HEAT TRANSFER COEFFICIENT AND MIXING TIME IN A CYLINDRICAL MIXING VESSEL WITH AXIAL JET NOZZLES

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Heat transfer coefficient and mixing time were measured in an axial-jet mixing vessel equipped with a single or combined nozzle of various diameters.

The equivalent nozzle diameter d_e for such a nozzle was defined as

$$d_e=\{(1/2)(4/\pi)^2(\rho Q^3/P)\}^{1/4}$$

where Q and P are total jet flow rate and total power input, respectively. The equivalent nozzle diameter was usable for the correlation of heat transfer coefficient and mixing time for the combined jet nozzle.

The heat transfer coefficient measured was well correlated with nozzle Reynolds number $Re_n(\equiv u_e d_e \rho/\mu)$ by using the equivalent nozzle diameter, as

$$\frac{hD}{\kappa}=0.27\left(\frac{u_e d_e \rho}{\mu}\right)^{3/4} Pr^{0.4}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

$$4.2\times10^3<Re_n<5.5\times10^4$$

The mixing time measured was well correlated with the nozzle Reynolds number, based on the correlation method of Hiby *et al.* The minimum dimensionless mixing time was obtained at $Re_n=1.0\times10^4\sim1.3\times10^4$, and its value satisfied the correlation equation proposed in our previous paper.

1. Introduction

Jet mixing is well-known as an alternative to mechanical agitation in liquid mixing[3,4,6,11,15]. Mixing time in a conventional jet mixing vessel has been measured by many investigators[3,4,7,8,10,12,14]. However, there had been no attempt to propose a geometrical design for jet mixing in a chemical process until 1978, when Hiby and Modigell[5] proposed the axial-jet mixing vessel, since Fossett and Prosser[3]

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proposed the inclined side-entry jet mixing vessel in 1949. On the other hand, axial jet mixing in space technology was investigated in 1972 by Poth and Van Hook[13] and in 1976 by Aydelott[1], although they used the axial jet to produce only a circulation flow of liquid. They investigated the method of effective cooling of the cryogenic propellant tank in which pressure increases due to various heat inputs during missions in space and examined the effects of the volume of liquid in the tank, the position of the outlet of the liquid jet and the liquid-jet velocity on the resulting liquid flow pattern in a spherical container in weightlessness.

Combined jet nozzles are usable for reaction/mixing of multiple fluids because each fluid can be fed to the reaction/mixing vessel by a specified jet nozzle. However, few investigations have been reported for the jet mixing vessel with combined jet nozzles.

In the previous papers[8,9] the heat transfer coefficient in the jet mixing vessel with a nozzle that rotates around the vessel axis was correlated in terms of Prandtl number and power input per unit volume[9]:

$$\frac{h}{\rho c_p}\left(\frac{\mu_w}{\mu}\right)^{0.14}=K\left(\frac{P_v\mu}{\rho^2}\right)^{1/4}Pr^{-0.6} \qquad (1)$$

The mixing time in the jet mixing vessel was expressed by the following equation[8]:

$$t_M=6.4\frac{D^{1.5}H^{0.5}}{\sqrt{n\,ud_n}} \qquad (2)$$

This paper deals with measurement of the heat transfer coefficient and the mixing time in a cylindrical mixing vessel equipped with a single or combined axial-jet nozzle of various diameters, and evaluates the effects of the jet flow rate, viscosity of liquid and combination of jet nozzles of different diameters on both the heat transfer coefficient and mixing time.

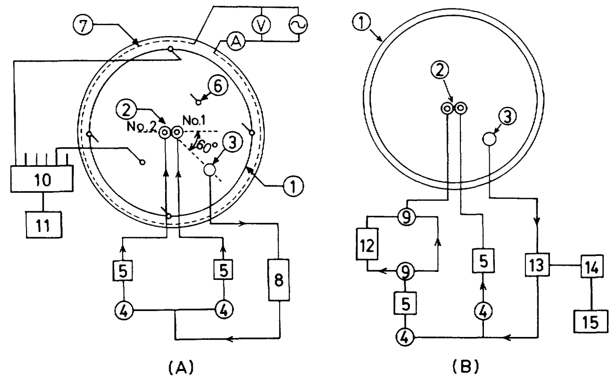

Fig. 1. Schematic diagram of experimental apparatus; (A) heat transfer experiments, (B) mixing experiments

