FLOW CHARACTERISTICS AND CIRCULAR PIPE FLOW OF PULP-SUSPENSION

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An experimental investigation of a laminar plug flow of pulp-suspension in a circular pipe was made. The pressure loss was measured by pressure transducers, and the velocity gradient at pipe wall and the local velocity were measured by an electrochemical technique.

From experimental relations between the pressure loss and the flow rate, and between the shear stress and the velocity gradient at pipe wall, the pulp-suspensions should be regarded as Newtonian liquids, though the values of viscosity determined from those relations were undoubtedly different from each other. On the other hand, both the measured velocity profile and the experimental relation between the velocity gradient at pipe wall and the flow rate were not those for a Newtonian liquid.

This inconsistency could be explained by the consideration that the pulp-suspension had a particular average viscosity to the pulp fiber concentration prepared and behaved as a Newtonian liquid at every radial position with different viscosity which was distributed in the radial direction in a circular pipe. Furthermore, an appropriate formula for the radial distribution of viscosity was presented. The calculated velocity profiles based on the formula coincided well with the measured ones.

Introduction

The flow mechanism and rheological properties of pulp-suspensions have not been understood sufficiently, though they greatly influence papermaking processes.

Circular pipe flows of pulp-suspension have been classified into three flow steps: plug flow, mixed flow and turbulent flow.[6] However, the details of these flow steps have not been made clear. Velocity profiles were measured,[4,5] by using impact probes sizes that do not seem to be small enough compared with pipe inner diameters. In addition, the measurements were carried out mainly near the pipe wall under the preconception that the velocity profiles near the pipe axis were perfectly flat. As noted above, the velocity profile, even if it is of a plug flow, has not been investigated sufficiently. This insufficiency is due to underdevelopment of methods for measuring local velocity of inhomogeneous pulp-suspensions.

Moreover, the pressure loss of the pulp-suspension flow in a circular pipe has been treated as a function of cross-averaged velocity as follows. When the velocity is relatively small, the pressure loss takes a larger value than that of water, and the value shows wavy changes with increasing velocity. On the other hand, when the velocity becomes as large as that for mixed or turbulent flow, the pressure loss becomes smaller than that of water. But there have been no systematic studies of the pressure loss using the Reynolds number and the like, because the viscosity of the pulp-suspension has not been defined clearly.

The purpose of this paper is to propose a model for the flow characteristics of a laminar plug flow of pulp-suspensions in a circular pipe based on experiments involving the relationship among pressure loss, velocity gradient at wall and local velocity.

1 Experimental

Test pulp-suspension consisted of beaten hardwood bleached kraft pulp, ion-exchanged water, and electrolytes for the electrode reaction controlled by mass transfer rate (3 mol/m^3 potassium ferrocyanide and potassium ferricyanide and 100 mol/m^3 potassium chloride).[2,3,7] **Table 1** shows the weight-concentration of the pulp fiber and the index of degree of beating CSF (Canadian Standard of Freeness) of the test pulp-suspensions. In addition, the filtrate of the suspension having a pulp fiber concentration of apparently 0 wt% and ion-exchanged water with the electrolytes were also used. The filtrates were confirmed to be Newtonian liquids by use of capillary viscometers.

Figure 1 shows the experimental apparatus

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Table 1. Pulp fiber concentration and CSF

