

# RELATIONSHIP BETWEEN APPARENT BED VISCOSITY AND FLUIDIZATION QUALITY IN A FLUIDIZED BED WITH FINE PARTICLES

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**Key Words:** Fluidization, Fluidized Bed, Apparent Viscosity, Pressure Fluctuations, Bubble Diameter

The apparent viscosity was measured in a fluidized bed with fine particles. The measurement was performed in the bed expanded uniformly at gas velocities between minimum fluidizing velocity and minimum bubbling velocity by means of the falling-sphere method. As the apparent viscosity obtained decreased with the diameter of the falling spheres, the viscosity value was determined by extrapolation. The apparent viscosity obtained was correlated with particle diameter, density and bed voidage. An equation for predicting the apparent viscosity in the emulsion phase in a bubbling fluidized bed was introduced by combination with the correlation of the voidage of the emulsion phase. Comparison of the values predicted from this equation with the data in the literature shows good agreement. To investigate the influence of the apparent viscosity on the fluidization quality, deviations of pressure fluctuations and bubble diameter were correlated with the apparent viscosity. It was shown that the proposed correlation for the apparent viscosity was useful for predicting the fluidization quality.

## Introduction

The fluidity of a fluid-bed reactor affects the reactor performance and mechanical stability of a reactor. The fluidity in a reactor cannot be predicted from the results in a cold model, because temperature level and fluidizing gas are different from those under reaction conditions. A parameter is needed to relate the fluidity and a system parameter such as gas and particle properties. The concept of apparent viscosity is useful for this purpose.

The apparent viscosity of a fluidized bed has been measured by various methods: a paddle-type viscometer<sup>7,18,20,27</sup>, rotating spheres<sup>18</sup>, a falling

sphere<sup>3,16,17</sup>, a Couette-type viscometer<sup>5,6,25</sup> and a torsion pendulum<sup>9</sup>. Since it is very difficult to measure the apparent viscosity of the emulsion phase directly in a bubbling fluidized bed, in the present study the viscosity was measured by varying the bed voidage under conditions where the bed expanded uniformly. Although the falling-sphere method was criticized by Schügerl<sup>26</sup> and Grace<sup>8</sup>, we chose this method because of the simplicity of the system. Kikuchi<sup>16</sup> has reported that the measured apparent viscosity decreased with the diameter of a falling sphere. We also reduced the influence of the disturbance caused by the sphere and the wall by extrapolating the data obtained by varying the sphere diameter.

It is known that the fraction of gas in the emulsion phase (the voidage of the emulsion phase)  $\varepsilon_{fe}$ , in a

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bubbling fluidized bed is larger than the voidage at minimum fluidizing velocity,  $\epsilon_{mf}$ , in a fluidized bed of fine particles. Since  $\epsilon_{fe}$  affects the apparent viscosity, it is important to investigate the effect of the voidage on the apparent viscosity. In the first step, the influence of particle diameter, particle density and bed voidage on the apparent viscosity of a bed of fine particles expanded uniformly was studied.

To predict the apparent viscosity of the emulsion phase, we introduced the correlation<sup>14)</sup> of  $\epsilon_{fe}$  and properties of fluidizing gas and particles. In the analysis it was assumed that the apparent viscosity of the emulsion phase was equal to that of the bed expanded uniformly when  $\epsilon_{fe}$  was equal to the voidage of the bed expanded uniformly. The apparent viscosity of the emulsion phase was used for investigating the relationship between apparent viscosity and fluidization quality. Pressure fluctuations were measured to evaluate the fluidization quality. In addition, the influence of the apparent viscosity on bubble diameter was studied.

## 1. Experimental

### 1.1 Apparent viscosity

A schematic diagram of the measuring system is shown in Fig. 1. The experiments were carried out in a 0.055 m-dia., 1.5 m-high fluidized bed with a sintered brass distributor. The column was made of acrylic resin. The physical properties of the powders used in the experiments are given in Table 1.

