# NUMERICAL ANALYSIS OF TRANSPORT PHENOMENA BETWEEN HEATED VERTICAL PARALLEL PLATES

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*Key Words*: Natural Convection, Heat Transfer, Telephone Exchange Unit, Velocity Distribution, Numerical Analysis, Vertical Parallel Plates

**To analyze the natural convective heat transfer from an electronic system, the natural convective heat transfer phenomena between uniformly heated vertical parallel plates were analyzed.**

**The equations of motion and the energy equation were solved numerically under suitable boundary conditions. The velocity and temperature distributions between the plates were obtained for the case where the distributor plates were attached at the top or bottom of the plates.**

**The calculated velocity and temperature distributions agreed approximately with the experimental ones.**

## Introduction

Recently, along with the remarkable development of electronic engineering, the microminiaturization of electronic devices has progressed. Electronic circuit cards such as integrated circuits (I.C.) and large-scale integrated circuits (L.S.I.) are quite sensitive to temperature. If their temperature is higher than 110°C, they do not work normaly. In various electronic systems such as computers and telephone exchange units, the circuit cards are aligned on a plate. The plates are arranged to form vertical channels. These electronic systems are usually cooled by natural convection. Therefore, quantitative information about the transport phenomena between the plates is necessary.

There have been many experimental or theoretical studies[1-3,6-10] of natural convective heat transfer between heated vertical parallel plates. In the theoretical approach to this problem, equations of continuity, motion and energy or boundary-layer equations were solved numerically under suitable boundary conditions. The velocity and temperature distributions between the plates were calculated numerically and the local heat transfer coefficient from the heated plate was calculated.

In the experimental approach to the problem, the gas or liquid temperature distributions between heated vertical parallel plates were measured. The local or average heat transfer coefficient was obtained under several experimental conditions. The measured temperature distribution and the heat transfer coefficient were compared with the calculated results.

However, the calculated velocity distributions between the plates were not compared with experimental ones in the previous works.

The transport phenomena between heated vertical parallel plates in the case where the distributor plates were attached at the top or bottom of the plates seemed not to have been analyzed yet.

The velocity and temperature distributions between the plates in the case where the distributor plates were attached at the top or bottom of the plates were measured experimentally in work reported in previous papers.[4,5]

In this study, to analyze the transport phenomena between heated vertical parallel plates in the case where the distributor plates were attached at the top or bottom of the plates, the fundamental equations were derived under suitable assumptions and were solved numerically with suitable boundary conditions. The calculated velocity distribution between the plates and the heat transfer coefficient from the heated plate were compared with experimental results.[4,5]

## 1. Experimental Apparatus and Procedure

Since the experimental apparatus and the experimental procedure for measurements of temperature distribution and velocity distribution between the plates were described in the previous papers,[4,5] these facts are summarized briefly.

A schematic diagram of the experimental apparatus is shown in **Fig. 1(A)** and the main features of the vertical parallel plates are shown in **Fig. 1(B)**. The plates were 60 cm high, 45 cm wide and 2 mm thick.

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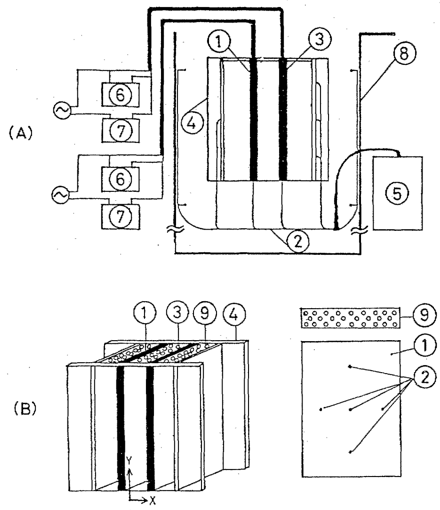


**Fig. 1.** Experimental apparatus

Kanthal A wire was installed in the plates as heat source and was heated isothermally. Distributor plates with suitable open ratio were attached at the top or bottom of the channel.