Concentration [wt%]	CSF [ml]
0.13	290
0.24	250
0.33	340
0.53	350

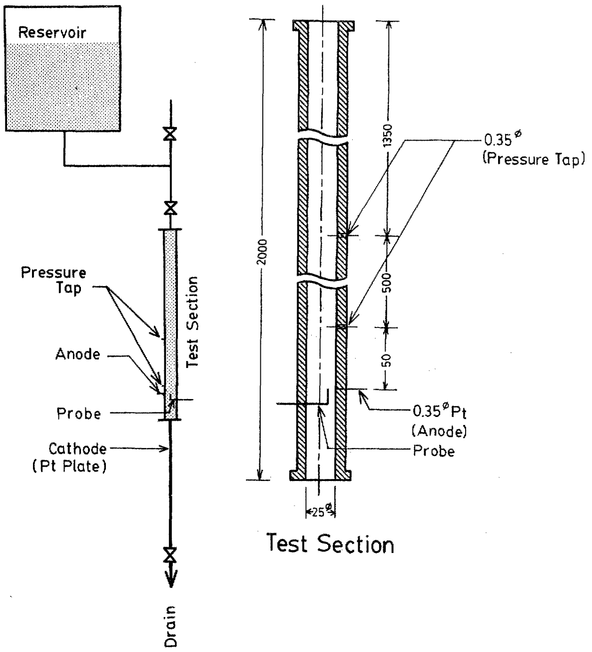

Fig. 1. Experimental apparatus

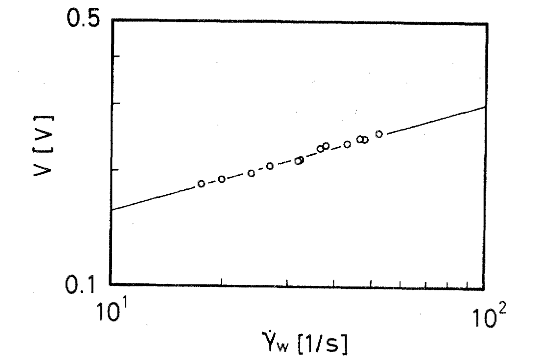

Fig. 2. Relation between output voltage and velocity gradient at wall

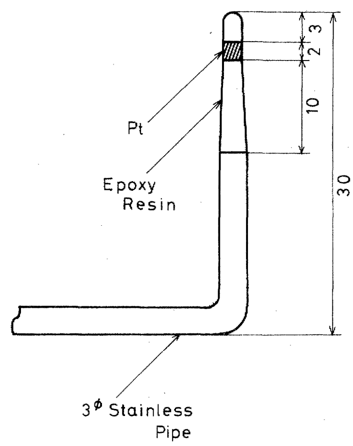

Fig. 3. Velocity-measuring probe

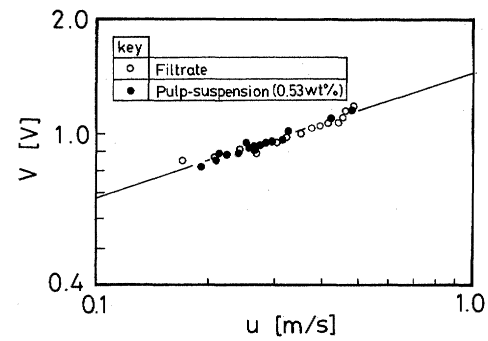

Fig. 4. Calibration line of velocity-measuring probe

schematically. The test section, made of acrylic resin (25 mm inner diameter and 2 m length) was set vertically. The flow rate was regulated by the valve connected to the end of the test section.

Two pressure taps were set 1350 mm and 1850 mm downstream of the entrance of the test section, respectively, and the pressure loss was measured by pressure transducers.

The velocity gradient at the wall was measured by a platinum anode (0.35 mm diameter) which was plugged into the test pipe wall and was flush with the inner wall surface.[2,3,7] This measuring method was useful because it was confirmed by observation that there was no pulp fiber near the wall in the case of the pulp-suspension flows also, and that only the filtrate covered it. An example of the experimental relation between the output voltage converted from diffusional electric current and the velocity gradient at the wall for the case of the filtrate is shown in **Fig. 2**. The solid line correlates the results sufficiently, and this relation was used to calibrate the electrode thereafter.

To measure the local velocity of the pulp-suspension flows, a new type of electrode probe was designed as shown in **Fig. 3**. A belt-shaped platinum electrode was set in the rear of the blunt nose tip. Every pulp fiber flows along the blunt nose tip and the belt-shaped electrode was scarcely covered by pulp fibers. The probe was set 1900 mm downstream of the entrance of the test section and the local velocity was measured at six radial positions by traversing the probe. Calibration of the probe was carried out by using a rotating annular open channel[1] filled with the same pulp-suspension as that for the pipe flow. The output voltage converted from diffusional electric current and the velocity were correlated well by a straight line as shown in **Fig. 4**. Additionally, it was confirmed clearly that the filtrate gave the same result as that of the pulp-suspension.

2 Flow Characteristics of Pulp-Suspension

Under the assumption that the flow characteristics of the pulp-subspension are uniform in a circular pipe flow, its flow characteristics are discussed as follows.

