INVESTIGATION OF A RUNNING CONDITION FOR A HEAT EXCHANGER NETWORK

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A method comprising two steps was developed to judge the feasibility and to determine a running condition when an operating condition is presented for an existing heat exchanger network. One step is to determine temperatures of each process stream at inlet and outlet of each unit by solving a linear programming problem which can be easily formulated from the operating condition and the network structure, and the other is to compare the necessary heat transfer areas calculated from these temperatures with those of the existing network. For a heat exchanger network which has freedom in assigning heat loads to the units, these two steps are repeated if necessary. The effectiveness of this method is demonstrated by solving example problems.

Introduction

The cost of heating and cooling utilities is often the dominant item in many process industries such as the petroleum and petrochemical industries. In these processes, heat exchanger networks are widely used because they play important roles in energy saving and cost reduction. Many studies of heat exchanger networks have been made, and it is useful for details to refer to the latest review by Gundersen and Naess.[3] Most of them have focused attention on synthesis methods under constant operating conditions where parameters (stream flowrates and supply/target temperatures) are specified with fixed values (for example, Muraki and Hayakawa,[5] Linnhoff and Hindmarsh,[4] Floudas *et al.*[2]). Unfortunately, in an industrial environment these parameters are subject to significant uncertainty with changes in the operating and economic environments of a process, so it is necessary to solve two problems: how to synthesize flexible heat exchanger networks for parameter changes, and how to run existing networks when different operating conditions are presented. For the former problem, several synthesis methods have been proposed (for example, Saboo and Morari,[6] Floudas and Grossmann[1]), but the latter has not been an active research topic in spite of its importance. Stream flowrates and supply/target temperatures specified by the environment, and the variables (for example, heat loads of units) controlled to satisfy the given specification, are respectively included in the operating and running conditions.

The purpose of this study is to develop a practical method for determining the running condition when an operating condition is given to an existing heat exchanger network, and judging whether or not the given operating condition is feasible. If units (heat exchanger, heater, and cooler) can handle variable heat loads by adjusting bypass valves, operators can run the heat exchanger network by knowing inlet and outlet temperatures of each unit. In this study a method is developed to determine these temperatures because the feasibility can be also examined by making use of them. But it is difficult to determine them directly from the specification of the heat exchanger network and the operating condition, because this is a multivariable, nonlinear problem with complicated constraints. Considering the heat exchanger network from two viewpoints—network structure and unit capacity (heat transfer area)—it is found that the problem is reduced to two subproblems. One is to determine inlet and outlet temperatures under a given operating condition from the viewpoint of a network structure, and the other is to compare the areas of the existing units and the necessary ones calculated from these temperatures. It is found under the assumption of constant heat capacity flowrates, which has been commonly used in studies of heat exchanger networks, that linear programming is useful for the former, and that a model can be easily formulated from a network structure and problem definition whether or not stream splits and pinch points exist in it. This study proposes a practical method in which these two subproblems are solved, and repeated if necessary. Its effectiveness is demonstrated through example problems of five process streams: two hot and three cold streams ($Sh_j, j=1$–2) ($Sc_j, j=1$–3).

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1. Pre-analysis

Before considering the solution of this problem, it is necessary to make clear the relation between the network structure and the degrees of freedom in assigning heat loads to the units when an operating condition (heat capacity flowrates, supply/target temperatures) is given to an existing heat exchanger network. Two networks of different structure, shown in **Fig. 1**, are adopted to investigate this relation.

Network (1) is composed of five units, and a heat balance equation is set up for each of five process streams by assigning heat loads to the units as shown in Fig. 1.

$$q_{E,1}+q_{E,2}=F_{H,1}(T_{H,1}(1)-T_{H,1}(2))$$
$$q_{E,3}+q_{E,4}=F_{H,2}(T_{H,2}(1)-T_{H,2}(2))$$
$$q_{E,3}=F_{C,1}(T_{C,1}(2)-T_{C,1}(1))$$
$$q_{E,1}=F_{C,2}(T_{C,2}(2)-T_{C,2}(1))$$
$$q_{H,1}+q_{E,4}+q_{E,2}=F_{C,3}(T_{C,3}(2))-T_{C,3}(1))$$

