PREDICTING BED VOIDAGE IN SOLID-LIQUID FLUIDIZATION

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Previously published solid-liquid fluidization data and new experimental data were correlated to develop an explicit expression for bed voidage, ε, in solid-liquid systems. Based on the observed deviation between the voidage predicted by the Richardson-Zaki[18] equation (ε_{RZ}) and the actual voidage, this new correlation assumes the form

$$\varepsilon=\varepsilon_{PK}+(1-\varepsilon_{PK})\varepsilon_{RZ}^{A}\exp(B(1-\varepsilon_{RZ}))$$

where $A=2.2n+8d_P/D$ and $B=2.1n$. ε_{PK} is the "packed" bed or the static bed voidage (i.e., ε when the superficial liquid velocity, U_L, is zero), n is the Richardson-Zaki[18] exponent, and d_P/D is the particle diameter-to-column diameter ratio. A total of 811 data points were correlated with Reynolds numbers based on terminal velocity, Re_t, ranging from 0.7 to 3200. The correlation satisfactorily predicts voidage with a standard deviation of less than 5%.

Introduction

Numerous correlations have been proposed in the literature aimed at establishing the velocity-voidage relationship observed during solid-liquid fluidization and sedimentation. Most[1,7,9,11-16,18,20-23] of these have been summarized and compared by Garside and Al-Dibouni.[6] More recent models include those of Ganguly,[5] which directly gives the expanded bed height for heterogeneous systems; Foscolo *et al.*,[4] which consists of different equations for the laminar, intermediate, and turbulent flow regimes; and Jean & Fan,[10] which presents a fluid mechanics-based model limited to low Reynolds number conditions. Generally, these correlations have been presented in forms where the superficial liquid velocity-to-terminal velocity ratio, U_L/U_t, is expressed as a function of the bed voidage, ε, the Reynolds number at the terminal velocity, Re_t, and/or other physical properties with the complexity varying from simple $U_L/U_t=\varepsilon^n$ forms (where the parameter, n, may be a constant or a function of some other properties) to relatively complex and, in some cases, discontinuous functions. The prediction of ε, therefore, necessitates the use of iterative solutions in most[1,4,6,7,12,15,16,20,22] of these models. Also, many[7,10,11,13,15,16,20] of these correlations are applicable only over a restricted range of Re_t and are limited to specific particle types (usually spherical particles) in addition to the wide discrepancies observed[6] between the predictions by these models. Taking into account these considerations, it appears that the "ideal" model must be fairly simple yet accurate with a significantly wide range of applicability.

This investigation has been undertaken to develop an empirical correlation which explicitly predicts bed voidage in solid-liquid fluidization systems over a wide range of application.

1. Experimental

The set-up whose schematic diagram is shown in **Fig. 1** essentially consists of a 5-cm i.d. fluidized-bed column 3.2 meters high, a water pump, a reservoir, liquid flowmeters, and control valves. A stainless steel screen fitted above the calming section of the bed supports the solid particles.

Water at 25°C was used to fluidize glass beads whose properties are summarized in **Table 1**. The average diameters were measured using a Personal Image Analyzer Sysem (PIAS LA-525).

For each particle size, fluidization runs were performed using 500, 700, and 900 grams of solid particles. Expanded bed height data were obtained using both increasing and decreasing liquid flow rates.

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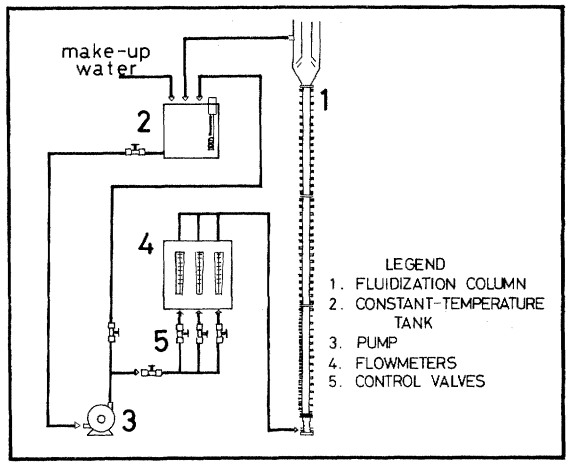


