CALCULATION OF DRAG COEFFICIENTS AND MASS TRANSFER OF TWO ADJACENT SPHERES OF VARIOUS DIAMETER RATIOS

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In our previous papers, both numerical[6] and experimental[2] approaches to the effect of interaction between two spheres on drag coefficients and diffusion fluxes of two spheres were made for the case of coaxially arranged spheres of nearly the same diameter. The purpose of the present paper is to make a further numerical approach to this effect for the more general case of the interaction between two spheres of various diameters by applying the same technique.

1. Calculation Method

The governing equations and the boundary conditions are summarized in **Table 1**. The outer boundary of the grid systems, R_∞, in Table 1 is calculated by the following equations.

$D_B/D_A>1$:

$$R_\infty=\frac{1}{|\sinh\eta_B|}\left\{(K+1)+\frac{|\sinh\eta_B|^2}{(\cosh\eta_B+1)}\right\} \tag{4}$$

$D_B/D_A<1$:

$$R_\infty=\frac{1}{|\sinh\eta_A|}\left\{(K+1)+\frac{|\sinh\eta_A|^2}{(\cosh\eta_A+1)}\right\} \tag{5}$$

Calculations were made in a similar way as in the references.[3,6] The drag coefficients and the diffusion fluxes were calculated by using Eqs. (19)–(29) of the reference.[6]

2. Results and Discussion

1) Drag coefficients Total drag coefficients of the front and rear spheres are affected by the diameter ratios, D_B/D_A, and the distance between two spheres, L/D_A, as was shown in Figs. 2 and 3 of the reference.[3]

Considering the results of the previous numerical analysis[6], numerical data for the drag coefficients of the front and rear spheres were well correlated by the following equations with maximum deviation less than 5%.

Front sphere:

$$\frac{C_{DA}}{C_{D0}}=\frac{1}{1+0.32(D_B/D_A)^{2.22}\cdot(L/D_A)^{-1.40Re_{PA}^{0.42}}} \tag{6}$$

Rear sphere:

$$\frac{C_{DB}}{C_{D0}}=\frac{1}{1+0.53Re_{PA}^{0.31}(D_B/D_A)^{-0.43}\cdot(L/D_A)^{-0.90Re_{PA}^{-0.06}}} \tag{7}$$

The ranges of the variables for the correlations are:

$$Re_{PA}:\quad 1.0-30.0$$

Table 1. Governing equations and boundary conditions

Navier-Stokes equation

$$\sin\xi\cdot(\cosh\eta-\cos\xi)\cdot\left(\frac{\partial\psi}{\partial\eta}\cdot\frac{\partial}{\partial\xi}-\frac{\partial\psi}{\partial\xi}\cdot\frac{\partial}{\partial\eta}\right)\cdot\frac{(\cosh\eta-\cos\xi)^2}{\sin^2\xi}E^2\psi=\frac{1}{Re_c}E^2E^2\psi \tag{1}$$

Diffusion equation

$$\frac{\partial\psi}{\partial\eta}\frac{\partial\theta_c}{\partial\xi}-\frac{\partial\psi}{\partial\xi}\frac{\partial\theta_c}{\partial\eta}=\frac{1}{Re_cSc}\left\{-\frac{\sin\xi\sinh\eta}{(\cosh\eta-\cos\xi)^2}\frac{\partial\theta_c}{\partial\eta}+\frac{\cos\xi\cosh\eta-1}{(\cosh\eta-\cos\xi)^2}\frac{\partial\theta_c}{\partial\xi}+\frac{\sin\xi}{\cosh\eta-\cos\xi}\frac{\partial^2\theta_c}{\partial\eta^2}+\frac{\sin\xi}{\cosh\eta-\cos\xi}\frac{\partial^2\theta_c}{\partial\xi^2}\right\} \tag{2}$$

Boundary conditions

$$\xi=0\ :\ \psi=0 \tag{3.a}$$
$$\partial\theta_c/\partial\xi=0 \tag{3.b}$$
$$\xi=\pi\ :\ \psi=0 \tag{3.c}$$
$$\partial\theta_c/\partial\xi=0 \tag{3.d}$$
$$\eta=\eta_A,\eta_B\ :\ \psi=0 \tag{3.e}$$
$$\theta_c=1 \tag{3.f}$$