To measure cross-sectional temperature distribution between the plates, the gas temperature between the plates was measured by using a thermocouple probe with very small diameter (0.25 mm$\phi$).

Gas velocity between the plates was measured with a laser doppler velocitimeter (L.D.V.).[5] Experimental conditions were as follows. The temperature of the heated plate was 100°C or 150°C. The distance between the heated plate and the opposite plate was 1.3 cm, 2 cm, or 3 cm. The open ratio of the perforated plate was in the range of 0.08–1.0.

## 2. Fundamental Equations and Numerical Calculation Procedure

Let us consider the transport phenomenon between vertical parallel plates with asymmetric heating as shown in **Fig. 2**. The following assumptions are made. The physical properties of the fluid are constant and independent of temperature. The buoyancy is directly proportional to the temperature difference between TH and TA. The fundamental equations and boundary conditions are obtained as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{1}{\rho}\frac{\partial P}{\partial x}=\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \qquad (2)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\frac{1}{\rho}\frac{\partial P}{\partial y}=\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-g\beta(T-TA) \qquad (3)$$

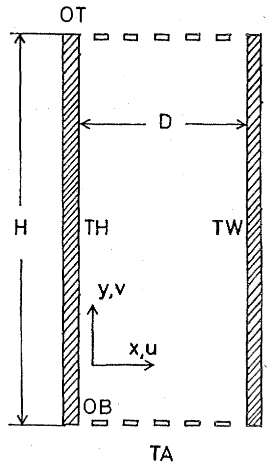


**Fig. 2.** Coordinates system

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{Pr}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \qquad (4)$$

(B.C.)

$$\begin{aligned} x&=0\ ; \quad u=v=0, \quad T=TH \\ x&=D\ ; \quad u=v=0, \quad T=TW \\ y&=0\ ; \quad u=0, \quad v=Vo, \quad T=TA \\ y&=H; \quad \partial u/\partial y=\partial v/\partial y=\partial T/\partial y=0 \end{aligned} \qquad (5)$$

where entrance velocity $Vo$ was obtained as follows. In two-dimensional laminar flow between the parallel plates, the pressure drop is given by Eq. (6):

$$\Delta P_f=4f\left(\frac{\rho Vo^2}{2}\right)\cdot(H/D)\cdot 10^{-1} \qquad (6)$$

For free-convective flow, this flow resistance must be balanced by the buoyant potential head as

$$\Delta P=(\rho_A-\bar{\rho})gH\cdot 10^{-1} \qquad (7)$$

where

$$\bar{\rho}=\frac{1}{H\cdot D}\int_0^H\int_0^D \rho dxdy$$

Equating Eqs. (6) with (7), $Vo$ is obtained. When the distributor plates are attached at the top or bottom of the channel, the pressure drops due to the distributor plates must be added to $\Delta P_f$.

$$\Delta P=\Delta P_f+\Delta P_T+\Delta P_B \qquad (8)$$

where $\Delta P_T$ and $\Delta P_B$, respectively, are the pressure drops for the distributor plates at the top and bottom of the channel and are expressed as

$$\Delta P_T=C_1\frac{\rho Vo^2}{OT^2}, \qquad \Delta P_B=C_2\frac{\rho Vo^2}{OB^2} \qquad (9)$$

Friction coefficient $f$ is expressed as

$$f=\frac{20}{Re}$$

$C_1$ and $C_2$ were obtained from the experimental results and were $(0.32\times10^{-4}/\rho)D^2$ and $(0.24\times10^{-4}/\rho)D^2$, respectively. The dimensionless variables were introduced and the dimensionless stream function $\psi$ and vorticity $\xi$ were defined. Each equation was written in a finite difference form (upwind method) and then was solved numerically by a relaxation method. The coefficient of local heat transfer from the heated plate was calculated as

$$h_{\mathrm{loc}}=\frac{\lambda(\partial T/\partial x)_{x=0}}{(TH-TA)} \tag{10}$$

The vertical gas velocity and gas temperature distributions between the plates were not easily measured. In this study, the heat transfer coefficient was calculated based upon $(t_H-t_A)$.