A stainless steel sphere was dropped from a point  $L_a$  [m] above the bed surface with an initial velocity of zero. The value of  $L_a$  was adjusted to agree with the velocity of a falling sphere at the bed surface with the terminal velocity  $V_t$  in the bed. The diameter of the steel sphere was varied from 3 to 8 mm. The steel sphere falling through the bed was detected by two metal detectors fixed at the outer surface of the column. The distance between the metal detectors,  $L_d$ , was 0.296 m.

Assuming that fluidized beds behave as a Newtonian fluid, we analyzed the behavior of a falling sphere and obtained the apparent viscosity of the bed. Since the relationship between the drag coefficient and the Reynolds number is given in the table and curve<sup>10)</sup>, the Reynolds number could be obtained from the value of the drag coefficient calculated from Eq. (1).

$$C_D = \frac{4d_s(\rho_s - \rho_b)g}{3\rho_b V_t^2} \tag{1}$$

where  $d_s$  is the diameter of a steel sphere,  $\rho_s$  is the density of the sphere and  $\rho_b$  is the density of a bed which is represented by the bed voidage and particle density:  $\rho_p(1 - \epsilon)$ . Then the apparent viscosity,  $\mu_a$ , was obtained from the relationship:

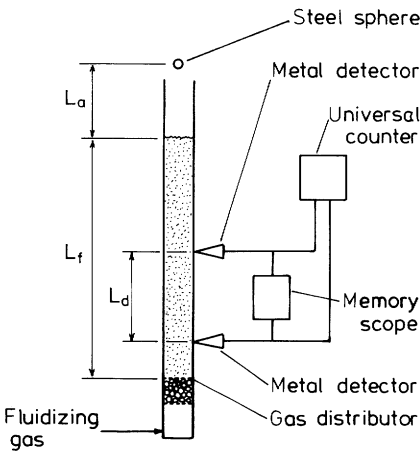


Fig. 1. Schematic diagram of experimental apparatus and measuring system

Table 1. Properties of powders

Powder	$d_p \times 10^{-6}$ [m]	$\rho_p$ [kg m <sup>-3</sup> ]	$U_{mf} \times 100$ [m s <sup>-1</sup> ]	$U_{mb} \times 100$ [m s <sup>-1</sup> ]	Key
FCC-1	35.8	920	0.059	0.17	□
FCC-2	37.4	1050	0.066	0.20	▣
FCC-3	48.4	1750	0.10	0.37	▤
FCC-4	49.0	1690	0.19	0.36	■
FCC-5	56.3	980	0.12	0.39	◇
Al-1	55.0	770	0.095	0.31	○
Al-2	83.2	770	0.21	0.50	●
Si-1	69.3	570	0.15	0.40	△
Si-2	105	1300	0.49	0.53	▲
Cat-1	49.0	1330	0.14	0.35	◐
Cat-2	59.3	990	0.15	0.40	⊙
Cat-3	40.1	2210	0.17	0.38	▼
Cat-4	51.1	2360	0.27	0.46	▽

$$Re = \frac{d_s \rho_b V_t}{\mu_a} \tag{2}$$

Data taken when the Reynolds number was smaller than 800 were used for the analysis.

To ascertain the reliability of the measuring technique and analysis system, the viscosities of aqueous solutions of sucrose were measured at 293 K. The concentration of sucrose was varied from 28% to 60%. The viscosity of water was not precisely obtained even where the smallest sphere was used, since the Reynolds number became more than 2000. The viscosity of the aqueous solutions measured in this system almost agreed with the data in the table<sup>10)</sup>, thus confirming that this method was suitable at least for measuring the viscosity of aqueous solutions.

### 1.2 Pressure fluctuations

Pressure fluctuations were measured in a column 0.082 m in diameter and 1.7 m in height with an expansion zone. The measurements were carried out under ambient conditions using air as fluidizing gas. For Al-1 powder, five other kinds of gas were used.

Pressure fluctuations were detected using a tap at the bottom of the bed above a sparger. The electrical signals from a pressure transducer were converted to digital signals. Then they were treated by a computer, and the average deviation of the amplitude of pressure fluctuations was calculated.