Outer boundary $X=R_\infty$

$$:\ \psi=\frac{\sin^2\xi}{2(\cosh\eta-\cos\xi)^2} \tag{3.g}$$
$$\theta_c=0 \tag{3.h}$$

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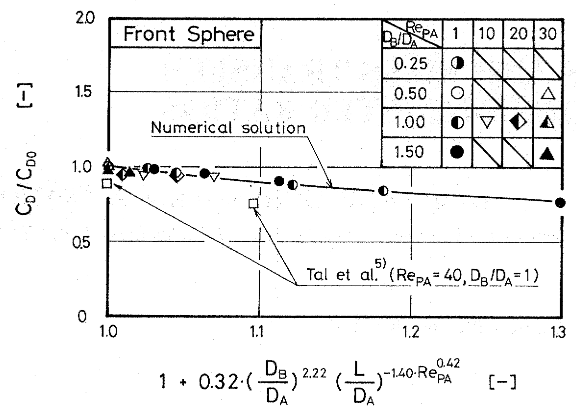

Fig. 1a. General correlation of the effect of distance between two spheres and of diameter ratios on the drag coefficients of the front sphere

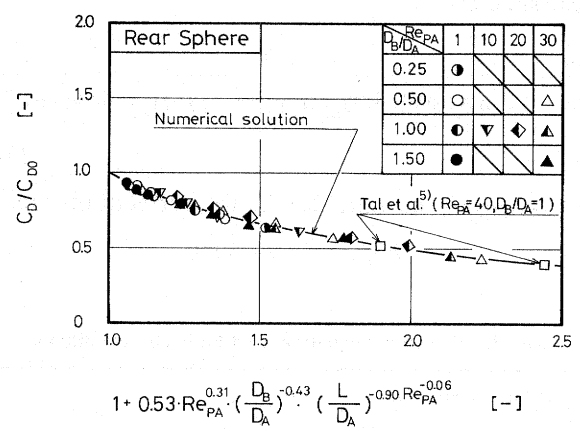

Fig. 1b. General correlation of the effect of distance between two spheres and of diameter ratios on the drag coefficients of the rear sphere

$$L/D_A : \ 1.5 - 10.0$$

$$D_B/D_A : \ 0.25 - 1.50$$

Figures 1a and **b** show a comparison of the numerical data for the total drag coefficients of the front and rear spheres with the proposed correlation. Good agreement is observed between the numerical data and the correlation. For comparison, numerical data by Tal *et al.*[5] at $Re_{PA}=40$ and $D_B/D_A=1$ are also shown in the figures.

The effect of L/D_A on the drag coefficients of the rear sphere, estimated at $Re_{PA}=30$ by using the above correlation showed good agreement with the numerical data by Tal *et al.*[5] at $Re_{PA}=40$ and Chen and Tong[4] at $Re_{PA}=200$. This may indicate that the effect of Reynolds number on the interaction of rear sphere become less considerable if Re_{PA} is greater than 30.

2) Diffusion fluxes The effect of D_B/D_A and L/D_A on the diffusion fluxes of the front and rear spheres showed a similar tendency to that of the drag coefficients.

All the numerical data for the diffusion fluxes of the front and rear spheres were well correlated by the following equations with maximum deviation less than 5%:

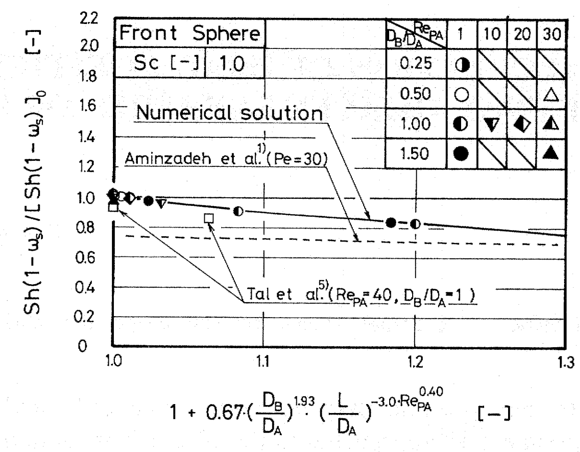

Fig. 2a. General correlation of the effect of distance between two spheres and of diameter ratios on the diffusion fluxes of the front sphere