Fig. 1. Schematic diagram of experimental set-up

Table 1. Properties of Glass Beads Used

Average diameter d_P [cm]	Particle density ρ_s [g/cm^3]	Particle Reynolds No. Re_t [—]	Packed Voidage ε_{PK} [—]
0.0267	2.470	10.0	0.411
0.0339	2.470	17.2	0.413
0.0631	2.458	62.7	0.405
0.0674	2.483	72.2	0.415
0.1876	2.487	481.3	0.415
0.2357	2.485	706.1	0.415

The bed height when the liquid flow rate is zero is recorded as the static bed height. Typical velocity-voidage results are shown in **Fig. 2.**

2. Additional Experimental Data

Previously published experimental results were collected from the literature to cover wider Re_t ranges and different particle types. Particle diameters and densities varied from 0.097 to 6.35 mm and from 1.603 to 11.03 g/cm^3, respectively. Values of Re_t ranged from 0.7 to about 3200. The solid particles were mostly spherical (glass beads, Socony beads, steel balls, and lead shot) but also included nonspherical types (sea sand, crushed rock). The sources of these experimental data together with details of their respective experimental systems are summarized in **Table 2.**

3. Preliminary Discussion

The equation proposed by Richardson and Zaki[18] shown in **Table 3** is probably the most widely used correlation for solid-liquid systems. It may be observed from **Fig. 3**, however, that the actual voidage values are generally higher than those predicted by this equation, with the behavior of the deviation apparently following a characteristic curvature. The search for a model which accurately describes this curvature is the basis for the development of the new correlation.

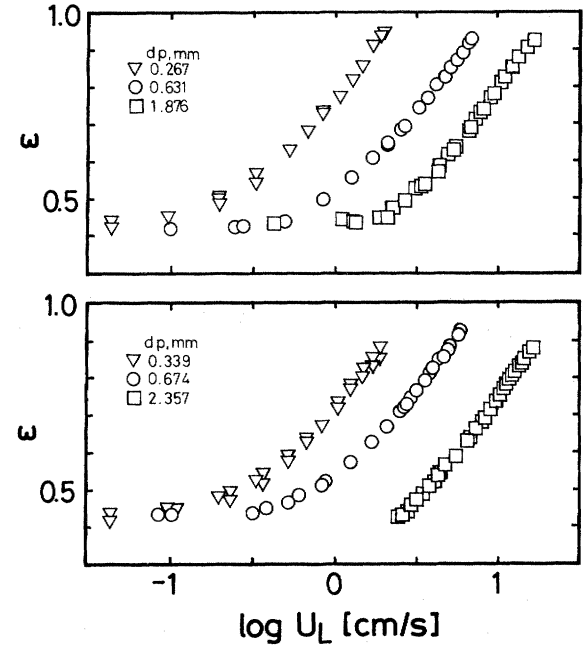


Fig. 2. Velocity-voidage results in this work

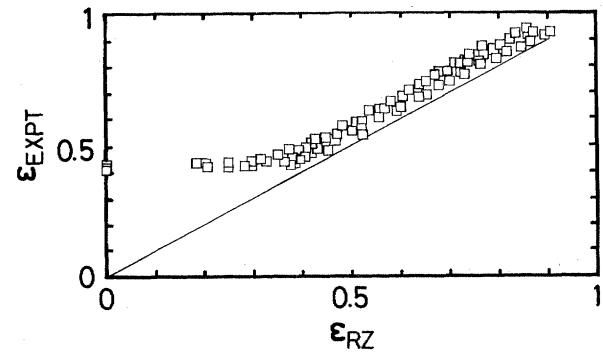


Fig. 3. Comparison of experimental values of bed voidage in this work and values predicted by the equation of Richardson and Zaki[18]