where $q_{E,i}(q_{H,i}, q_{C,i})$, $F_{H,j}(F_{C,j})$, $T_{H,j}(1)(T_{C,j}(1))$, and $T_{H,j}(2)(T_{C,j}(2))$ are respectively heat load of i-th heat exchanger (heater, cooler), heat capacity flowrate, and supply and target temperatures of j-th hot (cold) stream. If there are n process streams in the network, then n heat balance equations are set up. The values of the right-hand sides of the above five equations can be calculated from the operating condition. Then the number of variables becomes five, which corresponds to the number of units composing network (1). Only certain definite values of the variables can satisfy these equations if the numbers of equations and variables are the same. That is, the heat loads of the units are specified by the operating condition if the number of units is equal to that of process streams. This type of network has often been synthesized as the optimal heat exchanger network under a constant operating condition without pinch point.

Network (2) is composed of eight units, and the following five heat balance equations are set up similarly.

$$q_{E,1}+q_{E,2}+q_{E,5}+q_{C,1}=F_{H,1}(T_{H,1}(1)-T_{H,1}(2))$$
$$q_{E,3}+q_{E,4}=F_{H,2}(T_{H,2}(1)-T_{H,2}(2))$$
$$q_{E,3}=F_{C,1}(T_{C,1}(2)-T_{C,1}(1))$$
$$q_{H,2}+q_{E,1}=F_{C,2}(T_{C,2}(2)-T_{C,2}(1))$$
$$q_{H,1}+q_{E,4}+q_{E,5}+q_{E,2}=F_{C,3}(T_{C,3}(2)-(T_{C,3}(1))$$

The same as for network (1), the right-hand sides of the equations are fixed by the operating condition. Then the number of variables is eight, larger than that of the equations. These variables cannot be specified by the operating condition, and some optimization

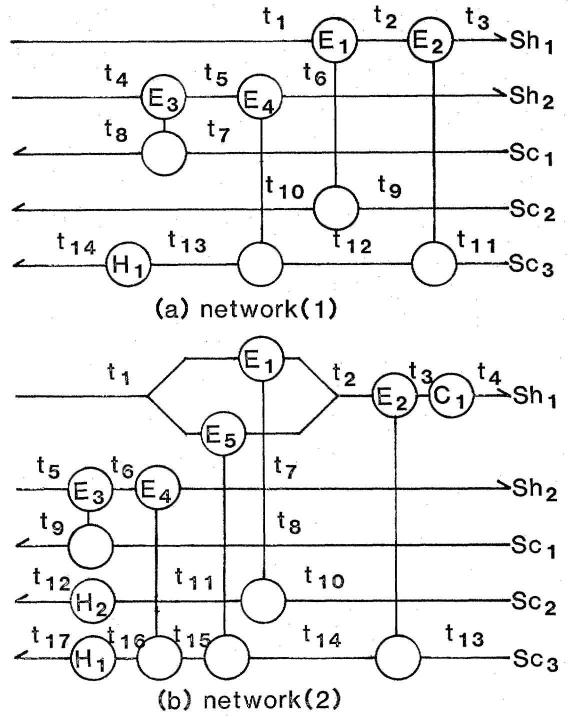

Fig. 1. Network structures of example problems

solution is necessary to assign heat loads to the units. That is the feature of this type of network, in which the number of units is larger than that of process streams.

Heat loads obtained by solving heat balance equations mentioned above cannot always be transferred through the units composing the existing heat exchanger network, because the constraints of temperature difference and heat transfer area are not considered in these equations. It is found in this study that linear programming is useful in determining inlet and outlet temperatures under the constraints of temperature difference when the structure of a heat exchanger network is known, and that necessary areas to transfer the heat loads estimated from these temperatures can be easily calculated from the equations defining the overall heat transfer coefficient. In the next section, linear programming models for networks (1) and (2) are explained separately.

2. Linear Programming Models

Considering the heat balance equations, it is found that heat exchanger networks should be classified into two groups according to the difference in number between the units and the process streams. One has freedom in assigning heat loads to the units when the operating condition is given, and the other does not. Then it is expected that the linear programming models of networks (1) and (2) would be different from each other, and that they would be used in a different manner.

