DESIGN OF GAIN SCHEDULING CONTROL SYSTEM WITH REQUIRED TOLERANCE

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Gain scheduling is comparable to feedforward compensation and is highly sensitive to modeling errors. In this paper, the errors are represented by variations in the uncertain physical parameter θ under changes in process dynamics. For a state of the process specified by the auxiliary variable α, the variational region in the θ-space can be mapped into the space of transfer function parameter p. An orthotope R_p is defined in the p-space as the region of required tolerance through such mapping. On the other hand, the inherent region of system tolerance S_p is the set of p at which the proposed schedule rule can satisfy the admissible system performance. An iterative design algorithm was developed to find a schedule rule so that S_p includes R_p for a selected set of α. Application of robust gain scheduling to a reactor system with catalyst decay was studied to illustrate the effectiveness of the proposed design method.

Introduction

In many processes the dynamic behavior changes during operation. If on-line identification of the process dynamics is achieved, adaptive control such as a self-tuning regulator can be applied. However, since on-line identification under disturbances has not been established, there are few implementations of the adaptive control to the process control area.

In some processes the changes in process dynamics can be estimated from an auxiliary process variable. Based on this variable, controller parameters can be adjusted by a simple rule. This primitive adaptation is called gain scheduling.[1,7] It is a scheme widely used for industrial processes because of simplicity of implementation. However, it has the drawback that the control system is highly sensitive to model error. This is a result of open-loop compensation. Therefore, remedies for model-process mismatch should be considered.

In this paper the robustness of the gain scheduling control system is analyzed in the uncertain parameter space.[5,6] This tolerance analysis is used to design a gain scheduling control system with required robustness under model-process mismatch.

2. Gain Scheduling and Its Drawback

Suppose that a catalytic reactor system with deactivation is the objective of gain scheduling control. The dynamic behavior of the reactor is represented by

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nonlinear differential equations,[3] summarized in **Appendix 1**.

A linear model is derived from the nonlinear differential equations under some assumptions. The transfer function between instructed signal of heater and reactor temperature is represented as follows[3]:

$$G(s) = \frac{K_p(\tau_3 s + 1)e^{-\tau_4 s}}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{1}$$

where K_p is process gain, τ_i, $i \in \{1, 2, 3\}$ are the time constants and τ_4 is the dead time. Derivation of Eq. (1) is shown in **Appendix 2**.

As the catalyst deactivates, the operating temperature of the reactor must be raised to keep the conversion of the reactant in the main reaction constant.[2] Then, changes in deactivating factor and reactor temperature cause variations in the process dynamics. **Figure 1** shows an example of the changes in deactivating factor a by catalyst sintering and changes in the reactor temperature. These changes are very slow in comparison with the process responses.

At four representative operating points 1–4 in Fig. 1, the deactivating factor and process parameters have the values shown in**Table 1**. As the catalyst fouls, the value of process gain K_p increases considerably, but the other process parameters remain almost constant.

Since the deactivating factor of the catalyst is measured by the composition analysis of product gas, the variable can be used as an auxiliary process variable to represent the transition of process dynamics. The process gain is represented by a function of the deactivating factor and physical parameters such as the frequency factor k_1 in the rate equation of the main reaction as follows:

$$K_p = \phi(k_1, a) \tag{2}$$

The function ϕ is derived in Appendix 2. The other parameters τ_i, $i \in \{1, 2, 3, 4\}$, are treated as constants for the controller design.

Consider the gain scheduling of a PI controller to cope with process gain transition.

$$C(s) = K_c(1 + 1/T_I s) \tag{3}$$

where K_c is the controller gain and T_I is the integral time.