The empirical equations for the Richardson-Zaki[18] exponent, n, covering different Re_t ranges are given as Eqs. (1a) to (1e) in Table 3. These equations, however, are a little awkward to use because of the discontinuities at the transition points between the ranges. Rowe[19] proposed a continuous empirical equation for estimating the exponent n when the particle diameter is very small relative to the bed diameter as follows:

$$n=\frac{2(2.35+0.175Re_t^{0.75})}{(1+0.175Re_t^{0.75})} \qquad (2)$$

Due to its simplicity as compared to the original Richardson-Zaki[18] correlations, Eq. (2) has been used in all subsequent calculations of n in this study.

The terminal velocity, U_t, of a single particle corrected for wall effect was calculated using the equation proposed by Richardson and Zaki,[18]

Table 2. Summary of Previously Published Experimental Systems Used in This Study

Author(s) (Year)	Symbol	Liquid	Particle	Particle diameter d_P [cm]	Column diameter D [cm]	Particle density ρ_s [g/cm^3]	Particle Reynolds No. Re_t [—]	Packed Voidage ε_{Pk} [—]
Wilhelm and Kwauk[23] (1948)	■	Water	Glass Beads	0.0287	14.61	2.492	10.5	0.384
				0.0511	7.54	2.492	37.1	0.384
				0.5207	7.54	2.351	2136.2	0.385
			Socony Beads	0.3277	7.54	1.603	675.6	0.368
				0.4420	7.54	1.603	1090.7	0.368
				0.4420	14.61	1.603	1164.2	0.368
			Sea Sand	0.0373	7.54	2.639	20.4	0.407
				0.0556	7.54	2.639	47.2	0.408
				0.0998	7.54	2.639	148.7	0.407
			Lead Shot	0.1283	7.54	10.792	715.1	0.375
			Crushed Rock	0.1415	7.54	2.636	284.4	0.447
Lewis *et al.*[13] (1949)	▲	Water	Glass Beads	0.0452	6.35	2.515	24.0	0.390*
				0.0284	6.35	2.435	9.1	0.410*
				0.0155	6.35	2.451	2.3	0.410*
Loeffler and Ruth[15] (1959)	▼	Water	Glass Beads	0.0659	2.60	2.630	71.5	0.430*
				0.3360	5.51	2.573	1289.8	0.410*
				0.5060	5.51	2.514	2249.7	0.410*
Hoffman *et al.*[9] (1960)	◎	Water	Glass Beads	0.0465	2.54	2.524	32.4	0.435
				0.0269	2.54	2.486	9.6	0.435
				0.0097	2.54	2.435	0.7	0.450
				0.0121	2.54	2.467	1.3	0.450
				0.0191	2.54	2.465	4.2	0.450
Wen and Yu[21] (1966)	◆	Water	Glass Beads	0.2032	10.16	2.450	569.8	0.390
				0.5004	10.16	2.460	2395.0	0.394
				0.6350	10.16	2.360	3208.7	0.399
			Steel Balls	0.2380	10.16	7.840	1777.2	0.403
Bhatia, V. K.[2] (1973)	●	Water	Glass Beads	0.1080	2.00	2.824	181.8	0.473
				0.0273	5.00	2.938	9.9	0.423
				0.0456	5.00	2.935	29.5	0.390*
				0.1080	5.00	2.949	183.6	0.410*
			Sand	0.0458	2.00	2.578	29.8	0.541
			Lead Shot	0.2180	5.00	11.030	1428.8	0.380*

* Estimated by extrapolation.