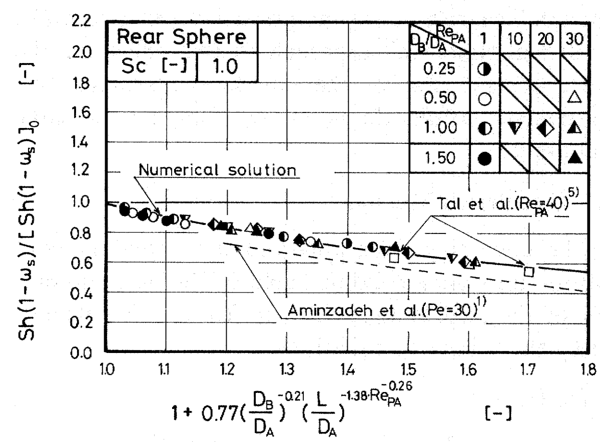

Fig. 2b. General correlation of the effect of distance between two spheres and of diameter ratios on the diffusion fluxes of the rear sphere

Front sphere:

$$\frac{Sh_A(1-\omega_S)}{[Sh(1-\omega_S)]_0} = \frac{1}{1+0.67(D_B/D_A)^{1.93} \cdot (L/D_A)^{-3.0 Re_{PA}^{0.40}}} \tag{8}$$

Rear sphere:

$$\frac{Sh_B(1-\omega_S)}{[Sh(1-\omega_S)]_0} = \frac{1}{1+0.77(D_B/D_A)^{-0.21} \cdot (L/D_A)^{-1.38 Re_{PA}^{-0.26}}} \tag{9}$$

The ranges of the variables for the correlations are:

$$Re_{PA} : \ 1.0 - 30.0$$

$$L/D_A : \ 1.5 - 10.0$$

$$D_B/D_A : \ 0.25 - 1.50$$

Sc : 1.0

Figures 2a and **b** show a comparison of the numerical data for the diffusion fluxes of the front and rear spheres with the proposed correlation. The solid lines in the figures represent the correlation. Good agreement between the present numerical data and the correlation is observed. For comparison, numerical data by Tal *et al.*[5] at $Re_{PA}=40$ and $D_B/D_A=1$ and numerical data by Aminzadeh *et al.*[1] at $Pe=30$ and $D_B/D_A=1$ are also shown in the figure.

Nomenclature

C_D	= total drag coefficient	[—]
c	= focal length of bipolar coordinate system	[m]
D_A	= diameter of front sphere	[m]
D_B	= diameter of rear sphere	[m]
$\mathscr{D}$	= binary diffusion coefficient	[m^2/s]
K	= constant for outer boundary	[—]
L	= distance between centers of the two spheres	[m]
Pe	= Peclet number	[—]
R_∞	= dimensionless distance for outer boundary defined by Eqs. (4), (5)	[—]
Re_c	= Reynolds number $(=cU_\infty/\nu)$	[—]
Re_{PA}	= Reynolds number $(=D_AU_\infty/\nu)$	[—]
Sc	= Schmidt number $(=\nu/\mathscr{D})$	[—]
Sh	= Sherwood number	[—]
U_∞	= free stream velocity	[m/s]
X	= x component of rectangular coordinate	[—]
η	= bipolar coordinate, normal to ξ	[—]
θ_c	= dimensionless concentration $(=(\omega_s-\omega_\infty)/(1-\omega_\infty))$	[—]
ν	= kinematic viscosity of gas	[m^2/s]
ξ	= bipolar coordinate, angle	[rad]
ψ	= dimensionless stream function	[—]
ω	= mass fraction	[—]

⟨Subscripts⟩

A	= front sphere
B	= rear sphere
S	= surface of sphere
∞	= free stream
0	= single sphere

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