The experimental relation between the pressure loss per unit length and the flow rate for each pulp fiber concentration is shown in **Fig. 5**. The pressure loss for any pulp fiber concentration seems to be proportional to the flow rate, allowing for experimental error. From this figure, the pulp-suspensions should be regarded as Newtonian liquids, and it is possible to determine the viscosity μ_a for each pulp-suspension from the slope of each straight line based on Hagen-Poiseuille's law.

$$\mu_a = \frac{(\Delta P/L)\pi R^4}{8Q} \tag{1}$$

By using the viscosity μ_a, the Reynolds number can be defined as

$$Re = \frac{2\rho u_a R}{\mu_a} \tag{2}$$

where u_a is the cross-average velocity and R is the pipe inner radius. The experimental relation between the Reynolds number and the friction factor is shown in **Fig. 6**. All the data can be represented satisfactorily by the following equation, the same as in the case of laminar flow of Newtonian liquids.

$$f = 16/Re \tag{3}$$

From this result, the pulp-suspension flows under these experimental conditions were confirmed to be sufficiently laminar.

3 Velocity Gradient at Wall

In the case of a Newtonian liquid, the relation between the velocity gradient at the wall and the flow rate should be described as

$$\dot{\gamma}_w = \frac{4}{\pi R^3} Q \tag{4}$$

The experimental relation between the velocity gradient at the wall and the flow rate for each pulp fiber concentration is shown in **Fig. 7**. The solid line shows the relation written with Eq. (4). Except for open and solid circle keys, which are for the water and the filtrate (0 wt%), respectively, almost all keys are clearly above the line. That is to say, the measured values of the velocity gradient at the wall are larger than those of Newtonian liquids. From this result it seems that the pulp-suspensions cannot be Newtonian liquids. Therefore, this result contradicts the experimental relation between the pressure loss and the flow rate described above. However, considering that no pulp fiber can be observed near the wall and that the flow characteristics of the pulp-suspension are not uniform in the radial direction when it flows through a circular pipe, the following hypotheses effectively explain the inconsistency described above.