In such a case as network (1), heat loads are specified by the operating condition as described in the previous section. Then it is possible to adopt the sum total of the absolute difference between target and output temperatures of each process stream as the objective function of the linear programming problem to determine inlet and outlet temperatures when a network structure is given. By assigning variables (temperature) as shown in Fig. 1, the following linear programming model (Type 1) can be set up.

$$
\begin{aligned}
J_1 = \min(&t_3+t_6-t_8-t_{10}-t_{14})\\
&-(T_{H,1}(2)+T_{H,2}(2)\\
&-T_{C,1}(2)-T_{C,2}(2)-T_{C,3}(2))
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t.}\quad & t_1-t_{10}\geq 0\,, && t_2-t_9\geq 0\\
& t_2-t_{12}\geq 0\,, && t_3-t_{11}\geq 0\\
& t_4-t_8\geq 0\,, && t_5-t_7\geq 0\\
& t_5-t_{13}\geq 0\,, && t_6-t_{12}\geq 0\\
& F_{H,1}(t_1-t_2)-F_{C,2}(t_{10}-t_9)=0\\
& F_{H,1}(t_2-t_3)-F_{C,3}(t_{12}-t_{11})=0\\
& F_{H,2}(t_4-t_5)-F_{C,1}(t_8-t_7)=0\\
& F_{H,2}(t_5-t_6)-F_{C,3}(t_{13}-t_{12})=0\\
& t_1=T_{H,1}(1)\,, && t_3\geq T_{H,1}(2)\\
& t_4=T_{H,2}(1)\,, && t_6\geq T_{H,2}(2)\\
& t_7=T_{C,1}(1)\,, && t_8\leq T_{C,1}(2)\\
& t_9=T_{C,2}(1)\,, && t_{10}\leq T_{C,2}(2)\\
& t_{11}=T_{C,3}(1)\,, && t_{14}\leq T_{c,3}(2)
\end{aligned}
$$

J_1 is the objective function, and the following equations are respectively constraints of temperature difference and heat balance on all units, and supply and target temperatures on all streams. $J_1>0$ indicates that some output temperatures cannot reach the target temperatures, and that this operating condition is rejected in this step because there is no room for improvement in this case. On the other hand, $J_1=0$ indicates that this operating condition is satisfied by this network structure, and that the next step proceeds in which the necessary heat transfer areas ($a_{E,i}$, $i=1$–4) and $a_{H,1}$) calculated by substituting inlet and outlet temperatures obtained in this linear programming problem into the equations defining the overall heat transfer coefficient are compared with those ($A_{E,i}$, $i=$ 1–4 and $A_{H,1}$) of the existing units. If there exists no unit of $a>A$, then this operating condition can be satisfied by adjusting the bypass valves of the units having areas larger than necessary ($a<A$). Otherwise, reject this operating condition because some units have areas insufficient to transfer the corresponding heat loads.

In such a case as network (2), as described in the previous section there exists freedom in assigning heat loads to the units. Then instead of J_1 of network (1), the sum total of the amount of utility (equivalent to the operating cost in running a heat exchanger network) is adopted as the objective function of the linear programming problem (Type 2). In this case, most of the constraints are similar to those of network (1). But constraints of target temperatures on all streams only change from $\geq$ to $=$, and temperature orders in heaters and coolers are added to the constraints to distinguish their functions. The objective function (J_2) and the replaced and added constraints are as follows.

Objective function

$$
\begin{aligned}
J_2=\min F_{H,1}(t_3-t_4)+F_{C,2}(t_{12}-t_{11})\\
+F_{C,3}(t_{17}-t_{16})
\end{aligned}
$$

Heat balance

$$
\begin{aligned}
& r_1F_{H,1}(t_1-t_2)-F_{C,2}(t_{11}-t_{10})=0\\
& F_{H,1}(t_2-t_3)-F_{C,3}(t_{14}-t_{13})=0\\
& F_{H,2}(t_5-t_6)-F_{C,1}(t_9-t_8)=0\\
& F_{H,2}(t_6-t_7)-F_{C,3}(t_{16}-t_{15})=0\\
& (1-r_1)F_{H,1}(t_1-t_2)-F_{C,3}(t_{15}-t_{14})=0
\end{aligned}
$$

Target temperature

$$
\begin{aligned}
& t_4=T_{H,1}(2)\,, && t_7=T_{H,2}(2)\\
& t_9=T_{C,1}(2)\,, && t_{12}=T_{C,2}(2)\\
& t_{17}=T_{C,3}(2)
\end{aligned}
$$