The average heat transfer coefficient and Nusselt number were obtained as

$$\bar{h}=\frac{1}{\mathrm{H}}\int_0^H h_{\mathrm{loc}}\,dy \tag{11}$$

$$\overline{Nu}=\frac{D\bar{h}}{\lambda} \tag{12}$$

The calculation was performed over the ranges of $Gr=2\times(10^3\text{–}10^6)$, $Pr=0.7$, $TH=100\text{–}150°\mathrm{C}$, $TW=20\text{–}100°\mathrm{C}$, $D=0.7\text{–}3.0\,\mathrm{cm}$, $OT=0.06\text{–}1.0$ and $OB=0.06\text{–}1.0$.

## 3. Calculation Results and Comparison of Calculated with Experimental Ones

The calculated velocity and temperature distributions and average Nusselt number were obtained for various conditions.

**Figures 3(a)** and **3(b)**, show the velocity distributions in the channel in the case where the distributor plate is not attached and where, respectively, one plate was heated and both plates were heated. In Fig. 3, the lines show the calculated results and the plots are the experimental results[5] under the same conditions as those of this numerical calculation. As can be seen from Fig. 3, the calculated velocity distributions are in a sufficiently good approximation to the experimental ones.[5]

**Figure 4** shows the vertical velocity distribution at $y=10$ cm with $D$ as a parameter. From Fig. 4, the calculated results agree approximately with the experimental ones.[5]

**Figures 5(a), 5(b)** and **5(c)**, respectively, show the calculated velocity distributions with experimental ones[5] in the case of distributor plate attached at the top or bottom or both top and bottom in the channel. The calculated vertical velocity distribution agrees approximately with the experimental ones.

**Figures 6(a)** and **6(b)** show the effect of open ratio of the distributor plate upon the calculated or

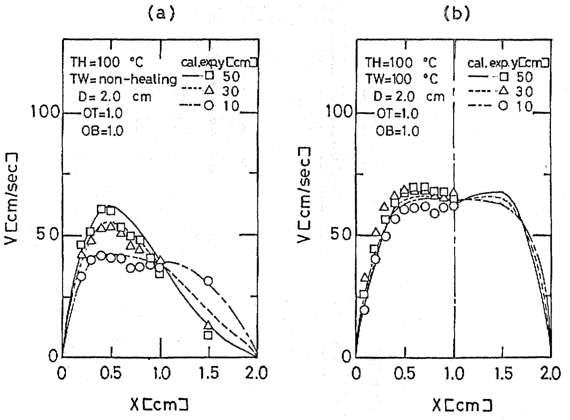


Fig. 3. Comparison of calculated vertical velocity distribution with experimental values

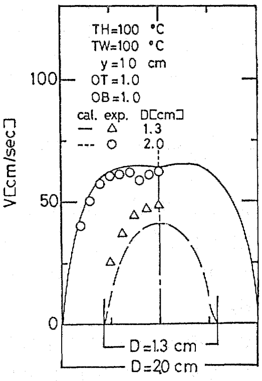


Fig. 4. Comparison of calculated vertical velocity distribution with experimental values

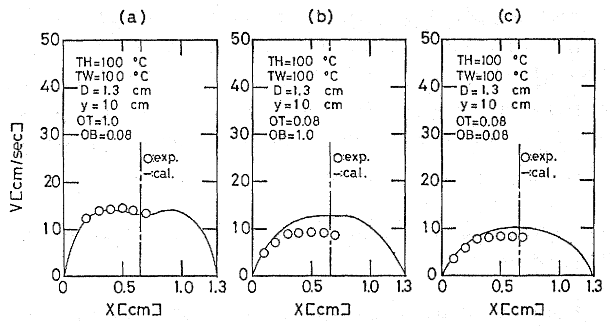


Fig. 5. Comparison of calculated vertical velocity distribution with experimental values