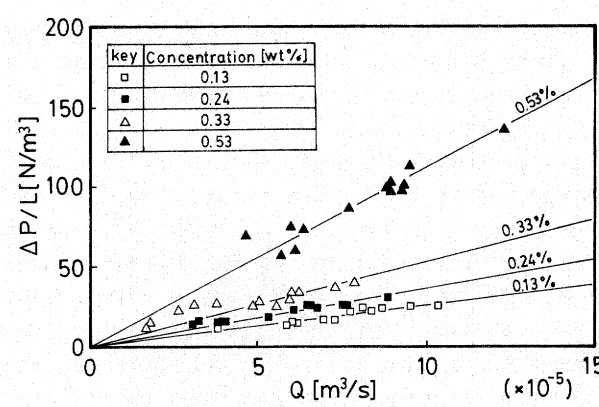

Fig. 5. Relation between pressure loss and flow rate

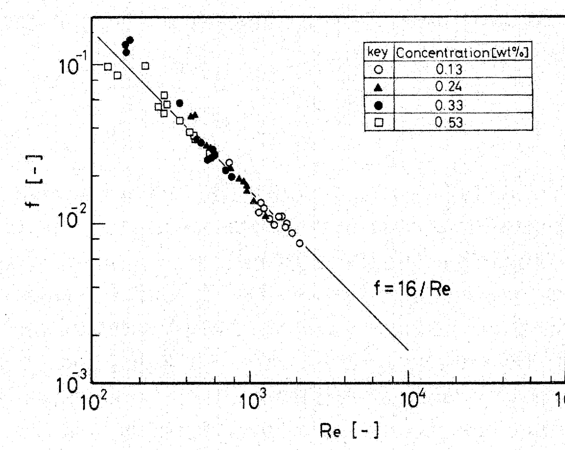

Fig. 6. Relation between Re and f

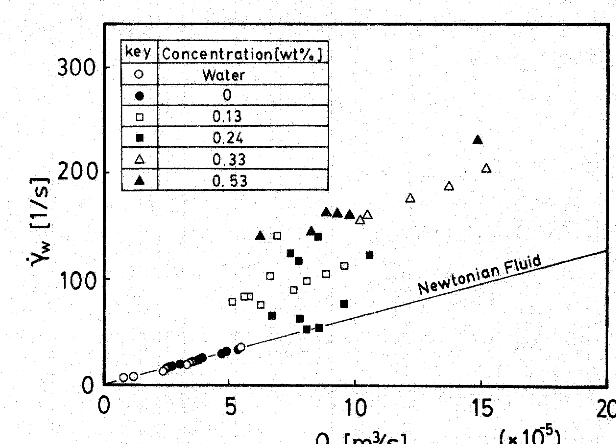

Fig. 7. Relation between velocity gradient at wall and flow rate

1. The local concentration of the pulp fiber increases as the radial position approaches the pipe axis by a kind of Magnus effect on the pulp fibers, and the values of the concentration in Table 1 can be recognized to be the cross-average values of the local concentration.
2. The viscosity of the pulp-suspension at a radial position is different from that at another position

according to the local concentration of the pulp fiber. However, at every radial position the pulp-suspension behaves as a Newtonian liquid with respective local viscosity.

On the basis of these hypotheses, the viscosity μ_a obtained from Fig. 5 can be recognized as the cross-average viscosity.

It might be thought that the rheological behavior of the pulp-suspension is expressed by a power-law model. However, this way of thinking is not appropriate because it is impossible to obtain the results shown in Figs. 5 and 6 for the liquids of the power-law model.

On the other hand, the shear stress at the wall is expressed by using pressure loss per unit length and the pipe innter radius as

$$\tau_w = \frac{\Delta P}{L} \cdot \frac{R}{2} \tag{5}$$

In the case of Newtonian liquids, the velocity gradient at the wall is proportional to this shear stress at the wall. **Figure 8** shows the experimental relation between the shear stress expressed by Eq. (5) and the velocity gradient at the pipe wall. Almost all data for each pulp fiber concentration are obviously on the respective straight lines through the origin and it is confirmed again that the pulp-suspensions behave as Newtonian liquids in the vicinity of the wall. The Viscosity at the wall μ_w for each pulp fiber concentration can be decided from the slope of each straight line as

$$\mu_w = \frac{(\Delta P/L)R}{2\dot{\gamma}_w} \tag{6}$$

The viscosity μ_w was compared with the cross-average viscosity μ_a described before as shown in **Fig. 9**. As is obvious from this figure, μ_a takes larger values than μ_w at any pulp fiber concentration. Additionally, both μ_a and μ_w tend to increase with increasing pulp fiber concentration.

Using both viscosities, μ_a and μ_w, the velocity gradient at the wall can be related to the flow rate as

$$\dot{\gamma}_w = \frac{4}{\pi R^3} \cdot \frac{\mu_a}{\mu_w} Q \tag{7}$$

The experimental relation between $\dot{\gamma}_w$ and $(\mu_a/\mu_w) \cdot Q$ is shown in **Fig. 10**. Almost all the data can be expressed by the straight line through the origin with theoretical slope of $4/(\pi R^3)$.

4 Velocity distribution

Figure 11 shows examples of the measured local velocity. It is obvious that the velocity profiles estimated are very different from the parabolic ones for laminar Newtonian liquids. However, existence of the complete plug is not recognized.