Table 3. Richardson-Zaki[18] equation

$$U_L/U_t=\varepsilon^n \quad (1)$$

where

$n=4.65+19.5d_P/D$: $Re_t<0.2$ (1a)

$=(4.35+17.5d_P/D)Re_t^{-0.03}$: $0.2<Re_t<1$ (1b)

$=(4.45+18d_P/D)Re_t-0.10$: $1<Re_t<200$ (1c)

$=4.45Re_t^{-0.10}$: $200<Re_t<500$ (1d)

$=2.39$: $Re_t>500$ (1e)

$$\log(U_{ti}/U_t)=d_P/D \quad (3)$$

where U_{ti} is the terminal velocity or free-fall velocity of spheres in an infinite medium and d_P/D is the particle diameter-to-bed diameter ratio.

The values of U_{ti} were calculated using a method recently proposed by Hartman *et al.*[8] involving the Archimedes number, Ar, and Re_t, as presented in **Table 4**. This equation circumvents the iterative solutions accompanying standard equations[3,17] for predicting the free-fall velocities, U_{ti}, involving the drag coefficient, C_D. The terminal velocities predicted by this method agree very well with those calculated from standard equations.

Table 4. Hartman *et al.*[8] equation for predicting the free-fall velocity in an infinite medium

$$\log Re_t=P(A)+\log R(A) \quad (4)$$

where

$P(A)=[(0.0017795A-0.0573)A+1.0315]A-1.26222$ (4a)

$R(A)=0.99947+0.01853\sin(1.848A-3.14)$ (4b)

$A=\log Ar$ (4c)

4. Development of the Model

Defining ε_{PK} as the "packed" or static bed voidage (i.e., the bed voidage when U_L is zero), a model is desired which satisfies the following conditions:

$\varepsilon=\varepsilon_{PK}$ when $\varepsilon_{RZ}=0$ $(U_L=0)$,

$\varepsilon=1$ when $\varepsilon_{RZ}=1$ $(U_L=U_t)$, and

$d\varepsilon/d\varepsilon_{RZ}=0$ when $\varepsilon_{RZ}=0$ $(U_L=0)$

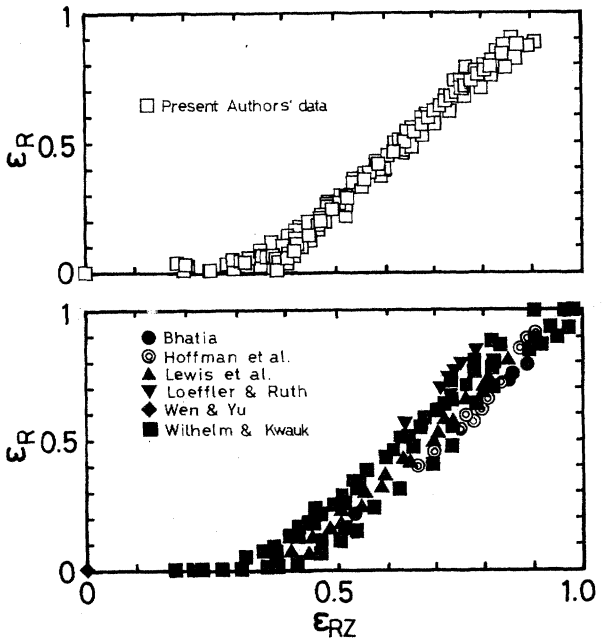


Fig. 4. Plot of ε_R vs. ε_{RZ} for all experimental data used in this study

where ε_{RZ} is the voidage calculated by the "Richardson-Zaki type" equation (Eq. 1) with n estimated by Eq. (2) and U_t by Eq. (3). The value of ε_{PK} is basically characterized by the particle type and may, for practical purposes, be treated as another physical property of the solid particle. Due to the dependence of ε_{PK} on particle type, varying lower limits are approached by the dependent variable, ε. To circumvent this, a new variable, ε_R, defined as

$$\varepsilon_R=\frac{(\varepsilon-\varepsilon_{PK})}{(1-\varepsilon_{PK})} \tag{5}$$

is introduced and used as the new dependent variable whose limits, in effect, approach zero and 1 as ε_{RZ} approaches zero and 1, respectively. The transformed ε_R vs. ε_{RZ} curve, shown in **Fig. 4**, generally exhibits the same behavior as the ε vs. ε_{RZ} curve.