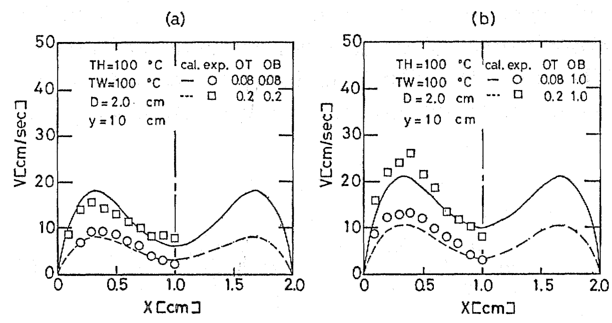


Fig. 6. Comparison of calculated vertical velocity distribution with experimental values

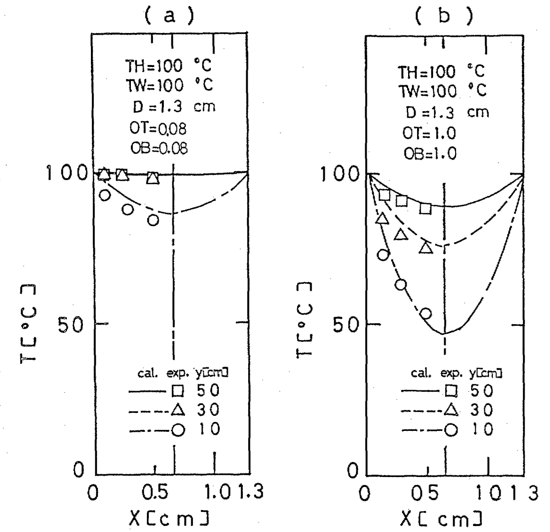


Fig. 7. Comparison of calculated temperature distribution with experimental values

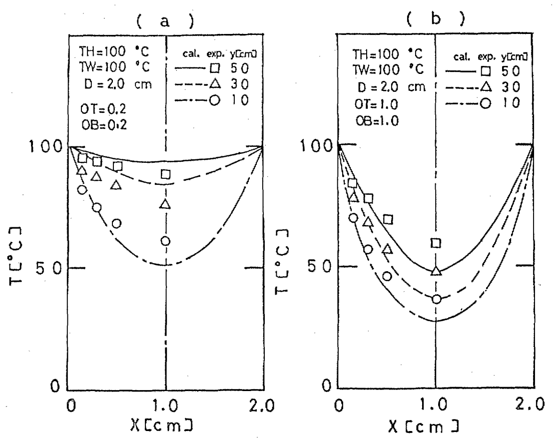


Fig. 8. Comparison of calculated temperature distribution with experimental values

measured velocity distribution[5] in the case of $D=2.0$ cm. From these figures, the local velocity between the plates is strongly affected by the open ratio of the distributor plates.

**Figures 7** and **8**, respectively, show comparison of calculated temperature distributions[4] with experimental ones with the distance from the bottom of the plate, $y$, as the parameter in the case of $D=1.3$ cm and $D=2.0$ cm. From these figures, the calculated temperature distributions agree approximately with experimental ones. If the distributor plate is attached at the top or bottom of the channel, the gas temperature in the channel increases remarkably as shown in Figs. 7(a) and 8(a).

The vertical average gas velocity between the plates can be calculated from the local gas velocity, $v$. **Figure 9** compares the calculated average gas velocity with experimental one[5]. From Fig. 9, the calculated average gas velocity agrees well with the experimental values.[5]