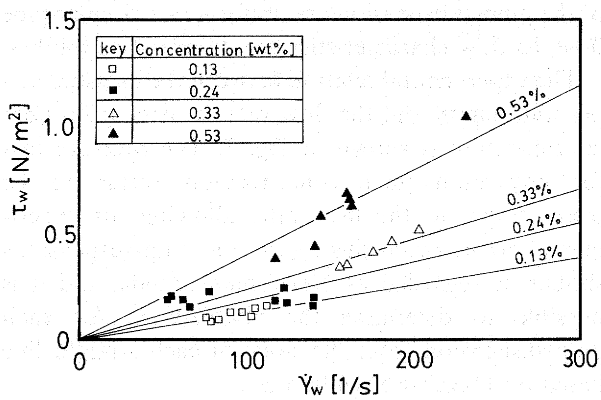

Fig. 8. Relation between shear stress and velocity gradient at wall

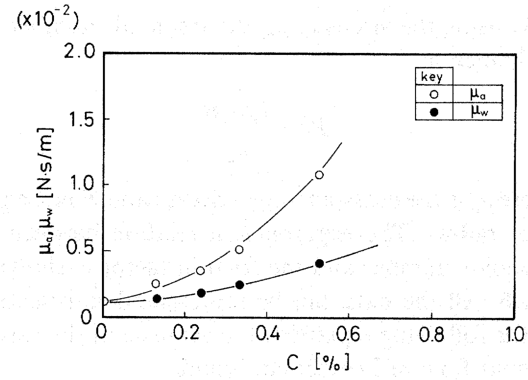

Fig. 9. Relation between μ_a or μ_w and pulp fiber concentration

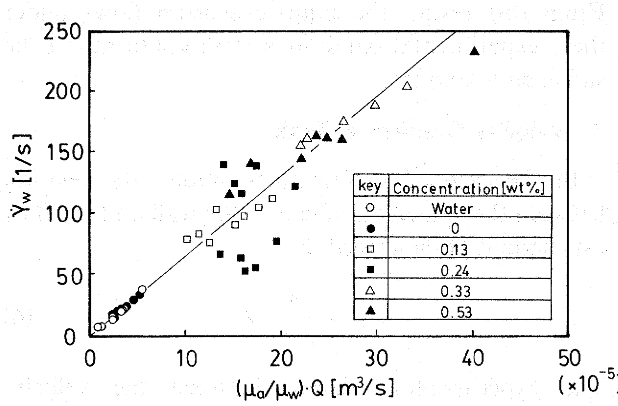

Fig. 10. Relation between $\dot{\gamma}_w$ and $(\mu_a/\mu_w) \cdot Q$

To propose a suitable expression of the velocity profiles, the radial distribution of the local viscosity should be estimated. Considering the hypothesis that the pulp-suspension behaves as a Newtonian liquid at every radial position with respective local viscosity, the velocity gradient at an arbitrary radial position r/R is expressed as

$$\dot{\gamma} \equiv \frac{du}{dr} = -\frac{\Delta P}{2\mu L} r \tag{8}$$

where μ is the local viscosity at radial position r. If the local viscosity is expressed as a function of r/R,

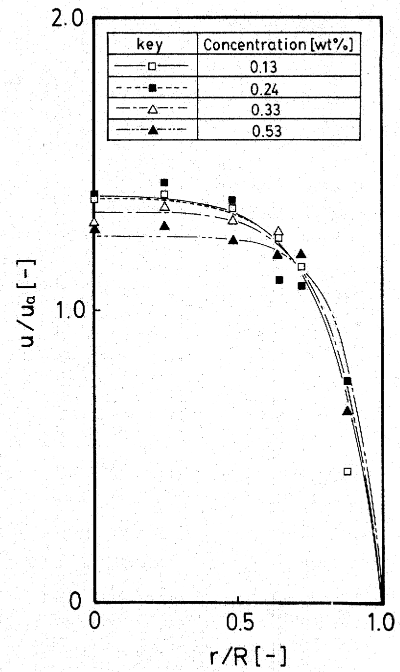

Fig. 11. Radial distribution of velocity

Eq. (8) can be solved. As a tentative trial, it is assumed that the radial distribution of the local viscosity changes exponentially with radial position as

$$\mu = \mu_w \exp\{A(1-r/R)\} \tag{9}$$

where A is the parameter which varies with the characteristics of a prepared pulp-suspension, e.g., pulp fiber concentration, CSF and so on. When the parameter takes zero value, the local viscosity takes constant values regardless of the radial position, the same as in the case of a Newtonian liquid. By substituting Eq. (9) into Eq. (8) and integration with respect to r, the velocity at an arbitrary radial position can be given as

$$u = -\frac{\Delta PR^2}{2A^2\mu_w L}[\{A(r/R)-1\}\exp\{-A(1-r/R)\}-A+1] \tag{10}$$

The integration of Eq. (10) over the pipe cross section gives the flow rate as

$$Q = \int_0^R 2\pi rudr = \frac{\Delta P\pi R^4}{A^2\mu_w L}\left[-\frac{3}{2}+\frac{3}{A} - \frac{3}{A^2}\{1-\exp(-A)\}+\frac{A}{2}\right] \tag{11}$$

On the other hand, the following equation based on Hagen-Poiseuille's law also gives the flow rate.

$$Q = \frac{\Delta P\pi R^4}{8\mu_a L} \tag{12}$$

Comparing Eqs. (11) and (12), it is clear that the following relation exists between the viscosities μ_a and μ_w.

