of sediment}} \quad (1)$$

The local compression characteristics of a solid-liquid mixture have been represented in several forms; the following forms<sup>15)</sup> are used in this paper.

$$\left. \begin{aligned} 1 - \varepsilon &= E p_s^\beta & : p_s \geq p_{si} \\ 1 - \varepsilon &= E p_{si}^\beta & : p_s < p_{si} \end{aligned} \right\} \quad (2)$$

where  $E$ ,  $\beta$  are empirical constants,  $p_s$  is the local solid compressive pressure, and  $p_{si}$  is the solid compressive pressure below which the local porosity is constant. In this paper it is assumed that the compression characteristics of activated sludge in which suspended solids have much bound water may be represented by Eq. (2).

## 1.2 Determination of empirical constants of compression characteristics

After sludge is settled in a centrifugal tube or a gravitational tube for sufficient time, the height of the surface of the sediment becomes constant. Figure 1 shows (a) the equilibrium thickness  $H_N$  of the sediment in a centrifugal tube at  $N$  r.p.m., and (b) the equilibrium thickness  $H_G$  of the sediment in a gravitational tube. The coordinates in Fig. 1 are defined as  $H_G = R - r_i$  and  $H_N = R - r_i$ , to describe in the same manner the conditions in both of the centrifugal tube and the gravitational tube.

The force balance for an infinitesimal layer in the centrifugal and the gravitational sediments can be written as

(a) centrifugal sediment

$$dp_s/dr = (\rho_s - \rho_w)(1 - \varepsilon)r\Omega^2 \quad (3)$$

(b) gravitational sediment

$$dp_s/dr = (\rho_s - \rho_w)(1 - \varepsilon)g \quad (4)$$

where  $\rho_s$ ,  $\rho_w$  are the density of wet solid and supernatant, respectively;  $\Omega$  is the angular velocity of the centrifuge.

Murase *et al.*<sup>11)</sup> have derived the following equation from Eqs. (2) and (3) and the condition of  $R \gg (R - r_i)$  for a centrifugal sediment.

$$R - r_i = \omega_0^{(1-\beta)} \{ (\rho_s - \rho_w) R \Omega^2 \}^{-\beta} / \{ E(1 - \beta) \} \quad (5)$$

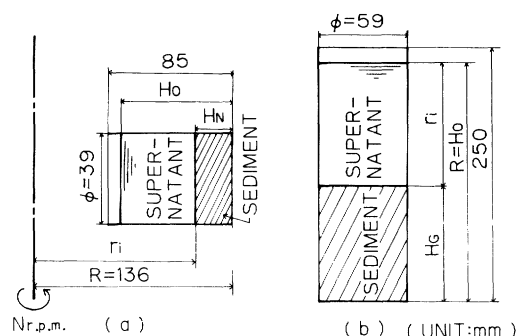


Fig. 1. Schematic illustration of (a) the centrifugal test, (b) the gravitational test

where  $\omega_0$  is the total wet solid volume per unit cross-sectional area of the tube. Plotting  $(R - r_i)$  vs.  $R\Omega^2$  on logarithmic paper based on Eq. (5), one can determine the values of  $E$  and  $\beta$  in Eq. (2).

The value of  $\omega_0$  is determined as follows. In the centrifugal settling experiments, an equilibrium surface height of sediment  $H_N$  is linear to the reciprocal of the rotational speed  $1/N$ , as reported previously<sup>7)</sup>. If  $N$  becomes infinity, the value of  $p_s$  is considered to become infinity. Thus, it is assumed that there is no void space in equilibrium sediment and accordingly that the equilibrium sediment is filled only with wet sludge solids which include bound water. The value of  $H_N$  at  $N = \infty$  (*i.e.*  $H_\infty$ ), which is equal to  $\omega_0$ , is obtained by the ordinate intercept in the graph of  $H_N$  vs.  $1/N$ .