After testing and comparing several models to describe the behavior of ε_R with ε_{RZ} it was found that an equation of the form

$$\varepsilon_R=\varepsilon_{RZ}^{A}\exp(B(1-\varepsilon_{RZ})) \tag{6}$$

with parameters A and B satisfactorily fits the specific ε_R vs. ε_{RZ} curves. Using this work's data and the previously published experimental data of various researchers, values of A and B were determined by regression and then evaluated. A and B were found to vary with the Richardson-Zaki exponent, n, with A slightly exhibiting an added dependence on d_P/D. These variations are shown in **Fig. 5**. The simplified models

$$A=an+bd_P/D \tag{7}$$

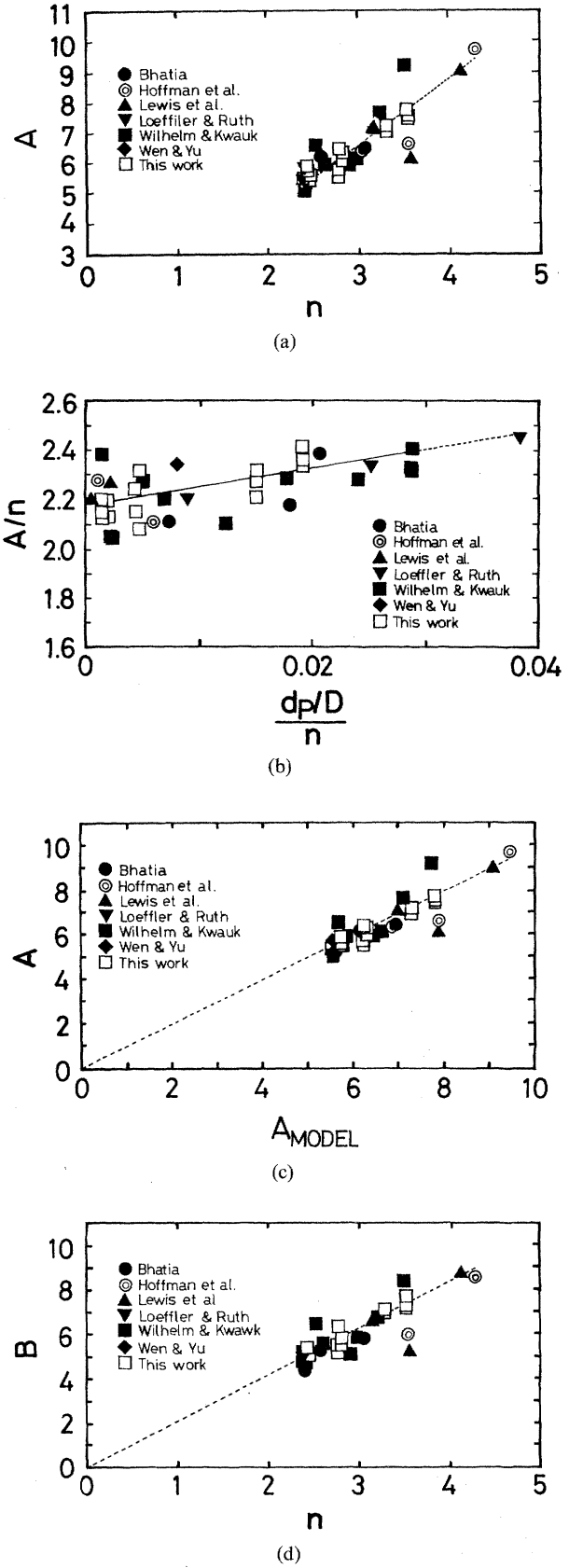


Fig. 5. (a) Variation of A with n, (b) Variation of A with d_P/D, (c) Comparison of actual values of A with those predicted by $[A=an+bd_P/D]$, (d) Variation of B with n $[B=cn]$