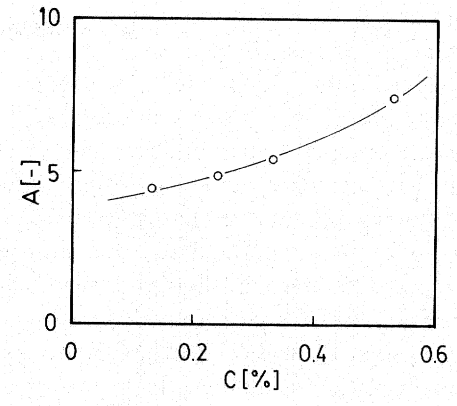

Fig. 12. Relation between A and pulp fiber concentration

$$\mu_a = \frac{A^2\mu_w}{-12+\frac{24}{A}-\frac{24}{A^2}\{1-\exp(-A)\}+4A} \tag{13}$$

Therefore, the parameter A for each pulp fiber concentration can be determined by substituting respective experimental values of μ_a and μ_w into Eq. (13). Additionally, by substituting the value of A determined for each pulp fiber concentration into Eq. (10), the radial distribution of velocity can be calculated. Each line in Fig. 11 shows the radial distribution of velocity for each pulp fiber concentration obtained according to the above steps. The calculated distribution agrees approximately well with experimental data. According to this agreement, the hypothesis that the pulp-suspension behaves as a Newtonian liquid at every radial position with respective viscosity and the assumption that the radial distribution of local viscosity is expressed by Eq. (9) is considered to be adequate. There is no clear, definite reason why the radial distribution of local viscosity should be expressed by Eq. (9). However, considering that the degree of cohesion of the pulp fiber seems to increase exponentially from the wall to the center, Eq. (9) may have a significant relation to the local pulp fiber concentration at each radial position.

Figure 12 shows the relation between the parameter A and the pulp fiber concentration. The parameter A decreases gradually as the pulp fiber concentration decreases, the same as in the cases of μ_a and μ_w, and takes zero value in the extreme when the concentration approaches zero. It is possible to estimate the velocity profile for an arbitrary pulp fiber concentration if the values of μ_a and μ_w of the pulp-suspension are measured. However, the experimental range of the pulp fiber concentration in these experiments is not sufficiently wide and the radial distribution of the viscosity should be further studied to obtain clearer information about the pulp-suspension flow.

Conclusion

The plug flow of the pulp-suspension is investigated by measuring the pressure loss, the velocity gradient at the wall and the local velocity.

Though the pulp-suspensions are considered to be Newtonian liquids from the experimental relations between the pressure loss and the flow rate and between the shear stress and the velocity gradient at the wall, the experimental relation between the velocity gradient at the pipe wall and the flow rate was not that for a Newtonian liquid. These different experimental results are shown to be consistent by considering that the viscosity of the pulp-suspension at a radial position is different from that at another position according to the radial distribution of pulp fiber concentration. In addition, by assuming a tentative formula for the radial distribution of the viscosity, the calculated velocity profiles agree with the experimental ones.

Nomenclature

A	= parameter	[—]
C	= pulp fiber concentration	[wt%]
f	= friction factor	[—]
L	= axial distance	[m]
Q	= flow rate	[m^3/s]
R	= pipe inner radius ($=D/2$)	[m]
Re	= Reynolds number	[—]
r	= radius	[m]
u	= velocity	[m/s]
u_a	= cross-average velocity	[m/s]
V	= output voltage	[V]
$\dot{\gamma}$	= velocity gradient	[1/s]
$\dot{\gamma}_w$	= velocity gradient at pipe wall	[1/s]
ΔP	= pressure loss	[N/m^2]
μ	= viscosity	[kg/(m·s)]
μ_a	= cross-average viscosity	[kg/(m·s)]
μ_w	= viscosity at pipe wall	[kg/(m·s)]
ρ	= density	[kg/m^3]

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