A schedule rule for the PI controller is determine by keeping the loop gain constant as follows:

$$K_c \cdot K_p = K_{c0} \cdot K_{p0} \tag{4}$$

$$T_I = T_{I0} \tag{5}$$

where K_{p0} and K_p are the process gains for the fresh catalyst and deactivated catalyst respectively. These values are calculated from Eq. (2) with the nominal value of k_1. The parameters K_{c0} and T_{I0} are the design variables, which are the initial parameter values of the PI controller. They are determined so that the control system always satisfies the following design specifications under catalyst deactivation:

$$\left.\begin{aligned} &6.0\,\text{dB} \le GM\,(\text{Gain Margin}) \le 10.0\,\text{dB} \\ &25.0\,\text{deg} \le PM\,(\text{Phase Margin}) \le 34.0\,\text{deg} \end{aligned}\right\} \tag{6}$$

This control system has three components: the process $G(s)$, controller $C(s)$ and schedule rule of Eqs. (4) and (5). System performance depends on the process parameter K_p and the design variables K_{c0} and T_{I0}. The solution for the design variables is not always unique. A solution was obtained under no variation in the physical parameter as follows:

$$K_{c0} = 0.384\,\text{V}\cdot\text{K}^{-1}, \quad T_{I0} = 36.887\,\text{s}$$

This schedule rule, Eqs. (4) and (5), with these values can satisfy admissible system performance at four points in Table 1. However, if the frequency factor k_1 of the main reaction has -20% error, the reactor temperature oscillates considerably even at the operating point 2. In this case a gain scheduling control system which is robust enough for the specified parameter variation should be designed.

3. Uncertainty in Model

A model is generally represented by a set of nonlinear ordinary differential equations (NODEs) as follows:

$$\frac{dx}{dt} = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\theta}, \boldsymbol{\alpha}) \tag{7}$$

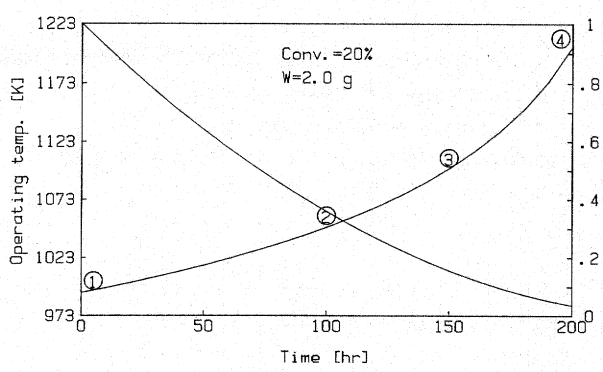

Fig. 1. Transitions of operating temperature and deactivating factor

Table 1. Transfer function parameters at four operating points

Operating condition	a [—]	K_p [K·V^{-1}]	τ_1 [s]	τ_2 [s]	τ_3 [s]	τ_4 [s]
1	1.00	26.8	333.9	153.8	249.1	15.0
2	0.36	30.0	332.6	152.4	249.4	15.0
3	0.16	33.2	331.1	150.9	249.6	15.0
4	0.04	40.0	328.8	147.9	250.1	15.0

where $\boldsymbol{x}$: state vector

$\boldsymbol{u}$: manipulated variable vector

$\boldsymbol{\theta}$: uncertain parameter vector

$\boldsymbol{\alpha}$: auxiliary process variable

The controlled variable vector is represented as follows:

$$y = C \cdot x \tag{8}$$

It is assumed that the auxiliary process variable α is measurable and changes slowly in comparison with the state variables.

For the control system design the NODEs are linearized around a set of values of $\boldsymbol{x}^s$, $\boldsymbol{u}^s$ at fixed $\boldsymbol{\theta}$ and $\boldsymbol{\alpha}$, *i.e.*,

$$\frac{d\delta \boldsymbol{x}}{dt} = \boldsymbol{A}\delta \boldsymbol{x} + \boldsymbol{B}\delta \boldsymbol{u} \tag{9}$$

where $\delta \boldsymbol{x} = \boldsymbol{x} - \boldsymbol{x}^s$, $\delta \boldsymbol{u} = \boldsymbol{u} - \boldsymbol{u}^s$, $\boldsymbol{A} = \partial \boldsymbol{g}/\partial \boldsymbol{x}|_s$ and $\boldsymbol{B} = \partial \boldsymbol{g}/\partial \boldsymbol{u}|_s$. Usually, $\boldsymbol{u}^s$ is determined by some optimization, and $\boldsymbol{x}^s$ is from the steady-state relationships.