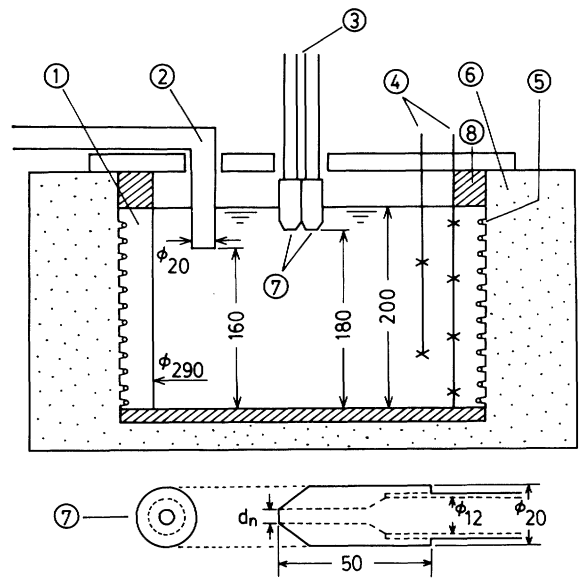

Fig. 2. Details of mixing vessel for heat transfer experiments

2. Experimental

Schematic diagrams of the experimental apparatuses and details of the mixing vessel are shown in **Figs. 1** and **2**, respectively. The aluminium cylindrical vessel for heat transfer experiments (Figs. 1(A) and 2) is 29 cm in inner diameter (I.D.) and 20 cm in height. The vessel height is increased by 4 cm with an acrylic resin and Teflon cylinder to prevent heat conduction to the top plate. The outer surface of the aluminium vessel is spiralled with grooves of 3.4 mm in width, 5.0 mm in depth and a 7.0 mm pitch for insertion of four electric heaters, each 3.2 mm in diameter and 6.8 mm in length, as a heat source. The outside of the vessel is insulated with glass wool, 30 cm in thickness.

To measure the temperature at the vessel wall, four thermocouples are placed in grooves, 0.5 mm deep by 1.4 mm wide, on the inner surface of the vessel. The tips of the thermocouples are at 3, 7, 12 and 17 cm vertically from the vessel top and at 90° intervals in the tangential direction. To measure the bulk liquid temperature, two thermocouples are set at depths of 7 and 14 cm and a radius of 10 cm.

A single or combined jet nozzle is set axially at 18 cm above the bottom of the mixing vessel. The suction pipe, 20 mm I.D., is set at a radius of 9.5 cm and is 16 cm above the bottom. For the combined jet nozzle the suction pipe is set at 60° in the tangential direction from the line drawn through the centers of the two jet nozzles, as shown in Fig. 1(A). The jet nozzle is connected to the suction pipe by a polyvinyl chloride tube via a liquid cooler, centrifugal pumps and rotameters.

The flow rate from each jet nozzle is controlled individually after liquid passes through the cooler. The combinations of jet nozzle diameters and the flow rate are shown in **Table 1**. No. 1 of the nozzle in Fig. 1(A) refers to the nozzle set near the suction pipe. The liquids used are deionized water and 40 *wt*% glycerine aqueous solution, which are poured into the vessel to a liquid height of 20 cm.

The heat transfer coefficient at the vessel wall, h, is calculated from the heat transfer rate Q_H and the temperature difference between vessel wall and bulk liquid, $T_w - T_b$, as

$$h = Q_H / \{A(T_w - T_b)\} \tag{3}$$

where A is the heat transfer surface area, and Q_H is calculated from measured voltage V and current I, as $Q_H = IV$. The vessel wall and bulk liquid temperatures, T_w and T_b, are obtained as the arithmetic mean values of the temperatures measured with four and two thermocouples, respectively. The values of I and V were read to three figures with an analog ampere meter and a voltage meter, respectively. The observed error in Q_H was about 0.4%. Heat loss was negligible because the heat loss through the insulator was only about 0.5% of input heat.

The dimensions of the acrylic resin cylindrical vessel for mixing experiments and the nozzle arrangement (Fig. 1(B)) are the same as those for heat transfer experiments. For observation of mixing progress the tracer technique is used, by additionally connecting the tracer filling line to the jet circulation system and setting an electric conductivity cell upstream from the tracer filling line. The liquid used is city water, poured into the vessel to a liquid height of 20 cm. A sodium chloride solution as tracer is injected through the tracer filling line into the vessel as a very short impulse by using three-way valves. From that moment the fluctuation of electric conductivity of recirculated liquid is observed with a conductivity meter and a mV recorder. The mixing time is obtained as the time in which the fluctuation of the electric conductivity decays less than 1% of the converged electric conductivity. The combinations of jet nozzle diameters and the flow rate are shown in **Table 2**. Details of the measurements of heat transfer coefficient and mixing time are reported in previous papers[8,9].