## 2. Results and Discussion

### 2.1 Apparent viscosity

Figures 2 and 3 show the relationship between the diameter of a steel sphere and the measured apparent viscosity of a bed. It was clearly seen that the apparent viscosity increased with sphere diameter. This tendency has been reported by Kikuchi<sup>16)</sup>, who used the falling-sphere method for a fluidized bed with FCC particles. This tendency was not observed in the measurement for the solution of sucrose, but was peculiar to the measurement in a gas-solid fluidized bed.

Figure 2 shows the relationship for the largest powder, Al-2, and Fig. 3 for the smallest powder, FCC-1, in this study. It was observed in these figures that the influence of sphere diameter was significant for the larger particles. This tendency was also observed in the case of the other powders. Figures 2 and 3 also show that the plot of  $\mu_a$  versus  $\ln(d_s)$  gave straight lines. To reduce the influence of disturbance caused by a sphere and a wall, we obtained the apparent viscosity by the extrapolation at  $d_s=0$ . In the following section, we call the viscosity obtained by the extrapolation the apparent viscosity for the sake of simplicity.

It has been reported that the apparent viscosity increased with  $d_p$  and  $\rho_p$ , and decreased with bed voidage<sup>26)</sup>. Figure 4 shows the relationship between bed voidage and the apparent viscosity for FCC-5 powder. The apparent viscosity increased with decreasing bed voidage. When the voidage approached unity, the viscosity became considerably small. Hence Pritchett *et al.*<sup>23)</sup> expressed the apparent viscosity by the function  $(1-\varepsilon)/(\varepsilon-\varepsilon_{mf})$ . Figure 4 shows, however, that the value of the apparent viscosity at  $\varepsilon_{mf}$  was not infinite. It is supposed that there exists a critical point at a voidage smaller than  $\varepsilon_{mf}$ , where the value of the apparent viscosity sharply increases. In the present study, the average value of the critical voidage obtained for all powders was calculated to be  $0.98\varepsilon_{mf}$ .

The dependence of  $d_p$ ,  $\rho_p$  and the voidage function,  $(1-\varepsilon)/(\varepsilon-0.98\varepsilon_{mf})$ , on the apparent viscosity was investigated for all the data obtained in this experiment. Consequently, the following equation was found to predict the apparent bed viscosity best:

$$\mu_a^0 = \frac{5.58 \times 10^{-5} d_p^{0.634} \rho_p^{1.39} (1-\varepsilon)}{(\varepsilon - 0.98\varepsilon_{mf})^{0.494}} \quad (3)$$