## 1.3 The relationship between average solid compressive pressure and average porosity of entire equilibrium sediment

In a centrifugal sediment, the average wet-basis porosity in equilibrium sediment  $(\varepsilon_N)_{av}$  can be written as follows.

$$\begin{aligned} (\varepsilon_N)_{av} &\equiv 1 - \frac{(\text{volume of all wet solid})}{(\text{volume of equilibrium sediment})} \\ &= 1 - H_\infty / H_N \\ &= 1 - \omega_0 / (R - r_i) \end{aligned} \quad (6)$$

Substitution of Eq. (5) into Eq. (6) yields

$$1 - (\varepsilon_N)_{av} = E(1 - \beta) \{ (\rho_s - \rho_w) R \Omega^2 \omega_0 \}^\beta \quad (7)$$

Rearranging Eq. (7) yields

$$1 - (\varepsilon_N)_{av} = E[(1 - \beta)^{1/\beta} \{ (\rho_s - \rho_w) R \Omega^2 \omega_0 \}]^\beta \quad (8)$$

If the average solid compressive pressure  $(p_{sN})_{av}$  in equilibrium centrifugal sediment is defined by Eq. (9), the relationship between  $(p_{sN})_{av}$  and  $(\varepsilon_N)_{av}$  can be expressed by Eq. (10), which is the same form as Eq. (2).

$$(p_{sN})_{av} = (1 - \beta)^{1/\beta} \{ (\rho_s - \rho_w) R \Omega^2 \omega_0 \} \quad (9)$$

$$1 - (\varepsilon_N)_{av} = E(p_{sN})_{av}^\beta \quad (10)$$

In a gravitational settling, one can obtain  $(p_{sG})_{av}$  by Eq. (11) and can obtain Eq. (12) in the same manner as in a centrifugal settling.

$$(\rho_{sG})_{av} = (1 - \beta)^{1/\beta} \{ (\rho_s - \rho_w) g \omega_0 \} \quad (11)$$

$$1 - (\varepsilon_G)_{av} = E (p_{sG})_{av}^\beta \quad (12)$$

Consequently, it is reasonable that the average solid compressive pressure, which corresponds to the average porosity, is defined by Eq. (9) or (11).

Using attapulgite clay or Hara Gairome clay in a centrifugal settling, Buscall<sup>11</sup> and Sambuichi *et al.*<sup>12</sup> also reported the average solid compressive pressure of an entire centrifugal sediment, defined as the integral mean of solid compressive pressure from the bottom to the surface of an centrifugal sediment. Referring to the foregoing, in the condition of  $R \gg R - r_i$  and Eq. (3), the average solid compressive pressure  $(p_{sN})'_{av}$  is obtained by Eq. (13). (see Appendix A)

$$(p_{sN})'_{av} = (\rho_s - \rho_w) \Omega^2 \omega_0 (3R + 5r_i) / 16 \quad (13)$$

In the condition of  $R \gg R - r_i$  (i.e.  $R \approx r_i$ ), Eq. (13) can be rewritten as

$$\begin{aligned} (p_{sN})'_{av} &\approx (\rho_s - \rho_w) R \Omega^2 \omega_0 / 2 \\ &= (\rho_s - \rho_w) R \Omega^2 \omega_0 (1 - \beta)^{1/\beta} / \{ 2(1 - \beta)^{1/\beta} \} \\ &= (p_{sN})_{av} / \{ 2(1 - \beta)^{1/\beta} \} \end{aligned} \quad (14)$$

Because the values of  $\beta$  for unfrozen and frozen activated sludge are almost 0.3 as described below (3.1), the value of  $1 / \{ 2(1 - \beta)^{1/\beta} \}$  is almost 1.6. Therefore  $(p_{sN})'_{av}$  is evaluated to be 1.6 times as large as  $(p_{sN})_{av}$  in this paper.