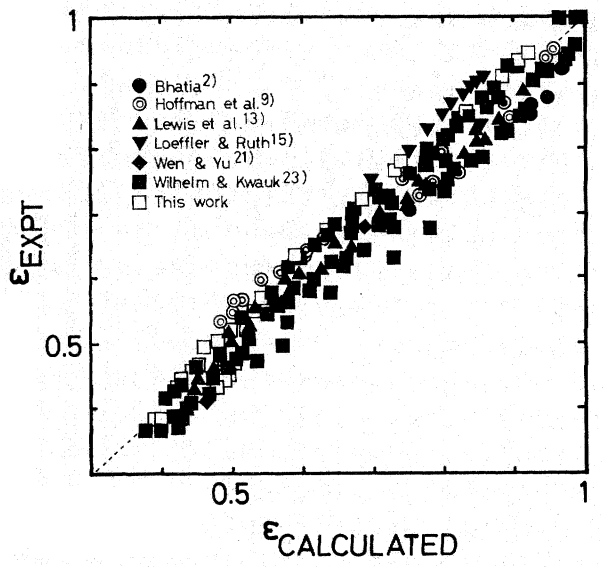


Fig. 6. Comparison of calculated values from this work's correlation and experimental values of bed voidage

and

$$B = cn \tag{8}$$

with correlation constants a, b, and c were, therefore, proposed for the estimation of A and B, respectively.

5. Final Correlation

A total of 811 data points from 51 experimental runs were carefully correlated and the best simplified values of the constants a, b, and c were calculated to be 2.2, 8, and 2.1, respectively. Thus, the final explicit correlation for ε may be stated as

$$\varepsilon = \varepsilon_{PK} + (1-\varepsilon_{PK})\varepsilon_{RZ}^{A} \exp(B(1-\varepsilon_{RZ})) \tag{9}$$

where

$$A = 2.2n + 8d_P/D\,, \tag{9a}$$

and

$$B = 2.1n \tag{9b}$$

Fig. 6 shows the agreement between the voidage values predicted by this correlation and the experimental values. The standard deviation of the predicted values was calculated to be 4.54% while the average absolute error was 3.55%. The relative frequency distribution of the percentage error is shown in **Fig. 7**.

Conclusion

A new empirical correlation represented by Eqs. (9), (9a), and (9b) which directly predicts bed voidage in solid-liquid fluidization systems has been proposed. The model is relatively simple and accurate and is applicable over a wide range of Re_t. Although the particles considered were mostly spherical, the behavior of the nonspherical particles used in this study was also well described by the correlation.