Table 1. Experimental ranges of nozzle diameters and jet flow rate for heat transfer

nozzle dia. [mm] d_{n1}-d_{n2}	jet flow rate [l/min] Q_1-Q_2		equiv. nozzle dia. [m] d_e
	water		
12–10	1.0– 8.0	2.0–6.0	0.0124–0.0156
12– 7	4.0– 8.0	3.0–6.0	0.0102–0.0137
12– 5	2.0– 8.0	1.0–5.8	0.0062–0.0130
12– 3	1.0– 9.0	1.0–3.0	0.0037–0.0120
10– 5	2.0– 8.0	1.0–6.0	0.0062–0.0111
10– 3	2.0– 8.0	1.0–3.0	0.0044–0.0103
12	3.0–10.0		
10	3.0–10.0		
7	3.0–10.0		
5	3.0– 9.0		
3	1.6– 3.6		
	40 wt% glycerine		
12–10	1.7– 5.7	2.8–6.6	0.0118–0.0156
12– 7	0.9– 5.7	1.9–7.6	0.0076–0.0139
12– 5	0.9– 5.7	0.9–6.6	0.0055–0.0130
12– 3	1.9– 5.7	0.9–2.8	0.0044–0.0111
10– 5	0.9– 5.7	1.9–5.7	0.0044–0.0100
10– 3	1.9– 5.7	0.9–2.8	0.0056–0.0110

Table 2. Experimental ranges of nozzle diameters and jet flow rate for mixing time

nozzle dia. [mm] d_{n1}-d_{n2}	jet flow rate water [l/min] Q_1-Q_2		equiv. nozzle dia. [m] d_e
12–10	3.0–8.6	3.0–8.4	0.0125–0.0156
12– 7	3.0–8.6	4.0–8.0	0.0096–0.0137
12– 3	3.0–8.6	1.4–3.4	0.0061–0.0114
12	3.0–8.6		
10	3.0–8.4		
7	2.0–8.0		
5	3.0–6.6		
3	1.4–3.4		

3. Results and Discussion

3.1 Definition of equivalent nozzle diameter

As shown in the previous paper[8], the mixing time in the jet mixing vessel was well correlated with Eq. (2) by introducing as a correction factor the square root of the number of nozzles; the equivalent nozzle diameter d_e for a combination of n jet nozzles of diameter d_n was expressed as

$$d_e = \sqrt{n}\, d_n \tag{4}$$

The general definition of the equivalent nozzle diameter for the combination of n jet nozzles of different diameters is expressed as[8]

$$d_e = \left\{ \frac{1}{2} \left(\frac{4}{\pi} \right)^2 \left(\frac{\rho Q^3}{P} \right) \right\}^{1/4} \tag{5}$$

where Q and P are the total jet flow rate and total power input, respectively, and they are expressed as

$$Q = Q_1 + Q_2 + \cdots + Q_n \qquad \left(Q_i = \frac{\pi}{4} d_{ni}^2 u_i \right) \tag{6}$$

$$P = P_1 + P_2 + \cdots + P_n \qquad \left(P_i = \frac{1}{2}\rho u_i^2 Q_i\right) \tag{7}$$

where Q_i and P_i are the jet flow rate and power input from the i-th jet nozzle, which has the diameter d_{ni}.

From the definition of d_e in Eq. (5), the equivalent jet velocity u_e can be defined as

$$u_e = \frac{4Q}{\pi d_e^2} \tag{8}$$

If $d_{n1} = d_{n2} = \cdots = d_{nn}$ and $Q_1 = Q_2 = \cdots = Q_n$, Eqs. (5) and (8) are reduced respectively to

$$\left.\begin{aligned} d_e &= \sqrt{n}\, d_{n1} \\ u_e &= u_{n1} \end{aligned}\right\} \tag{9}$$

The expression of the equivalent nozzle diameter in Eq. (9) is identical with Eq. (4). The variation of d_e in the present experiments is shown in Tables 1 and 2.

