

ANALYSIS OF FILTRATION RATE IN CLARIFICATION FILTRATION OF POWER-LAW NON-NEWTONIAN FLUIDS-SOLIDS MIXTURES UNDER CONSTANT PRESSURE BY STOCHASTIC MODEL

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Clarification filtration experiments are conducted under constant pressure, using a filter cloth and very dilute suspensions prepared by suspending diatomaceous earth in a power-law fluid comprising aqueous sodium polyacrylate. To analyze the clarification filtration process, a stochastic model that describes particle capture in a filtration process has been coupled with the conventional blocking filtration laws. Four types of blocking filtration equations are derived, including complete blocking, intermediate blocking, and standard blocking equations. The intermediate blocking law can be applied when the particle size is larger than the pore size or is almost the same as the pore size. In contrast, the standard blocking law can be applied when the particle size is considerably smaller than the pore size. In addition, the effects of the solid concentration in the feed suspension and of the applied filtration pressure on the clarification filtration characteristics are delineated.

Introduction

In filter-medium filtration (sometimes called blocking or clarification) which is typical in the separation of fine particles from very dilute suspensions, suspended solids may be captured both on the surface of the filter medium and within the inner pore passages. Over the past fifty years, there has emerged a body of studies on the mathematical modelling of clarification filtration of Newtonian fluids-solids mixtures.^{1-5,7,8,17)} More recently, clarification filtration of non-Newtonian fluids-solids mixtures has become increasingly important in such diverse fields as petrochemical and food processing industries, and there has been growing interest in information pertinent to the field. In spite of the very substantial progress in the theory of non-Newtonian cake filtration in the past several decades,^{10-12,14-16,19,21,22)} there are few developments in the theory dealing with non-Newtonian clarification filtration.^{6,18,20)}

In the previous paper,²⁰⁾ generalized blocking filtration equations applicable to both non-Newtonian and Newtonian fluids were presented by using the power-law model for flow of non-Newtonian fluids on the basis of blocking filtration laws presented by Hermans and Bredée⁵⁾ and Grace.⁴⁾

In the present work, these generalized blocking filtration equations are reexamined by viewing the clarification filtration process as a stochastic process,^{1-3,7)} derived through probability considerations.

The theory is tested rigorously by conducting clarification filtration experiments with a very dilute suspension prepared by suspending diatomaceous earth in non-Newtonian fluids and with woven cloth as the filter medium.

1. Experimental Apparatus and Procedure

A schematic diagram of the experimental apparatus is shown in Fig. 1. All experiments are performed in a filter with a filtration area of 12.6 cm². After the space from the outlet of the filtrate to the filter medium is filled with non-Newtonian fluids, non-Newtonian fluids-solids mixtures are poured into the filter and the pressurized vessel. Clarification filtration experiments are carried out by applying air pressures of 49 to 196 kPa, and the time variations of both the filtrate

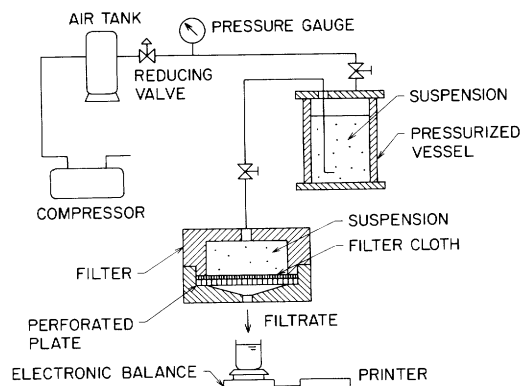


Fig. 1. Schematic diagram of experimental apparatus

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volume and the solid concentration of the filtrate are measured. The solid concentration of the filtrate is measured spectrophotometrically at 400 nm by using a specific calibration curve.

The characteristics of the filter medium and suspended solid particles are summarized in **Table 1**. Woven cloths (Izumi) of three mean pore sizes are used as the filter medium. The values of the mean pore size shown in the table are determined by the mercury-intrusion method. The suspended particles used are Radiolite (Showa Chemical Ind.), a typical diatomaceous filter aid, of various mean particle sizes. The values of mean particle size shown in the table are calculated from Feret's diameter determined using an image analyzer, QTM720 (Cambridge Instruments). Non-Newtonian fluids used in this work are 0.15 wt% sodium polyacrylate-deionized water solutions, which can be treated as typical power-law fluids over the whole range of shear rates that may be encountered in these experiments. The value of the fluid behavior index N is about 0.36. The solid particles are added to the polymer solutions with solid concentrations varying over a wide range (5×10^{-6} – 4×10^{-4} by weight).