Temperature order

$$
t_3\geq t_4\,,\qquad t_{12}\geq t_{11}\,,\qquad t_{17}\geq t_{16}
$$

where r_1 is a splitting ratio of Sh_1 given in the specification of the heat exchanger network. If there exists no solution, then reject this operating condition in this step because it indicates that some output temperatures cannot reach the target temperatures. Otherwise, go to the next step and compare the necessary heat transfer areas ($a_{E,i}$, $i=1$–5, $a_{H,i}$, $i=1$–2 and $a_{C,1}$) calculated in the same way adopted in network (1) with those ($A_{E,i}$, $i=1$–5, $A_{H,i}$, $i=1$–2 and $A_{C,1}$) of the existing units. If there exists no unit of $a>A$, then this heat exchanger network can satisfy this operating condition by adjusting the bypass valves of the units of $A>a$. Otherwise, areas of some existing units are insufficient to transfer the heat loads estimated from the result of this linear programming problem. But it is possible that this contradiction can be resolved by increasing the right-hand side values of the temperature difference constraints of the units of $a>A$, because this network has freedom in assigning heat loads to the units as mentioned above. Then return to the first step of linear programming with the

Table 1. Specification of networks

1) heat transfer areas

	$A_{E,1}$ [m^2]	$A_{E,2}$ [m^2]	$A_{E,3}$ [m^2]	$A_{E,4}$ [m^2]	$A_{E,5}$ [m^2]	$A_{H,1}$ [m^2]	$A_{H,2}$ [m^2]	$A_{C,1}$ [m^2]
(1)	72.0	16.0	32.0	19.0		13.0		
(2)	80.0	10.0	41.0	42.0	10.0	15.0	10.0	25.0

2) heat transfer coefficients

process stream—process stream	0.851 [kW/m^{2}K]
process stream—cooling water	0.851 [kW/m^{2}K]
process stream—steam	1.135 [kW/m^{2}K]

3) others

splitting ratio	$r_1 = 0.7$	
cooling water	supply temp.	311 [K]
	target temp.	355 [K]
steam	supply temp.	509 [K]

revised constraints and repeat these two steps until all the units satisfy the condition of $A \geq a$. If some units of $a > A$ remain no matter how these constraints are improved, then this operating condition must be rejected because there is no more room for improvement.

3. Example Problems

To illustrate the proposed method, three problems are investigated in this section. Their structures, network specifications, and operating conditions are shown in Fig. 1 and **Tables 1** and **2**, and the following nomenclature is used to designate them. For example, P(1, 2) is a problem composed of heat exchanger network (1) and operating condition (2).

1) P(1, 1): According to its network structure, a linear programming problem of Type 1 is solved, and its result is shown in **Table 3**. It is clear from $J_1 = 0$ that each of the process streams can reach its target temperature by this structure. As the next step, the necessary heat transfer areas (a) are calculated by substituting the temperatures in Table 3 into the equations defining the overall heat transfer coefficient, and they are shown in **Table 4**. Comparing the necessary areas (a) with the areas of the existing units (A) in Table 1, it is clear that there exists no unit of $a > A$. Then network (1) can be run under this operating condition, but it is necessary for this execution to adjust the bypass valves of the units having areas larger than necessary.

2) P(1, 2): In the same way as in P(1, 1), a linear programming problem of Type 1 is solved, and the necessary heat transfer areas are calculated because $J_1 = 0$. The results are shown in Tables 3 and 4. It is clear from the comparison of the necessary areas (a) with those of the existing units (A) that the area of heat exchanger E_1 is less than necessary. There is no freedom in this network. Thus network (1) cannot be run under this operating condition, and it is judged that this operating condition is infeasible.

3) P(2, 2): According to its network structure, a linear programming problem of Type 2 is solved, and the result is shown in the row of $J_2 = 929.5$ in Table 3. It is clear from $t_2 = t_{10}$ that a heat exchanger of infinite size is necessary for E_1, and that this network cannot be run as it is. Fortunately, this network has freedom in assigning heat loads to the units as mentioned above, and so it is possible that this contradiction can be removed by changing the temperature difference constraints of unit E_1 from $t_2 - t_{10} > 0$ to $t_2 - t_{10} > x$ (x is a variable of positive value). This is checked by solving a linear programming problem with this revised constraint. The change of constraints often produces another contradiction. In this case, the contradiction in E_1 disappears when $x = 5$, but this modification newly raises the contradiction of $t_6 = t_{16}$, which causes the necessity of infinite heat transfer area of unit E_4. Then