## 2. Experimental

The excess activated sludges produced at a municipal wastewater treatment plant were used as experimental materials. Their properties are listed in Table 1.

Sediment which was prepared by 18-hour gravitational settling in a refrigerator (5°C) is called "unfrozen sample". Sediments were also frozen under the conditions listed in Table 2. The temperature of sludge in a freezing tube was measured by a C-C thermocouple mounted at the center of the freezing tube. As shown in Table 2, the freezing rates of samples were  $9.1 \text{ mm} \cdot \text{h}^{-1}$  and  $1.9 \text{ mm} \cdot \text{h}^{-1}$ \*. The frozen samples were thawed in a 30°C constant-temperature bath and analyzed. In this paper, the sample frozen at  $9.1 \text{ mm} \cdot \text{h}^{-1}$  freezing rate is called "fast-frozen sample", and that at  $1.9 \text{ mm} \cdot \text{h}^{-1}$  freezing rate is called "slow-frozen sample". In the test,

\* As mentioned in a previous paper<sup>5</sup>, the slow freezing rate ( $1.9 \text{ mm} \cdot \text{h}^{-1}$ ) is within the range in which the solid liquid separation characteristics are most improved.

Table 1. Analysis of raw activated sludge (After 18 h settling)

pH	[—]	6.63
Sludge concentration $\times 10^2$	[ $\text{kg} \cdot \text{kg}^{-1}$ ]	0.459
MLVSS/MLSS	[—]	0.814
$\text{DS}_0^* \times 10^6$	[ $\text{kg} \cdot \text{kg}^{-1}$ ]	495
$\text{DOC}_0^{**} \times 10^3$	[ $\text{kg} \cdot \text{m}^{-3}$ ]	14.7
$\alpha_{av} \times 10^{-12}$	[ $\text{m} \cdot \text{kg}^{-1}$ ]	5.85

\* Dissolved solids concentration in the centrifugal supernatant

\*\* Dissolved organic carbon concentration in the centrifugal supernatant

experimental materials of various concentrations were obtained by mixing supernatant the sediment.

In the centrifugal sedimentation test, a centrifugal tube with an inner diameter of 39 mm and a depth of 85 mm was used. It was spun in a centrifuge with an arm length  $R$  of 136 mm at a rotational speed  $N$  of 1000–3500 r.p.m. for 60 min, because the thickness of the sediment became almost constant within 60 min. The equilibrium thickness of the sediment  $H_N$  was measured.

In the gravitational sedimentation test, an acrylate resin tube with an inner diameter of 59 mm and a height of 250 mm was used. The equilibrium height of the sediment  $H_G$  was measured.

The density of supernatant and dry suspended solid were measured by a pycnometer.

## 3. Results and Discussion

### 3.1 Determination of empirical constants of compression characteristics: $E$ , $\beta$ in Eq. (2)

Figure 2 presents the relation of  $(R - r_i)$  vs.  $R\Omega^2$  for the fast-frozen sample on logarithmic paper. Because the experimental data are fitted by straight lines, Eq. (5) is suitable for activated sludge. The relationships between  $(R - r_i)$  and  $R\Omega^2$  for the unfrozen sample and the slow-frozen sample were the same as that for the fast-frozen sample. The values of  $E$  and  $\beta$  are obtained from these figures and are shown in Table 3(A), together with  $\omega_0$ .

The values of  $E$  and  $\beta$  for each sample are almost constant; therefore  $C_0$  has little effect on the values of  $E$  and  $\beta$ . The average values of  $E$  and  $\beta$  are shown in Table 3(A), and are used for the following calculations of  $(p_{sN})_{av}$ .