2. Theoretical Analysis

2.1 Variations of number of trapped particles

The filtration process of very dilute suspensions is conveniently divided into two sequential stages: clarification filtration and cake filtration. In the initial stage the solid particles are captured both on the surface of the filter medium and within the inner pore passages.

In deriving the filtration equations in the clarification filtration process, it is assumed that the filter medium consists of parallel pores of constant diameter and length. The variations in number of particles captured by the filter medium tend to be stochastic in nature. Consequently, stochastic modelling has been found to be an effective tool for describing particle capture in a clarification filtration process.^{1–3,7)}

Not the number of particles sealing the pore or accumulating within the interstices of the medium, $X(\theta)$, at time θ but rather $X(v)$ at filtrate volume v per unit medium area should be taken as the random variable, because the number of particles approaching the filter medium is proportional to v in clarification filtration of very dilute suspensions. The conditional probability that a birth event will occur, i.e., that a particle will be trapped during the small filtrate volume interval $(v, v+h)$ is given by

$$\Pr\{X(v+h)-X(v)=1 \mid X(v)=n\} = \lambda_n h + o(h) \quad (1)$$

where λ_n is the intensity of transition, which is a function of n , and $o(h)$ signifies that

Table 1. Characteristics of filter medium and suspended particles

Filter medium:

Filter cloth	Material	Mean pore size [μm]	Thickness [μm]	Weave
TES5026	Tetoron	5.8	0.75	1/1 plain
TS401	Tetoron	15.8	1.35	2/2 twill
PF3036F	Pylon	21.5	1.00	2/2 twill

Suspended particles:

Radiolite	Mean particle size [μm]
FNF-B	5.9
#200	7.3
#600	10.6

$$\lim_{h \rightarrow 0} (o(h)/h) = 0.$$

Obviously the probability of no change in this volume interval is given by

$$\Pr\{X(v+h)-X(v)=0 \mid X(v)=n\} = 1 - \lambda_n h + o(h) \quad (2)$$

The probability that exactly n particles are trapped at the moment v will be denoted as $P_n(v)$ ($\equiv \Pr\{X(v)=n\}$). For two successive filtrate volume intervals of $(0, v)$ and $(v, v+h)$, the occurrence of exactly n particles being trapped during the interval $(0, v+h)$ can be realized in the following mutually exclusive ways.

1. All n particles are trapped in $(0, v)$, and none in $(v, v+h)$.
2. Exactly $(n-1)$ particles are trapped in $(0, v)$, and one particle is trapped in $(v, v+h)$.
3. Exactly $(n-j)$ particles are trapped in $(0, v)$, and j particles are trapped in $(v, v+h)$, where $2 \leq j \leq n$.

Considering all these probabilities and combining all quantities of order $o(h)$ on the basis of Eqs. (1) and (2), one obtains

$$\begin{aligned} P_n(v+h) &= \sum_{k=0}^{\infty} P_k(v) \Pr\{X(v+h)-X(v) \\ &= n-k \mid X(v)=k\} \\ &= P_n(v) \{1 - \lambda_n h + o(h)\} \\ &\quad + P_{n-1}(v) \{\lambda_{n-1} h + o(h)\} \\ &\quad + \sum_{k=0}^{n-2} P_k(v) o(h) \quad n \geq 1 \Bigg\} \\ P_0(v+h) &= P_0(v) \{1 - \lambda_0 h + o(h)\} \end{aligned} \quad (3)$$

Rearranging these equations and taking the limit as $h \rightarrow 0$ result in

$$\begin{aligned} P'_n(v) &= -\lambda_n P_n(v) + \lambda_{n-1} P_{n-1}(v) \quad n \geq 1 \\ P'_0(v) &= -\lambda_0 P_0(v) \end{aligned} \quad (4)$$

where $P'_n(v)$ is the derivative of $P_n(v)$.