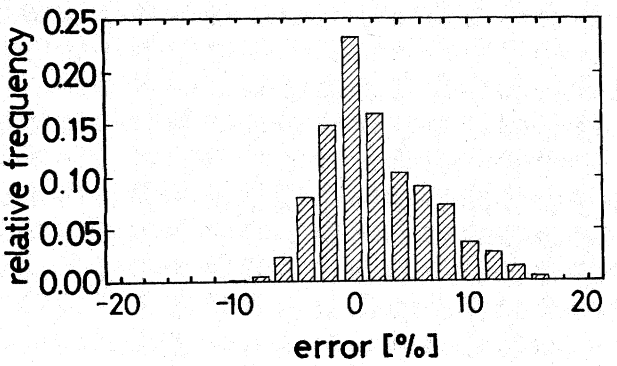


Fig. 7. Error distribution of this work's proposed correlation

Acknowledgement

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Nomenclature

a	=	correlation constant in Eq. (7)	[—]
A	=	parameter in Eq. (6)	[—]
Ar	=	$d_P^3 g\rho_L(\rho_s-\rho_L)/\mu^2$ = Archimedes number	[—]
b	=	correlation constant in Eq. (7)	[—]
B	=	parameter in Eq. (6)	[—]
c	=	correlation constant in Eq. (8)	[—]
C_D	=	drag coefficient	[—]
D	=	bed diameter	[cm]
d_P	=	particle diameter	[cm]
n	=	Richardson-Zaki exponent	[—]
Re_t	=	$d_P U_t \rho_L/\mu$ = particle Reynolds number at terminal velocity conditions	[—]
U_L	=	superficial liquid velocity	[cm/s]
U_t	=	particle terminal velocity corrected for wall effect	[cm/s]
U_{ti}	=	particle terminal velocity in an infinite fluid	[cm/s]
ε	=	bed voidage	[—]
ε_{PK}	=	static bed voidage	[—]
ε_R	=	$(\varepsilon-\varepsilon_{PK})/(1-\varepsilon_{PK})$ = voidage ratio	[—]
ε_{RZ}	=	bed voidage based on the "Richardson-Zaki type" equation	[—]
μ	=	liquid viscosity	[g/cm-s]
ρ	=	density	[g/cm^3]

〈Subscripts〉

L	=	liquid
P	=	particle
PK	=	"packed" or static
R	=	ratio
RZ	=	Richardson-Zaki
s	=	solid
t	=	terminal velocity conditions

Literature Cited

1) Barnea, E. and J. Mizrahi: *Chem. Eng. J.*, **5**, 171 (1973).

2) Bhatia, V. K.: Ph.D. Thesis, University of British Columbia (1973).
3) Clift, R., J. R. Grace and M. E. Weber: Bubbles, Drops and Particles, Academic Press, New York (1978).
4) Foscolo, P. U., L. G. Gibilaro and S. P. Waldram: *Chem. Eng. Sci.*, **38**, 1251 (1983).
5) Ganguly, U. P.: *Can. J. Chem. Eng.*, **58**, 559 (1980).
6) Garside, J. and M. R. Al-Dibouni: *Ind. Eng. Chem. Process Des. Dev.*, **16**, 206 (1977).
7) Happel, J.: *A.I.Ch.E. J.*, **4**, 197 (1958).
8) Hartman, M., V. Havlin, O. Trnka and M. Carsky: *Chem. Eng. Sci.*, **44**, 1743 (1989).
9) Hoffman, R. F., L. Lapidus and J. C. Elgin: *A.I.Ch.E. J.*, **6**, 321 (1960).
10) Jean, R. H. and L. S. Fan: *Chem. Eng. Sci.*, **44**, 353 (1989).
11) Jottrand, R.: *J. Appl. Chem.*, **1**, 317 (1952).
12) Letan, R.: *Chem. Eng. Sci.*, **29**, 621 (1974).
13) Lewis, W. K., E. R. Gilliland and W. Bauer: *Ind. Eng. Chem.*, **41**, 1104 (1949).
14) Lewis, E. W. and E. W. Bowerman: *Chem. Eng. Prog.*, **48**, 603 (1952).
15) Loeffler, A. L. and B. F. Ruth: *A.I.Ch.E. J.*, **5**, 310 (1959).
16) Oliver, D. R.: *Chem. Eng. Sci.*, **15**, 230 (1961).
17) Perry, R. H. and D. Green: Perry's Chemical Engineers' Handbook, 6th ed., McGraw Hill, New York (1984).
18) Richardson, J. F. and W. N. Zaki: *Trans. Inst. Chem. Eng.*, **32**, 38 (1954).
19) Rowe, P. N.: *Chem. Eng. Sci*, **42**, 2795 (1987).
20) Steinour, J. H.: *Ind. Eng. Chem.*, **36**, 618 (1944).
21) Wen, C. Y. and Y. H. Yu: *Chem. Eng. Prog. Symp. Ser. No. 62*, **62**, 100 (1966).
22) Wen, C. Y. and L. S. Fan: *Ind. Eng. Chem. Process Des. Dev.*, **13**, 194 (1974).
23) Wilhelm, R. H. and M. Kwauk: *Chem. Eng. Prog.*, **44**, 201 (1948).