Table 2. Specification of process streams

	stream	supply temp. [K]	target temp. [K]	heat cap. flow rate [kW/K]
(1)	Sh_1	478	339	13.29
	Sh_2	522	394	16.62
	Sc_1	366	478	13.03
	Sc_2	339	455	12.92
	Sc_3	311	478	11.40
(2)	Sh_1	469	330	13.29
	Sh_2	516	389	16.62
	Sc_1	366	483	13.03
	Sc_2	344	464	12.92
	Sc_3	319	478	11.40

Table 3. Solutions of example problems

		t_1 [K]	t_2 [K]	t_3 [K]	t_4 [K]	t_5 [K]	t_6 [K]	t_7 [K]
P(1, 1)	$J_1=0$	478.0	365.2	339.0	522.0	434.2	394.0	366.0
P(1, 2)	$J_1=0$	469.0	352.3	330.0	516.0	424.3	389.0	366.0
P(2, 2)	$J_2=929.5$	469.0	344.0	330.0	330.0	516.0	424.3	389.0
P(2, 2)	$J_2=1391.6$	469.0	349.0	347.4	330.0	516.0	424.3	389.0

t_8 [K]	t_9 [K]	t_{10} [K]	t_{11} [K]	t_{12} [K]	t_{13} [K]	t_{14} [K]	t_{15} [K]	t_{16} [K]	t_{17} [K]
478.0	339.0	455.0	311.0	341.6	400.2	478.0			
483.0	344.0	464.0	319.0	345.0	396.5	478.0			
366.0	483.0	344.0	446.9	464.0	319.0	335.3	364.5	415.9	478.0
366.0	483.0	344.0	430.4	464.0	319.0	320.9	362.8	414.3	478.0

Table 4. Necessary heat transfer areas

	$a_{H,1}$ [mB]	$a_{E,2}$ [mB]	$a_{E,3}$ [mB]	$a_{E,4}$ [mB]	$a_{E,5}$ [mB]	$a_{H,1}$ [mB]	$a_{H,2}$ [mB]	$a_{C,1}$ [mB]
P(1, 1)	71.65	15.90	31.06	18.44		12.61		
P(1, 2)	279.06	38.67	40.31	19.53		12.95		
P(2, 2)	79.80	0.89	40.31	41.00	9.57	11.22	6.35	19.36

both x and y ($t_6-t_{16}>y$) must be simultaneously searched. It is not so difficult to determine the feasible and optimal values of these variables. The search process is omitted here, but the feasible solution obtained when $x=5$ and $y=10$ is shown in the row of $J_2=1391.6$ of Table 3. The necessary heat transfer areas calculated from these temperatures are shown in Table 4. There esists no unit in which the necessary area is larger than the existing one, but it is necessary to adjust the bypass valves.

In this study, example problems where the overall heat transfer coefficients are common to the heat exchangers were adopted to illustrate the proposed method, but this method can also be applied to solve the problem where these coefficients are different from each other. In practice, it is possible that some target temperatures may be changed within limits. This type of problem can be also dealt with by modifying some of the constraints in the Type 2 linear programming problem.

Conclusion

A practical method is proposed to determine the running condition when an operating condition is given to an existing heat exchanger network. It is found that linear programming is useful in determing inlet and outlet temperatures under a given operating condition from the viewpoint of a network structure, and that its formulation can easily be made directly from the specification of the network and the operating condition. Then a two-step method is developed; one step is to determine these temperatures by linear programming, and the other is to compare the necessary heat transfer areas calculated from these temperatures with those of existing units. It is clear from the pre-analysis that networks can be classified into two groups; one has freedom in assigning heat loads to the units, and the other does not. In the proposed method, for the former these two steps are repeated to search out a feasible and optimal running condition if necessary. The effectiveness of this method is demonstrated through example problems.

Nomenclature

A	= heat transfer area (existing unit)	[m^2]
a	= necessary heat transfer area	[m^2]
F	= heat capacity flowrate	[kW/K]
q	= heat load	[kW]
r	= splitting ratio	[—]
$T(1)$	= supply temperature	[K]
$T(2)$	= target temperature	[K]
t	= temperature	[K]

⟨Subscript⟩

C	= cold stream, cooler
E	= heat exchanger
H	= hot stream, heater
i	= unit number
j	= stream number

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