### 3.2 Compression characteristics of each sample

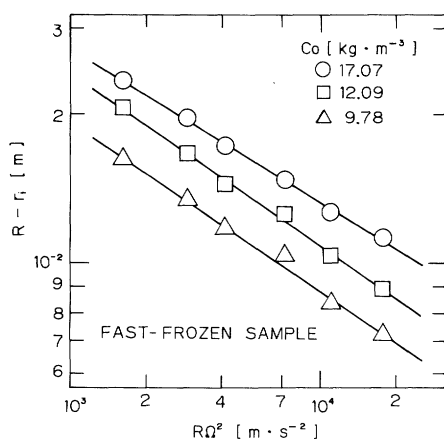
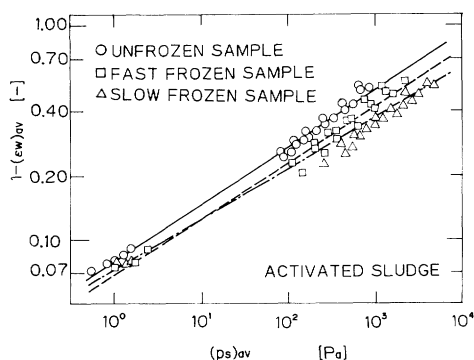
The relations between the average wet-basis porosity  $(\varepsilon_w)_{av}$  and  $(p_{sN})_{av}$  or  $(p_{sG})_{av}$  for all samples are shown together in Fig. 3. Here, the data near 1 Pa are obtained from the gravitational settling test and the data in the range  $10^2 \text{ Pa} < (p_s)_{av} < 10^4 \text{ Pa}$  are obtained from the centrifugal settling test. All experimental data for each kind of sample show good linearity over a wide range of  $(p_s)_{av}$  from 1 Pa to  $10^4 \text{ Pa}$ . These lines are also shown in Fig. 3 and the

**Table 2.** Freezing condition

	Freezing column diameter [mm]	Refrigerant	Time required for freezing [h]	Freezing rate [mm·h <sup>-1</sup> ]
Fast-frozen	137	Ethylene glycol	7.5	9.1
Slow-frozen	147	Air	39	1.9

**Table 3.** Values of  $E$  and  $\beta$ 

Unfrozen sample				Fast-frozen sample				Slow-frozen sample				
$C_0$ [kg·m <sup>-3</sup> ]	$\omega_0$ [m]	$E$ [Pa <sup>-β</sup> ]	$\beta$ [—]	$C_0$ [kg·m <sup>-3</sup> ]	$\omega_0$ [m]	$E$ [Pa <sup>-β</sup> ]	$\beta$ [—]	$C_0$ [kg·m <sup>-3</sup> ]	$\omega_0$ [m]	$E$ [Pa <sup>-β</sup> ]	$\beta$ [—]	
(A)	from centrifugal settling experiment only											
	6.63	0.00390	0.0516	0.324	17.07	0.00598	0.0497	0.308	39.67	0.01100	0.0424	0.298
	5.67	0.00314	0.0498	0.338	12.09	0.00428	0.0374	0.347	32.76	0.00914	0.0392	0.317
	4.88	0.00294	0.0560	0.326	9.78	0.00330	0.0411	0.338	20.24	0.00527	0.0362	0.328
	average		0.0525	0.329			0.0427	0.331			0.0393	0.314
(B)	from both centrifugal and gravitational settling experiments											
		0.0795	0.266			0.0677	0.264			0.0728	0.237	

**Fig. 2.** Change of  $(R - r_i)$  with  $R\Omega^2$  (Fast-rate frozen activated sludge)**Fig. 3.** Change of  $1 - (\epsilon_w)_{av}$  with  $(p_s)_{av}$ 

values of  $E$  and  $\beta$  for these lines are listed in **Table 3(B)**.

In **Table 3(B)**,  $\beta$  of slow-frozen sample is a little

lower than  $\beta$  of unfrozen sample. This means that the shape of suspended solid or floc in sludge becomes difficult to deform by the slow freezing- and -thawing process. This fact corresponds to the results that the sludge processed by freezing and thawing has larger values of density and dry solid fraction of floc than does the unfrozen sludge<sup>6,7)</sup>.