3.2. Correlation of heat transfer coefficient

Figure 3 shows the correlation of the heat transfer coefficient with the power input per unit volume, based on the correlation form of Eq. (1), where the power input is almost equal to the power consumption in the present experiments. The coefficient K in Eq. (1) is correlated well by using the equivalent nozzle diameter d_e regardless of the viscosity of liquid or the combination of nozzles of different diameters. In both a single and a combined axial jet nozzle of different diameters the coefficient K increases linearly with a slope of 1/4 on a log-log graph as d_e increases:

$$K = 0.39 d_e^{1/4} \tag{10}$$

Then, for the correlation of the heat transfer coefficient the combination of Eq. (1) with Eq. (10) gives the following equation with the help of Eqs. (5) to (8). (see **Appendix**)

$$\frac{hD}{\kappa} = K'\left(\frac{u_e d_e \rho}{\mu}\right)^{3/4} Pr^{0.4}\left(\frac{\mu}{\mu_w}\right)^{0.14} \tag{11}$$

where

$$K' = 0.33(D^2/H)^{1/4} \tag{12}$$

Figure 4 shows the correlation of the heat transfer coefficient with the nozzle Reynolds number Re_n ($\equiv u_e d_e \rho/\mu$) based on Eq. (11). As the values of D and H are 0.29 m and 0.20 m respectively in the present experiments, the coefficient K' in Eq. (12) is calculated as 0.27 from Eq. (12).

The solid line in Fig. 4 shows Eq. (11) at $K' = 0.27$, where the heat transfer coefficient in the cylindrical mixing vessel with axial jet nozzles is confirmed to correlate well with the equivalent nozzle diameter.

3.3 Correlation of mixing time

In the previous paper[8] it was shown that the mixing time in the jet mixing vessel with a rotating nozzle

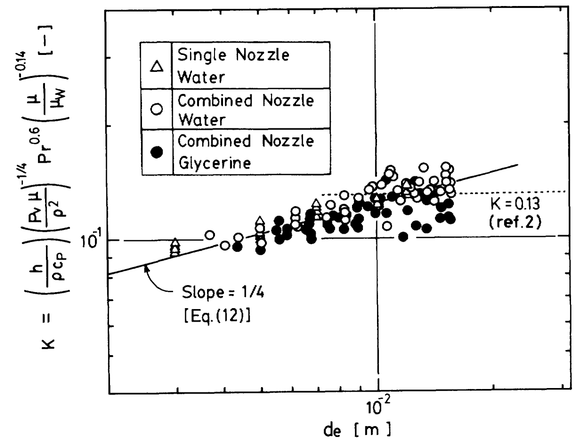

Fig. 3. Coefficient K in Eq. (1) with respect to equivalent nozzle diameter

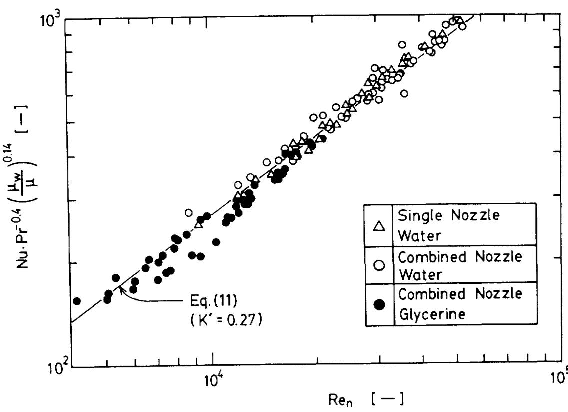

Fig. 4. Heat transfer coefficient with respect to nozzle Reynolds number

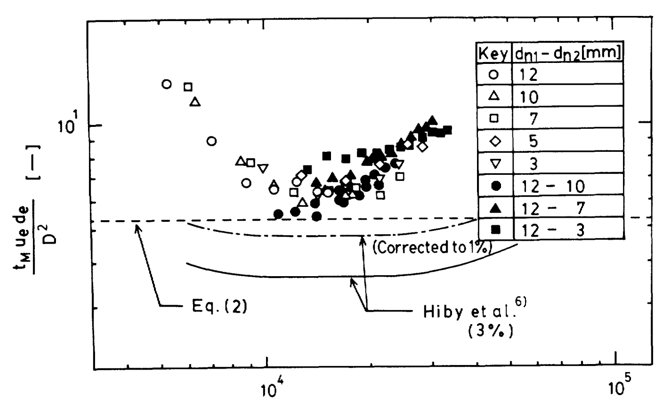

Fig. 5. Mixing time with respect to nozzle Reynolds number

was expressed with Eq. (2). **Figure 5** shows the correlation between the dimensionless mixing time and the nozzle Reynolds number Re_n, by using the equivalent nozzle diameter d_e and velocity u_e, in the same way as by Hiby *et al.*[6]. The dimensionless mixing time is well correlated with the nozzle Reynolds number regardless of nozzle diameter and combination of nozzles. Hiby *et al.*[6] showed there is about 30% difference between mixing times measured with 1% and 3% relative deviations from final concentration. The mixing time in the present experiments, however, does not agree with the corrected 3% relative deviation data of mixing time of Hiby *et al.*[6] as 1% relative deviation, though the fact that the dimensionless mixing time takes a minimum value is similar to that reported by them. The minimum dimensionless mixing time appears at $Re_n = 1.0 \times 10^4$–1.3×10^4. This behavior can be estimated from that of the dimensionless mixing time in a non-baffled agitated vessel; i.e., the dimensionless mixing time increases slightly with increasing Reynolds number in the fully turbulent region, because the turbulent diffusion coefficient is not proportional to the Reynolds number[16]. Hiby *et al.*[6] explained that the increase in dimensionless mixing time below $Re_n = 1.0 \times 10^4$ was due to the decrease in turbulence, while the increase at higher Re_n was caused by surface waves which slow down circulation.