$$\boldsymbol{g}(\boldsymbol{x}^s, \boldsymbol{u}^s, \boldsymbol{\theta}, \boldsymbol{\alpha}) = 0 \tag{10}$$

Therefore, matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ depend on $\boldsymbol{\theta}$ and $\boldsymbol{\alpha}$. Taking the Laplace transformation of the above linear ODEs, the following transfer function description is obtained:

$$\boldsymbol{y}(s) = \boldsymbol{G}(s)\boldsymbol{u}(s) \tag{11}$$

where

$$\boldsymbol{G}(s) = \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B}$$

and $\boldsymbol{y}(s)$ and $\boldsymbol{u}(s)$ are the output and input vectors respectively in the s-domain. The transfer function matrix $\boldsymbol{G}(s)$ depends on $\boldsymbol{\theta}$ and $\boldsymbol{\alpha}$ as the matrices do.

4. Gain Scheduling under Uncertainty

A general scheme of gain scheduling is shown in **Fig. 2**. The system is composed of the process, the controller, and the schedule rule. In this section, a single-input, single-output process is considered.

Assume that the process dynamics is approximated by a transfer function at any operating point as follows:

$$G = G(s, \boldsymbol{p}) \tag{12}$$

The parameter vector $\boldsymbol{p}$ in the transfer function changes gradually with time. The vector can be estimated from the auxiliary process variable $\boldsymbol{\alpha}$ and the physical parameter vector $\boldsymbol{\theta}$ as follows:

$$\boldsymbol{p} = \boldsymbol{\phi}(\boldsymbol{\theta}, \boldsymbol{\alpha}) \tag{13}$$

The function $\boldsymbol{\phi}$ can be derived from the linearized ordinary differential equations.

Although the parameter $\boldsymbol{\theta}$ has statistical variations, the nominal value $\boldsymbol{\theta}^n$ is used for the estimation of $\boldsymbol{p}$ at each operating point, *i.e.*,

$$\boldsymbol{p}^n = \boldsymbol{\phi}(\boldsymbol{\theta}^n, \boldsymbol{\alpha}) \tag{14}$$

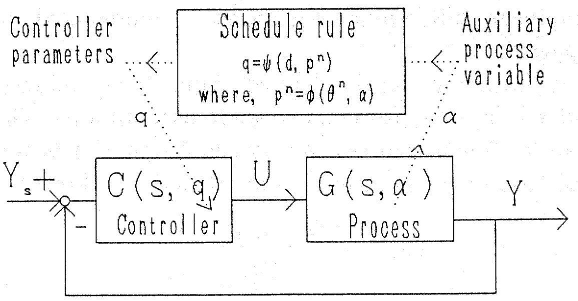

Fig. 2. Black diagram of gain scheduling

A controller is represented by the following transfer function:

$$C = C(s, \boldsymbol{q}) \tag{15}$$

where $\boldsymbol{q}$ is the vector of controller parameters. In the gain scheduling control system, the controller parameter vector $\boldsymbol{q}$ is adjusted as a function of $\boldsymbol{p}^n$, *i.e.*,

$$\boldsymbol{q} = \psi(\boldsymbol{d}, \boldsymbol{p}^n) \tag{16}$$

where $\boldsymbol{d}$ is the design variable vector of the schedule rule. Then the system performance depends on $\boldsymbol{d}$, $\boldsymbol{\theta}$ and $\boldsymbol{\alpha}$.

Usually, the physical parameter vector $\boldsymbol{\theta}$ varies around the nominal value $\boldsymbol{\theta}^n$. The region of possible variation is represented as follows:

$$\Delta\underline{\boldsymbol{\theta}} \le \boldsymbol{\theta} - \boldsymbol{\theta}^n \le \Delta\overline{\boldsymbol{\theta}} \tag{17}$$

The lower and upper bounds are determined by the analysis of accumulated data.