If the rate of particle capture, i.e., the intensity of transition, λ_n , is constant (will be denoted as λ), the mean number or the expected value of trapped particles at a given filtrate volume v , namely,

$$E[X(v)] \left(\equiv \sum_{n=0}^{\infty} n P_n(v) \right),$$

is given by the following equation on the basis of Eq. (4).⁹⁾

$$E[X(v)] = \lambda v \quad (5)$$

If the rate of particle capture is proportional to the number of particles susceptible to capture, λ_n is expressed as $\lambda_n = \eta(N' - n)$, where N' is the total number of particles susceptible to capture at the moment $v = 0$, and η is a proportionality constant. Hence $E[X(v)]$ is given by the following equation on the basis of Eq. (4).⁷⁾

$$E[X(v)] = N' \{1 - \exp(-\eta v)\} \quad (6)$$

2.2 Blocking filtration equations

To utilize the results obtained above, it is necessary to relate the expected value of trapped particles to filtration rate.

For power-law non-Newtonian fluids, the rheological equation is given by

$$\tau = K \dot{\gamma}^N \quad (7)$$

where τ is the shear stress, K the fluid consistency index, and $\dot{\gamma}$ is the shear rate. On the basis of this model, the average velocity u for laminar flow through a straight circular tube with radius r can be written as²⁰⁾

$$u = \frac{N}{3N+1} \left(\frac{p}{2KL} \right)^{1/N} r^{(N+1)/N} \quad (8)$$

where p is the pressure drop, and L is the tube length.

When the diameter of the suspended solids is large relative to that of the pore, it is assumed that a particle reaching an open pore in a filter medium of parallel pores of constant diameter and length seals the pore completely.^{4,5,13,20)} When λ_n equals λ , the mean number of pores which are not blocked at a given filtrate volume v equals $N' - \lambda v$ according to Eq. (5). Thus, on the basis of Eq. (8), an expression that relates the filtration rate $q (\equiv dv/d\theta)$ to v under constant-pressure conditions can be obtained by²⁰⁾

$$q = q_0 (1 - \lambda v / N') \quad (9)$$

where q_0 is the filtration rate at the beginning of the filtration process. When $\lambda_n = \eta(N' - n)$, the mean number of pores which are not clogged at a given filtrate volume v equals $N' \exp(-\eta v)$ on the basis of Eq. (6). Accordingly, with the use of Eq. (8) the filtration rate q is given by

$$q = q_0 \exp(-\eta v) \quad (10)$$

When the diameter of the suspended solids is much smaller than that of the pore, it is assumed that pore volume decreases proportionally to the volume of the particles deposited on the pore walls.^{4,5,13,20)} In the case that $\lambda_n = \lambda$, referring to Eqs. (5) and (8), the filtration rate q is given by

$$\left. \begin{aligned} q^{2N/(3N+1)} &= q_0^{2N/(3N+1)} \left(1 - \frac{K_s}{2} v \right) \\ K_s &= \frac{d_p^3 \lambda}{3N' L (1 - \varepsilon_p) r_0^2} \end{aligned} \right\} \quad (11)$$

where d_p is the diameter of the particle, ε_p the packing porosity of the particles deposited on the pore walls, and r_0 is the effective pore radius at the commencement of filtration. When $\lambda_n = \eta(N' - n)$, by using Eqs. (6) and (8) one obtains the following expression:

$$\left. \begin{aligned} q^{2N/(3N+1)} &= q_0^{2N/(3N+1)} [1 - a \{1 - \exp(-\eta v)\}] \\ a &= \frac{d_p^3}{6L(1 - \varepsilon_p)r_0^2} \end{aligned} \right\} \quad (12)$$

Equations (9)–(11) correspond to the complete blocking law, the intermediate blocking law, which has been developed empirically and the standard blocking law,^{4–6,18,20)} respectively. Equation (12) does not correspond to any conventional blocking law.

In general there are two sequential stages of filtration: clarification filtration and cake filtration. When $d_p > d_m$ or $d_p \simeq d_m$ in clarification filtration, a particle reaching an open pore seals the pore completely. Thus, the filtration rate is given by Eq. (9) or Eq. (10) depending on the form of the function of λ_n . When $d_p \ll d_m$, pore volume decreases gradually by particle deposit on the pore walls. In this case, Eq. (11) or Eq. (12) can be applied to describe the filtration rate depending on the form of the function of λ_n . In all cases, the filtration rate during the process of cake filtration is given by^{16,19)}

$$\left. \begin{aligned} (1/q)^N &= K_c v + (1/q_0)^N \\ K_c &= \frac{K \gamma_{av} \rho s}{p(1 - ms)} \end{aligned} \right\} \quad (13)$$

where γ_{av} is the average specific filtration resistance for power-law fluids, ρ the density of the liquid, and m is the ratio of wet to dry cake mass.