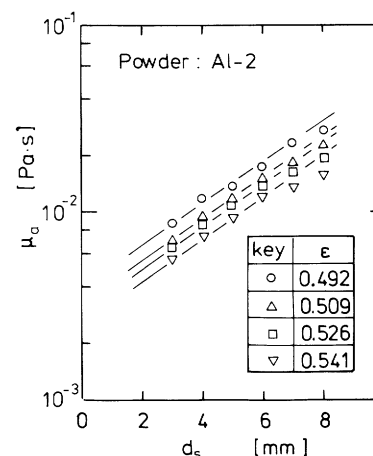


Fig. 2. Effect of sphere diameter on apparent viscosity for Al-2 powder

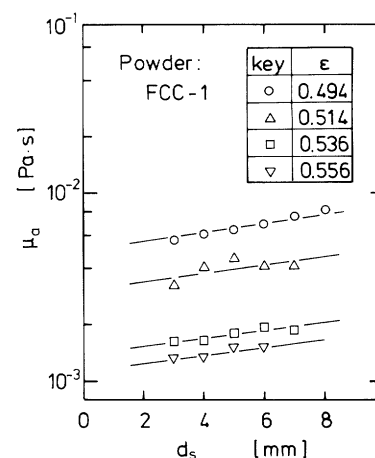


Fig. 3. Effect of sphere diameter on apparent viscosity for FCC-1 powder

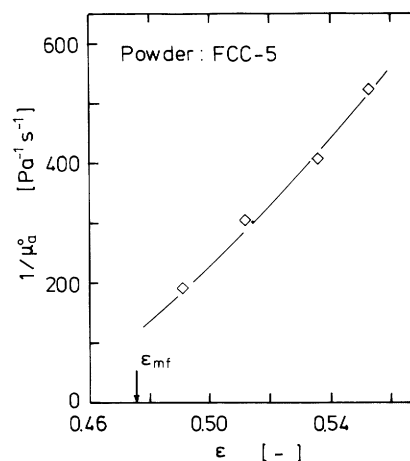


Fig. 4. Variation of apparent bed viscosity with bed voidage

The average deviation of the predictions by Eq. (3) from the experimental data was 7.7%. The comparison between the calculated and experimental values is shown in Fig. 5. As shown in this figure, the apparent viscosity was from  $10^{-3}$  to  $10^{-2}$  Pa s in this

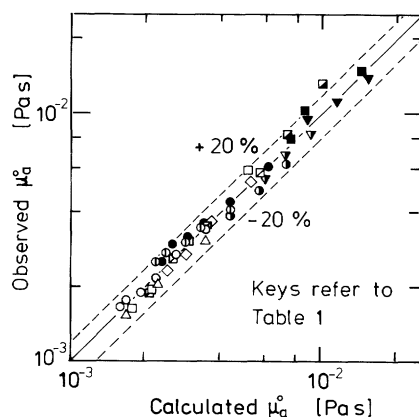


Fig. 5. Comparison of experimental values of apparent viscosity with predictions

experiment.

Figure 6 shows the comparison of these values with the data in the literature. This figure is basically the same as the figure plotted by Nozaki<sup>22)</sup>, but we considered particle diameter in the parameter of the lateral coordinate. The powders used in the present study were smaller and lighter than those used by the other investigators. It is seen that the apparent viscosity could be roughly correlated by this parameter, whereas the measuring methods were different among the investigators. The solid line in Fig. 6 is supposed to be the lower limit of the measured values. The experimental values tended to deviate to larger values than those presented by this line.

The values obtained by Grace<sup>8)</sup> were not so affected by the properties of particles and were larger than those by the other investigators. Grace estimated the apparent viscosity from the bubble shape indirectly. Nozaki<sup>22)</sup> has pointed out the difficulty in judging bubble shape from a photograph and the propriety of applying the correlation for a gas-liquid system to a gas-solid system. In addition to those reasons, it is supposed that the rising bubbles affected the voidage of the emulsion phase.

Kikuchi<sup>16)</sup>, King *et al.*<sup>17)</sup> and Daniels<sup>3)</sup> measured the viscosity by means of the falling-sphere method. Kikuchi obtained the apparent viscosity by extrapolation. Since Daniels varied the size of the falling sphere, we calculated the viscosity from his data. King *et al.* used spheres of only one size and their values were a little larger.

Since the bed density is given by  $\rho_p(1-\varepsilon)$ , the apparent kinetic viscosity of a fluidized bed is expressed as

$$v_a = \frac{5.58 \times 10^{-5} d_p^{0.634} \rho_p^{0.39}}{(\varepsilon - 0.98\varepsilon_{mf})^{0.494}} \quad (4)$$

In the previous paper<sup>14)</sup>, the voidage of the emulsion phase,  $\varepsilon_{fe}$ , was correlated by the particle and gas

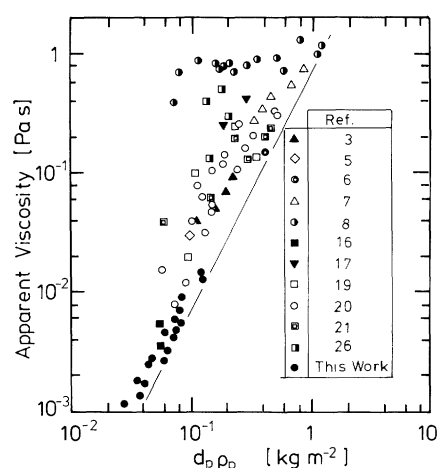


Fig. 6. Relationship between apparent viscosity and parameter  $d_p \rho_p$

properties:

$$\varepsilon_{fe} - \varepsilon_{mf} = 4.03/N^* \quad (5)$$

$$N^* = \sqrt{d_p^3 g (\rho_p - \rho)(\rho_{air}/\rho)^{0.12}/\mu} \quad (6)$$

From Eqs. (4) and (5), the apparent kinetic viscosity for the emulsion phase in a bubbling fluidized bed can be expressed as a function of the properties of the particles and fluidizing gas:

$$v_a = \frac{5.58 \times 10^{-5} d_p^{0.634} \rho_p^{0.39}}{(4.03/N^* + 0.02\varepsilon_{mf})^{0.494}} \quad (7)$$