The magnitude order of  $(\epsilon_w)_{av}$  in the sediment zone is as follows.

Unfrozen sample < Fast-frozen sample  
< Slow-frozen sample

On the other hand, the order of magnitude of the average specific filtration resistance  $\alpha_{av}$ , which is obtained from the constant-pressure filtration experiments and which shows the difficulty of filtration, is as follows<sup>5)</sup>.

Unfrozen sample > Fast-frozen sample  
> Slow-frozen sample

Because increase in porosity makes filtration easier, this order is reasonable.

**Figure 4** shows the relations between the average dry-basis porosity  $(\epsilon_d)_{av}$  and  $(p_{sN})_{av}$  or  $(p_{sG})_{av}$ , based on dry solid volume. (see **Appendix B**) From **Fig. 4** and the fact that the sludge cell and floc processed by freezing and thawing has a larger value of dry solid fraction of suspended solid than the unfrozen sludge<sup>6,7)</sup>, the order of magnitude of  $(\epsilon_d)_{av}$  in the entire sediment zone is as follows.

Unfrozen sample > Fast-frozen sample  
> Slow-frozen sample

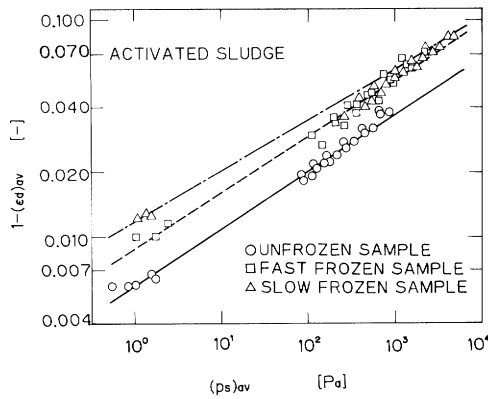


Fig. 4. Change of  $(1 - \epsilon_d)_{av}$  with  $(p_s)_{av}$

This order is equal to the order of  $\alpha_{av}^{.5}$  and this is unreasonable.

As mentioned above, it makes a wide difference in the value of  $\epsilon$  whether the basis of porosity is dry solid or wet solid. The result of Fig. 3 using wet-basis porosity is reasonable because the order of magnitude of  $(\epsilon_w)_{av}$  is consistent with that of  $\alpha_{av}$ .

Consequently, the wet-basis porosity should be used in a sedimentation or filtration process in which free water moves in the space except not occupied by wet solid.

## Conclusions

- (1) The new average solid compressive pressure  $((p_s)_{av})$  in the sediment zone was derived on the basis of the general equation for the local compression characteristics of a solid-liquid mixture. Also, the relationship between  $(p_s)_{av}$  and the average porosity  $((\epsilon)_{av})$  in the sediment zone  $(1 - (\epsilon)_{av} = E(p_s)_{av}^\beta; E$  and  $\beta$  being constants) was confirmed by centrifugal and gravitational settling experiments.
- (2) The magnitude order of average wet-basis porosity in the sediment zone was as follows.

$$\begin{aligned} \text{Unfrozen sample} &< \text{Fast-frozen sample} \\ &< \text{Slow-frozen sample} \end{aligned}$$

This order was consistent with that of the average specific filtration resistance obtained from constant-pressure filtration experiments.