The broken line in Fig. 5 shows Eq. (2), where the liquid height H in Eq. (2) is replaced by the vessel diameter D with the correction of $(H/D)^{0.5} = 0.83$. The minimum mixing time in the present experiments roughly satisfies Eq. (2).

Conclusion

The conception of the equivalent nozzle diameter for a combined nozzle of different diameters was usable for the correlation of heat transfer coefficient and mixing time in a cylindrical jet mixing vessel equipped with two axial jets.

The heat transfer coefficient observed was well correlated with the nozzle Reynolds number Re_n by using the equivalent nozzle diameter and jet velocity.

The mixing time observed was also well correlated with the nozzle Reynolds number, based on the correlation method by Hiby *et al.*, where the fact that the dimensionless mixing time took a minimum value was similar to their findings and to behavior in a non-battled agitated vessel. The minimum dimensionless mixing time appeared at $Re_n = 1.0 \times 10^4$–1.3×10^4, and its value roughly satisfied the correlation equation of the mixing time in the jet mixing vessel with a rotating nozzle proposed in our previous paper.

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Appendix Derivation of Equation (11)

Introducing Eq. (10) into Eq. (1), we get the expression

$$\frac{h}{\rho c_p} = 0.33\left\{\left(\frac{\rho u_e^3 d_e^2}{D^2 H}\right)\frac{\mu}{\rho^2}\right\}^{1/4} Pr^{-0.6}\left(\frac{\mu}{\mu_w}\right)^{0.14} \qquad \text{(A-1)}$$

because

$$P_v = \left(\frac{1}{2}\rho u_e^2\right)\left(\frac{\pi}{4}d_e^2 u_e\right)\Bigg/\left(\frac{\pi}{4}D^2 H\right) = \frac{\rho u_e^3 d_e^2}{2D^2 H} \qquad \text{(A-2)}$$

from the definitions of d_e and u_e in Eqs. (5) and (8).

Multiplying both side of Eq. (A-1) by Pr ($\equiv c_p\mu/\kappa$), Eq. (A-1) is rearranged to

$$\frac{hD}{\kappa} = 0.33\left(\frac{D^2}{H}\right)^{1/4}\left(\frac{\rho u_e d_e}{\mu}\right)^{3/4} Pr^{0.4}\left(\frac{\mu}{\mu_w}\right)^{0.14} \qquad \text{(A-3)}$$

This is Eq. (11) in the text.

Nomenclature

A	= heat transfer surface area	[m^2]
c_p	= heat capacity at constant pressure	[J·kg^{-1}·K^{-1}]
D	= vessel diameter	[m]
d_e	= equivalent nozzle diameter	[m]
d_n	= nozzle diameter	[m]
H	= liquid height	[m]
h	= heat transfer coefficient	[W·m^{-2}·K^{-1}]
I	= current	[A]
K	= coefficient in Eq. (1)	[—]
K'	= coefficient in Eq. (12)	[—]
n	= number of jet nozzles	[—]
P	= power input	[W]
Pr	= Prandtl number, $c_p\mu/\kappa$	[—]
P_v	= power input per unit volume	[W·m^{-3}]
Q	= jet flow rate	[m^3·s^{-1}]
Q_H	= heat transfer rate	[W]
Re_n	= nozzle Reynolds number, $u_e d_e \rho/\mu$	[—]
T_b	= bulk liquid temperature	[K]
T_w	= vessel wall temperature	[K]
t_M	= mixing time	[s]
u	= mean velocity of liquid at jet nozzle	[m·s^{-1}]
u_e	= equivalent velocity of liquid at jet nozzle	[m·s^{-1}]
V	= voltage	[V]
α	= correction factor in Eq. (10)	[—]
κ	= thermal conductivity	[W·m^{-1}·K^{-1}]
μ	= liquid viscosity	[Pa·s]
μ_w	= liquid viscosity at vessel wall	[Pa·s]
ρ	= liquid density	[kg·m^{-3}]

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