As the value of  $N^*$  is small for fine particles, the term  $N^*$  in Eq. (7) is dominant and  $v_a$  is affected by gas viscosity and density, because the parameter  $N^*$  contains the term of the gas properties. For coarse particles, on the other hand, the value of  $N^*$  is large and thus the effect of the expansion of emulsion phase can be ignored even if Eq. (7) cannot be applied directly to the fluidized beds of coarse particles. In this case, therefore, the apparent viscosity is little influenced by gas properties. These results correspond to the observation that bubble diameter was not influenced by temperature for large particles, but it was sensitive to temperature for fine particles.

The apparent bed viscosity increased with decreasing gas viscosity for fine particles. This is because  $\varepsilon_{fe}$  increases with gas viscosity. The correlations<sup>15,24,29)</sup> for liquid-solid systems have often been used as the apparent viscosity for gas-solid systems. But in these correlations the apparent viscosity is directly proportional to fluid viscosity. This tendency is different from the experimental behavior. The correlation proposed in the present study is rather suitable to investigate the effect of the gas properties for a gas-solid fluidized bed.

## 2.2 Influence of sphere diameter

The voidage of the fluidized bed expanded

uniformly is larger than  $\varepsilon_{mf}$ . However,  $\varepsilon_{mf}$  is not the smallest voidage of the bed. When a steel sphere was falling through a bed, it was affected by the residence force of the surrounding particle structure which was compacted by the sphere. This residence force caused a frictional force on the surfaces of the sphere. Considering the force balance on a steel sphere falling through a bed at its terminal velocity,  $V_t$ , gives the following relationship:

$$\frac{\pi C_D d_s^2 \rho_b V_t^2}{8} + F_f = \frac{\pi}{6} d_s^3 (\rho_s - \rho_b) g \quad (8)$$

Assuming that the residence force per unit area acting on a steel sphere is proportional to the sphere diameter, as the frictional force is proportional to the residence force, the horizontal net residence force is considered to be proportional to  $d_s^3$ .

The value of the frictional force for each value of  $d_s$  was obtained by calculating the Reynolds number from the apparent viscosity at  $d_s=0$ . **Figure 7** shows the relationship between  $d_s$  and the frictional force calculated at  $\varepsilon=0.5$  for some powders. It can be seen from this figure that the frictional force increased with sphere diameter and was almost proportional to  $d_s^3$  as predicted.

### 2.3 Relationship between $v_a$ and pressure fluctuations

**Figure 8** shows the relationship between  $U_f$  and the average deviation of pressure fluctuations for Al-1 powder. The average deviation increased with gas velocity, and after reaching a maximum point, it decreased with increasing gas velocity. As the maximum deviation was small for the powders that show good fluidization, it was used as the index of fluidization quality. As is also seen from Fig. 8, the pressure fluctuations were influenced by the type of fluidizing gas. This was caused by the change in apparent viscosity of the emulsion phase due to the change in gas properties. It was reported<sup>14)</sup> that the voidage of the emulsion phase increased with gas viscosity and density.

**Figure 9** shows the relationship between the apparent kinematic viscosity and the maximum average deviation of pressure fluctuations. The deviation of pressure fluctuations could be correlated by the apparent viscosity. Although this relationship is influenced by the type of apparatus, the fluidization quality for various gas-solid systems in a similar system can be predicted by  $v_a$  calculated from Eq. (7).