3. Experimental Results and Discussion

The experimental results of clarification filtration of a suspension containing particles which are quite small compared with the pores are plotted as $q^{2N/(3N+1)}$ and $(1/q)^N$ versus v with the suspension concentration s as the parameter in Fig. 2. As shown by Eq. (11)

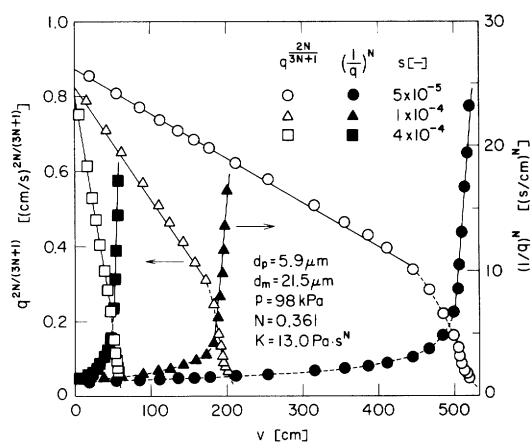


Fig. 2. Effects of suspension concentration on filtration rate (the case of particle size smaller than pore size)

$q^{2N/(3N+1)}$ versus v yields straight lines for the first step of filtration. As the solid concentration s decreases, the slope decreases and the clarification filtration process continues for a long time. In this case, a large quantity of particles bleed through the pores. As soon as a filter cake composed of deposited particles forms at the top surface of the filter medium, the value of $(1/q)^N$ increases precipitously and $(1/q)^N$ versus v yields straight lines in accordance with Eq. (13).

In Fig. 3, one adjustable parameter K_s present in Eq. (11) calculated on the basis of the results of Fig. 2 is plotted against s . The line is straight and goes through the origin. The intensity of the transition, λ , is proportional to s because K_s is in proportion to λ in Eq. (11).

Data of the filtration rate of a suspension containing particles with almost the same size as that of the pore are illustrated in the manner of Eqs. (10) and (13) for various solid concentrations in Fig. 4. For the first step of filtration the observed linearity of semilogarithmic plots of q versus v agrees with Eq. (10). As s decreases, the slope decreases and the period of clarification filtration becomes long. The concentration ratio of effluent to influent is much smaller than that in the experiments referred to in Fig. 2. With the onset of cake formation, $(1/q)^N$ versus v yields straight lines according to Eq. (13).

In Fig. 5, η calculated from the results of Fig. 4 is plotted against s . The plot shows the linear relationship through the origin.

The experimental results of the 5.9- μm particles for filter cloths of different pore sizes are presented in Fig. 6. In the case of $d_m = 5.8 \mu\text{m}$, which is almost the same as the particle size, semi-log plots of q versus v show the linear relationship in accordance with Eq. (10) for the period of clarification filtration. Conversely, when $d_m = 15.8$ or $21.5 \mu\text{m}$, which is considerably larger than the particle size, $q^{2N/(3N+1)}$ is directly proportional to v in accord with Eq. (11).

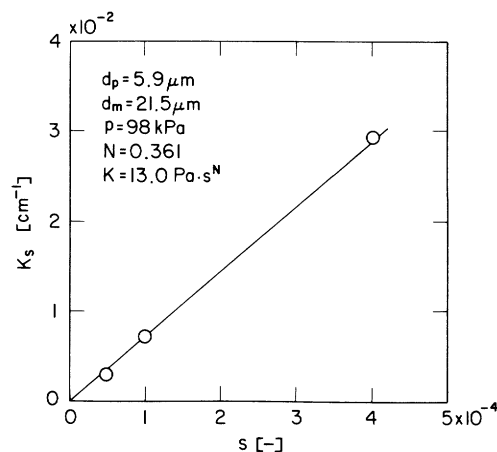


Fig. 3. Relation between K_s and s

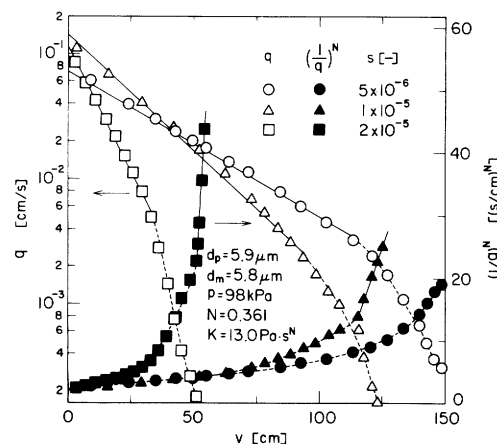


Fig. 4. Effects of suspension concentration on filtration rate (the case of almost the same particle size as pore size)