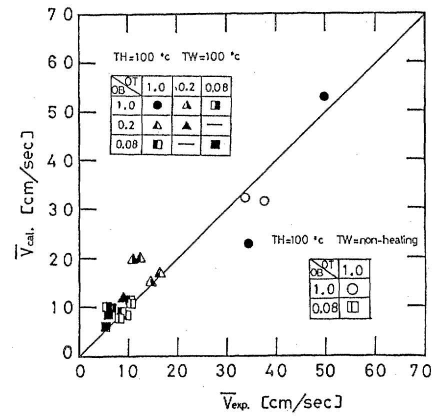


**Fig. 9.** Relation between $\bar{V}_{cal}$ and $\bar{V}_{exp}$

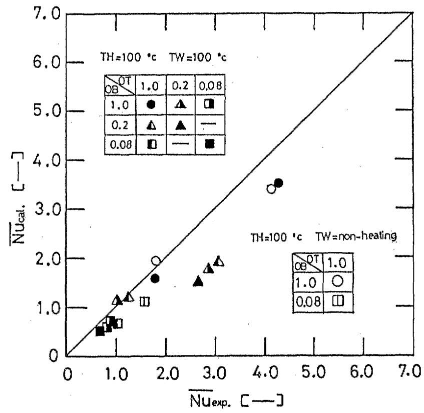


**Fig. 10.** Relation between $\overline{Nu}_{cal}$ and $\overline{Nu}_{exp}$

The average Nusselt number from the heated plate is calculated from Eqs. (10), (11) and (12). **Figure 10** compares the calculated $\overline{Nu}$ with experimental ones.[4] From this figure, the calculated $\overline{Nu}$ approximately agrees with experimental ones.[4]

## Conclusion

To analyze the transport phenomena between heated vertical parallel plates, the fundamental equations were derived and solved numerically.

The calculated gas velocity and gas temperature distributions between the plates and the average Nusselt number agreed approximately with the experimental ones.

### Nomenclature

| | | |
|---|---|---|
| $C_p$ | = specific heat of fluid | [J/g K] |
| $C_1$ | = friction coefficient of distributor plates attached at top of channel | [—] |
| $C_2$ | = friction coefficient of distributor plates attached at bottom of channel | [—] |
| $D$ | = distance between plates | [cm] |
| $f$ | = friction coefficient | [—] |
| $g$ | = gravitational acceleration | [cm/s²] |
| $H$ | = height of plate | [cm] |
| $\bar{h}$ | = average coefficient of heat transfer from heated plate | [W/cm² K] |
| $h_{loc}$ | = local coefficient of heat transfer | |

| | | |
|---|---|---|
| | from heated plate | [W/cm$^2$ K] |
| $\overline{Nu}$ | = average Nusselt number ($=D\bar{h}/\lambda$) | [—] |
| $\overline{Nu}_{cal}$ | = calculated average Nusselt number | [—] |
| $\overline{Nu}_{exp}$ | = experimental average Nusselt number | [—] |
| $OB$ | = open ratio of distributor plate attached at bottom of channel | [—] |
| $OT$ | = open ratio of distributor plate attached at top of channel | [—] |
| $Pr$ | = Prandtl number ($=Cp\,\mu/\lambda$) | [—] |
| $\Delta P$ | = static pressure difference due to density difference of fluid | [Pa] |
| $\Delta P_B$ | = pressure drop due to distributor plate attached at bottom of the channel | [Pa] |
| $\Delta P_f$ | = pressure drop due to hydrodynamic resistance of fluid across the plates | [Pa] |
| $\Delta P_T$ | = pressure drop due to distributor plate attached at top of channel | [Pa] |
| $p$ | = fluid pressure | [Pa] |
| $Re$ | = Reynolds number ($=D\,Vo/\nu$) | [—] |
| $T$ | = temperature of gas | [°C] |
| $TA$ | = room temperature | [°C] |
| $TH$ | = temperature of heated plate | [°C] |
| $TW$ | = temperature of opposite plate | [°C] |
| $u$ | = horizontal gas velocity | [cm/s] |
| $v$ | = vertical gas velocity | [cm/s] |
| $Vo$ | = vertical gas velocity at inlet of channel | [cm/s] |
| $\bar{V}_{cal}$ | = calculated vertical average gas velocity in channel | [cm/s] |
| $\bar{V}_{exp}$ | = measured vertical average gas velocity in channel | [cm/s] |
| $x$ | = horizontal coordinate | [cm] |
| $y$ | = vertical coordinate | [cm] |
| $\beta$ | = volumetric coefficient of expansion | [1/K] |
| $\lambda$ | = thermal conductivity of fluid | [W/cm K] |
| $\nu$ | = kinematic viscosity of fluid | [cm$^2$/s] |
| $\rho_A$ | = density of fluid at room temperature | [g/cm$^3$] |
| $\bar{\rho}$ | = average density of fluid in channel | [g/cm$^3$] |

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