### 2.4 Relationship between $v_a$ and bubble diameter

In the previous studies<sup>11-13)</sup> we measured bubble diameter in fluidized beds with powders almost all of which were the same as in this study. **Figure 10** shows the relationship between  $d_b$  and  $v_a$  determined from Eq. (7) by using the values of the properties of particles and fluidizing gases. The bubble diameter was

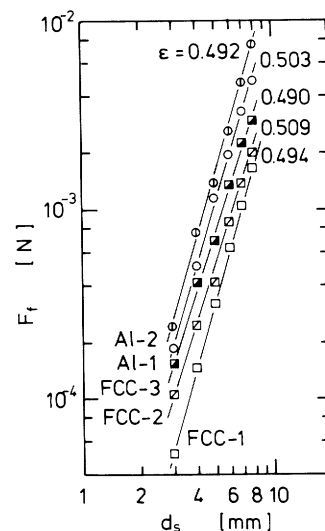


Fig. 7. Relationship between calculated frictional force and sphere diameter

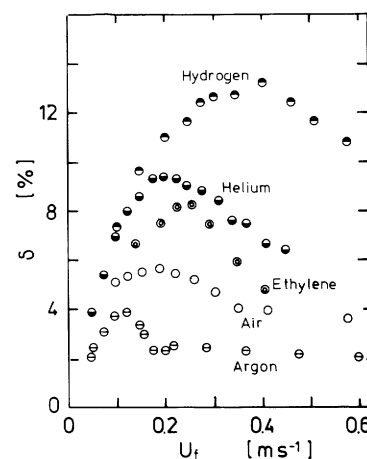


Fig. 8. Average deviation of pressure fluctuations for Al-1 powder

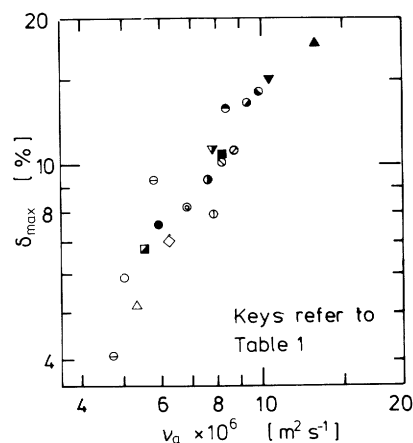


Fig. 9. Relationship between apparent kinetic viscosity and maximum deviation of pressure fluctuations

measured by using an optical probe placed 0.45 m above a distributor and  $U_f$  was about  $0.1 \text{ m s}^{-1}$ . In these experiments, not only powders but also

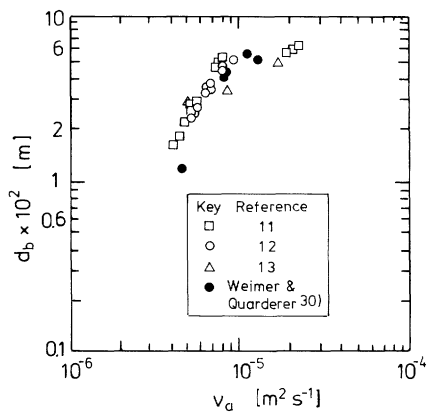


Fig. 10. Change in bubble size with apparent kinetic viscosity in fluidized beds

temperature<sup>11)</sup> and fluidizing gas<sup>12,13)</sup> were varied. Although the bubble diameter was limited by the column size, a directional trend of larger  $d_b$  with larger  $v_a$  was clearly seen. In a qualitative sense, a similar tendency between liquid viscosity and bubble diameter was observed in a bubble column<sup>21)</sup>.

Weimer and Quarderer<sup>30)</sup> measured bed voidage and bubble rise velocity in a high-pressure fluidized bed. They calculated  $d_b$  from the bubble rise velocity<sup>4)</sup> and examined the relationship between system parameters and apparent viscosity. However, they did not meet with good results because of the lack of data, correlations and theory relating  $v_a$  in high-pressure fluidized beds. They also reported that the required  $v_a$  calculated from  $d_b$  based on the Taylor instability criterion was significantly smaller than that predicted from the correlations of Kawase and Ulbrecht<sup>15)</sup>, and Thomas<sup>29)</sup>. The predictions from Eq. (7) in this study also did not satisfy the Taylor instability criterion.