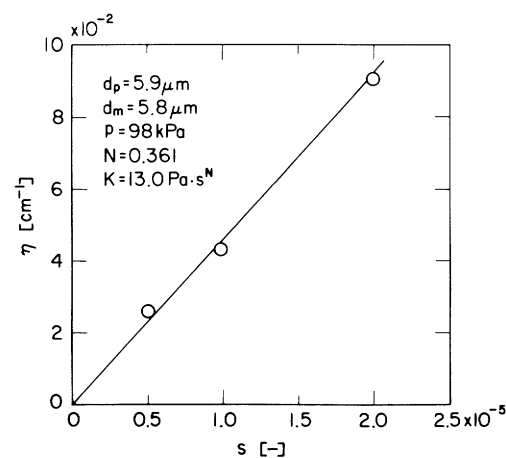


Fig. 5. Relation between η and s

The results of the experiments conducted at different average particle sizes and the filter cloth with the pore size of 5.8 μm are shown in Fig. 7. According to Eq. (10) semi-log plots of q versus v show the linear relationship for the period of clarification filtration since d_p is about the same as d_m or is larger than d_m . The larger the particle size the shorter the period of

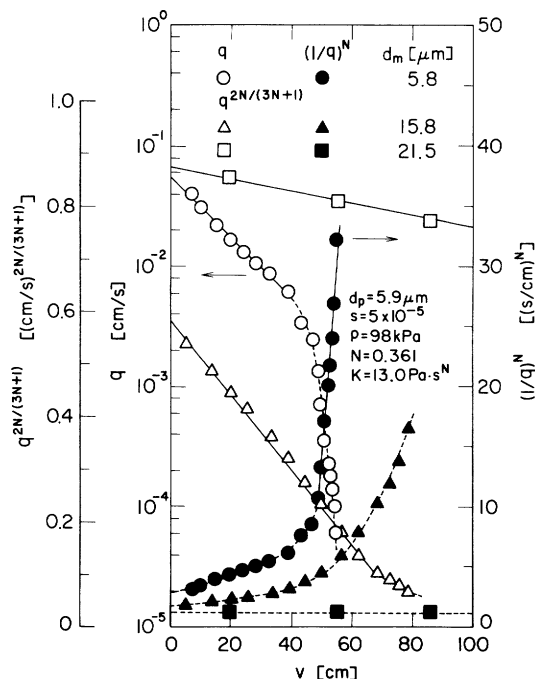


Fig. 6. Effects of pore size on filtration rate

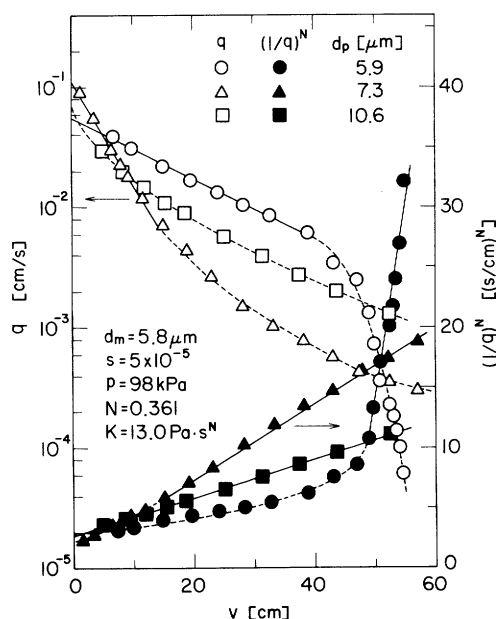


Fig. 7. Effects of particle size on filtration rate

clarification filtration. $(1/q)^N$ versus v shows the linear relationship in accordance with Eq. (13) as deposits grow. In the case where $d_p = 10.6 \mu\text{m}$, the plots agree with Eq. (13) from the start of the filtration process.