## [Appendix]

A) The value of integral mean solid compressive pressure in the entire sediment  $(p_{sN})_{av}$  is defined as follows.

$$(p_{sN})_{av} = \frac{1}{(R - r_i)} \int_{r_i}^R p_s(r) dr \quad (A1)$$

The local solid compressive pressure  $p_s(r)$  at any position  $r$  is obtained by the following equation.

$$p_s(r) = (\rho_s - \rho_w) \Omega^2 \int_{r_i}^r (1 - \epsilon) r dr \quad (A2)$$

Substituting Eq. (A2) into Eq. (A1) yields Eq. (A3).

$$(p_{sN})_{av} = \frac{(\rho_s - \rho_w)}{(R - r_i)} \Omega^2 \int_{r_i}^R \left\{ \int_{r_i}^r (1 - \epsilon) r dr \right\} dr \quad (A3)$$

If  $R \gg (R - r_i)$ , the variable  $r$  in Eq. (A3) can be approximated by  $(r + r_i)/2$  or  $(R + r_i)/2$ . Thus,

$$\begin{aligned} (p_{sN})_{av} &= \frac{(\rho_s - \rho_w)}{(R - r_i)} \Omega^2 \int_{r_i}^R \frac{(r + r_i)}{2} \left\{ \int_{r_i}^r (1 - \epsilon) dr \right\} dr \\ &= (\rho_s - \rho_w) \Omega^2 \omega_0 (3R + 5r_i) / 16 \\ &\left( \because \int_{r_i}^R (1 - \epsilon) dr = \omega_0 \right) \end{aligned} \quad (13)$$

B) The average dry-basis porosity in equilibrium sediment  $(\epsilon_d)_{av}$  which has been used conventionally can be computed by Eq. (B1), which is derived from the material balance for dry solid.

$$(\epsilon_d)_{av} = 1 - C_0 H_0 / \{ (R - r_i) \rho_k \} \quad (B1)$$

The relation between  $(\epsilon_w)_{av}$  and  $(\epsilon_d)_{av}$  is written as Eq. (B2).

$$1 - (\epsilon_w)_{av} = (\rho_k / C_0) (H_\infty / H_0) (1 - (\epsilon_d)_{av}) \quad (B2)$$

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## Nomenclature

$C$	= suspended solids concentration	$[\text{kg} \cdot \text{m}^{-3}]$
$E$	= empirical constant in Eq. (2)	$[\text{Pa}^{-\beta}]$
$g$	= gravitational acceleration	$[\text{m} \cdot \text{sec}^{-2}]$
$H$	= equilibrium height of sludge in a centrifugal or a gravitational settling tube	$[\text{m}]$
$N$	= rotational speed	$[\text{min}^{-1}]$
$p_s$	= local solid compressive pressure	$[\text{Pa}]$
$(p_{sN})_{av}$	= integral mean solid compressive pressure in Eq. (13)	$[\text{Pa}]$
$p_{si}$	= solid compressive pressure below which local porosity is constant	$[\text{Pa}]$
$R$	= distance between center of centrifuge and bottom of sediment	$[\text{m}]$
$r$	= radial coordinate of cylindrical coordinate system	$[\text{m}]$
$r_i$	= distance from center of centrifuge to surface of sediment	$[\text{m}]$
$\alpha_{av}$	= average specific filtration resistance	$[\text{m} \cdot \text{kg}^{-1}]$
$\beta$	= empirical constant in Eq. (2)	$[-]$
$\epsilon$	= local porosity	$[-]$
$\epsilon_d$	= dry-basis porosity	$[-]$
$\epsilon_w$	= wet-basis porosity	$[-]$
$\rho_k$	= density of dry sludge solids	$[\text{kg} \cdot \text{m}^{-3}]$
$\rho_s$	= density of wet sludge solids	$[\text{kg} \cdot \text{m}^{-3}]$
$\rho_w$	= density of supernatant	$[\text{kg} \cdot \text{m}^{-3}]$
$\Omega$	= angular velocity	$[\text{rad} \cdot \text{s}^{-1}]$
$\omega_0$	= total solid volume in the mixture per unit sectional area	$[\text{m}]$

## <Subscripts>

$av$	= average value
$G$	= value in a gravitational settling
$N$	= value at rotational speed, $N$ , in a

centrifugal settling  
 0 = initial value  
 $\infty$  = value at rotational speed,  $\infty$

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