For the 66  $\mu\text{m}$  powder, Weimer and Quarderer<sup>30)</sup> obtained the value of 0.692 Pa s as the apparent viscosity at 100 kPa and this value was two orders of magnitude larger than the values obtained in the present study. In the case of bubble columns, the value of 0.692 Pa s is supposed to be very large as the liquid viscosity. It is uncertain that the Taylor instability can be directly used for the analysis of the viscosity in fluidized beds.

We calculated  $v_a$  from the system parameters in the experiment of Weimer and Quarderer<sup>30)</sup>, and show the relationship between  $v_a$  and  $d_b$  in Fig. 10. As Eq. (5) is not usable for a high-pressure fluidized bed, the value of the voidage of the emulsion phase was directly put in Eq. (4). It is found from this figure that the data of Weimer and Quarderer approximately agreed with the results in the present study. This implies that the bubble diameter can be correlated by the  $v_a$  corresponding to the system parameters by the equation introduced in the present study.

## Conclusions

The apparent viscosity was measured by the falling-sphere method in a fluidized bed of fine particles expanded uniformly. The measured apparent viscosity increased with the diameter of the falling sphere. This was considered to be due to the frictional force caused by compaction of the emulsion phase near the sphere. Therefore, the apparent viscosity was obtained by the extrapolation to zero diameter of a falling sphere.

The apparent viscosity in the emulsion phase of a bubbling fluidized bed was correlated with the particle and gas properties by using the correlation of the voidage of the emulsion phase. It was explained by this correlation that the effect of fluidizing gas cannot be ignored for small powders, whereas the effect was insignificant for large powders.

The relationships between the pressure fluctuations and bubble diameter, and the apparent viscosity were investigated. It was found that the fluidization quality could be evaluated by the apparent viscosity predicted from the correlations obtained in the present study.

## Nomenclature

$C_D$	= drag coefficient	[—]
$D_r$	= diameter of column	[m]
$d_b$	= bubble diameter	[m]
$d_p$	= particle diameter	[m]
$d_s$	= diameter of steel sphere	[m]
$F_f$	= frictional force defined by Eq. (8)	[N]
$g$	= acceleration due to gravity	[m s <sup>-2</sup> ]
$L_a$	= distance from bed surface to a releasing point	[m]
$L_d$	= distance between two metal detectors	[m]
$L_f$	= height of fluidized bed	[m]
$N^*$	= parameter defined by Eq. (6)	[—]
$Re$	= Reynolds number defined by Eq. (2)	[—]
$U_f$	= superficial gas velocity	[m s <sup>-1</sup> ]
$U_{mb}$	= minimum bubbling velocity	[m s <sup>-1</sup> ]
$U_{mf}$	= minimum fluidizing velocity	[m s <sup>-1</sup> ]
$V_t$	= terminal velocity of falling sphere	[m s <sup>-1</sup> ]
$\delta$	= deviation of amplitude of pressure fluctuations	[—]
$\delta_{max}$	= maximum deviation of amplitude of pressure fluctuations	[—]
$\varepsilon$	= bed voidage	[—]
$\varepsilon_{fe}$	= volume fraction of gas in emulsion	[—]
$\varepsilon_{mf}$	= bed voidage at $U_{mf}$	[—]
$\rho$	= gas density	[kg m <sup>-3</sup> ]
$\rho_{air}$	= density of air under ambient condition	[kg m <sup>-3</sup> ]
$\rho_b$	= bed density	[kg m <sup>-3</sup> ]
$\rho_p$	= particle density	[kg m <sup>-3</sup> ]
$\rho_s$	= density of falling sphere	[kg m <sup>-3</sup> ]
$\mu$	= gas viscosity	[Pa s]
$\mu_a$	= apparent bed viscosity	[Pa s]
$\mu_a^\circ$	= apparent bed viscosity obtained by extrapolation	[Pa s]
$v_a$	= apparent kinetic viscosity	[m <sup>2</sup> s <sup>-1</sup> ]

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