The experimental results for various filtration pressures are described in Fig. 8. The particle size d_p of the suspended solid is $5.9 \mu\text{m}$, and the pore size d_m of the filter cloth is $5.8 \mu\text{m}$. In the period of clarification filtration, semi-log plots of q versus v show the linear relationship in accordance with Eq. (10). It is apparent that the period of clarification filtration becomes long with increase in pressure. This is because the ratio of

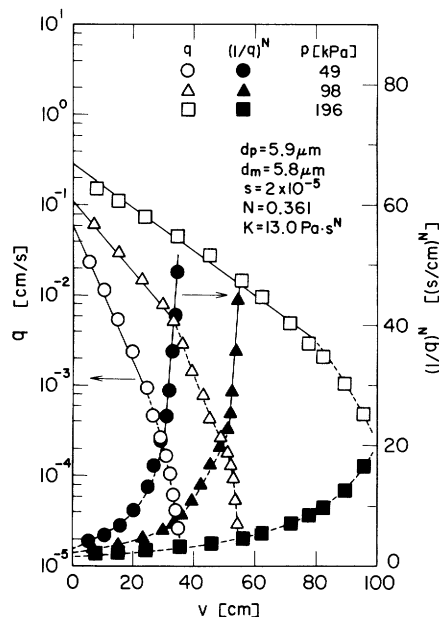


Fig. 8. Effects of applied filtration pressure on filtration rate

effluent to influent solid concentrations increases under the action of hydrodynamic drag forces due to the flow of filtrate through the medium, which increases with increase in pressure.

Conclusion

Clarification filtration characteristics under constant-pressure conditions were examined for power-law non-Newtonian fluids-solids mixtures. Four types of blocking filtration equations including complete blocking, intermediate blocking, and standard blocking equations were derived through the stochastic approach in which the number of trapped particles, $X(v)$, at filtrate volume v can be taken as the random variable. These blocking filtration equations are generalized ones which can be applied to both Newtonian and non-Newtonian fluids. Dilute suspensions prepared by suspending diatomaceous earth in non-Newtonian fluids were filtered by use of filter cloth to test the validity of the theory and to ascertain the principal modes of particle capture. Filtration results conformed to the intermediate blocking law when the particle size was larger than the pore size, or was almost the same as the particle size. On the other hand, the results conformed to the standard blocking law when the particle size was considerably smaller than the pore size. It was also found that the solid concentration of the feed suspension and the applied filtration pressure markedly affect the clarification filtration characteristics.

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Nomenclature

a	= constant defined in Eq. (12)	[—]
c	= mass fraction of solids in filtrate	[—]
d_m	= average pore size	[m]
d_p	= average particle size	[m]
$E[X(v)]$	= mean value of random variable $X(v)$	[—]
h	= infinitesimal filtrate volume per unit medium area	[m]
K	= fluid consistency index	[Pa · s ^N]
K_c	= plugging constant of cake filtration law	[s ^N /m ^{1+N}]
K_s	= plugging constant of standard blocking law	[m ⁻¹]
L	= thickness of filter medium	[m]
m	= ratio of wet to dry cake mass	[—]
N	= flow behavior index	[—]
N'	= total number of particles susceptible to capture	[—]
n	= number of trapped particles	[—]
$P_n(v)$	= probability that exactly n particles are trapped at filtrate volume v	[—]
$P'_n(v)$	= derivative of $P_n(v)$ ($= dP_n(v)/dv$)	[m ⁻¹]
p	= applied pressure	[Pa]
q	= filtration rate at any time θ	[m/s]
q_0	= filtration rate at start of filtration	[m/s]
r	= effective pore radius at any time θ	[m]
r_0	= effective pore radius at start of filtration	[m]
s	= mass fraction of solids in suspension	[—]
u	= average velocity of laminar flow of power-law fluids through straight circular tube	[m/s]
v	= filtrate volume per unit medium area	[m]
$X(v)$	= random variable which describes number of trapped particles at filtrate volume v	[—]
γ_{av}	= average specific filtration resistance for power-law fluid	[m ^{2-N} /kg]
$\dot{\gamma}$	= shear rate	[s ⁻¹]
ε_p	= packing porosity of particles deposited on pore walls	[—]
η	= constant defined by $\lambda_n = \eta(N' - n)$	[m ⁻¹]
θ	= filtration time	[s]
λ	= constant defined by $\lambda_n = \lambda$	[m ⁻¹]
λ_n	= intensity of birth transition	[m ⁻¹]
$o(h)$	= any function of h such that $\lim_{h \rightarrow 0} (o(h)/h) = 0$	[—]

ρ	= density of liquid	[kg/m ³]
τ	= shear stress	[Pa]

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