In the previous section a schedule rule was obtained without considering that the physical parameter k_1 has estimation error. Thus the rule is sensitive to the variation of k_1. A robust gain scheduling control system is designed to solve the problem of finding a schedule rule or the design variable vector $\boldsymbol{d}$ so that the control system satisfies the admissible system performance under any variation in the specified region in the uncertain parameter space.

5. Design of Gain Scheduling with Required Tolerance

5.1 Problem Statement

First, choose m representative operating points with respect to the auxiliary process variable. The feasibility at these points should guarantee the feasibility in the range of actual operational conditions. These points are called the critical points. In this paper the existence of a set of critical points, denoted by $\boldsymbol{\alpha}^1, \boldsymbol{\alpha}^2, \cdots, \boldsymbol{\alpha}^m$, is assumed.

For each $\boldsymbol{\alpha}^i$ the region of required tolerance can be defined in the $\boldsymbol{\theta}$-space as follows:

$$R^i_\theta = [\boldsymbol{\theta} \,|\, \Delta\underline{\boldsymbol{\theta}} \le \boldsymbol{\theta} - \boldsymbol{\theta}^n \le \Delta\overline{\boldsymbol{\theta}}] \tag{18}$$

This region can be mapped into the $\boldsymbol{p}$-space by the function $\boldsymbol{\phi}(\boldsymbol{\theta}, \boldsymbol{\alpha}^i)$. Then an orthotope to include the mapped region is made as follows:

$$R_p^i=[p\,|\,\Delta\underline{p}^i\le p-p^{ni}\le\Delta\bar{p}^i] \qquad (19)$$

where

$$p^{ni}=\phi(\theta^n,\alpha^i) \qquad (20)$$

On the other hand, the inherent region of system tolerance is defined in the p-space as follows:

$$S_p^i(d)=[p\,|\,f(p,\alpha^i,d)\in F] \qquad (21)$$

where f is the vector of system performances and F is the corresponding admissible region. A necessary condition for a gain scheduling control system to be robust for the specified parameter variations is described as follows:

$$R_p^i\subseteq S_p^i, \qquad i\in\{1,\cdots,m\} \qquad (22)$$

When the inherent region of system tolerance S_p^i is convex or one-dimensional convex, feasibility at all the vertices of R_p^i guarantees the feasibility at any point within the orthotope R_p^i.[5)]

5.2 Design Algorithm

An iterative design procedure to include R_p^i within $S_p^i(d)$ is composed of the following four steps.

Step 1: First, a set of typical operating point α^i's, $i\in\{1,\cdots,m\}$, is chosen. For each α^i, calculate the corresponding nominal value p^{ni}. From the information on statistical variation of the physical parameter θ, determine an orthotope of required tolerance in the p-space. All the vertices of R_p^i, $i\in\{1,\cdots,m\}$, must be tested for robustness.

Step 2: Give a schedule rule $\psi(d,p^n)$ and determine the initial value of design variable vector $d(1)$.

Step 3: (j-th iteration): Calculate the controller parameter $q(j)$ by the schedule rule at each α^i, $i\in\{1,\cdots,m\}$. The robustness test[6)] is achieved for all vertices of the orthotope R_p^i, $i\in\{1,\cdots,m\}$. Then a test variable is chosen among all uncertain parameters. The variable should give the inherent region of system tolerance of convex or one-dimensional convex. Checking of whether each vertex is inside the inherent region of system tolerance or not is achieved along the one-dimensional manifold parallel to the test variable axis. If the vertex is inside the inherent region, the corresponding variation in the p-space satisfies the admissible system performance.

If the design specification is satisfied for all vertices of the set of orthotopes R_p^i, $i\in\{1,\cdots,m\}$, the proposed schedule rule $\psi[d(j),p^n]$ is robust enough for any variation in the specified region of required tolerance. Otherwise, go to next step.

Step 4: The design variables in the schedule rule are retuned as $d(j+1)$ so that the farthest vertices from the inherent region of system tolerance along the one-dimensional manifold parallel to the test variable axis are contained in the new inherent region of system tolerance. Then go to Step 3. The sensitivities of the

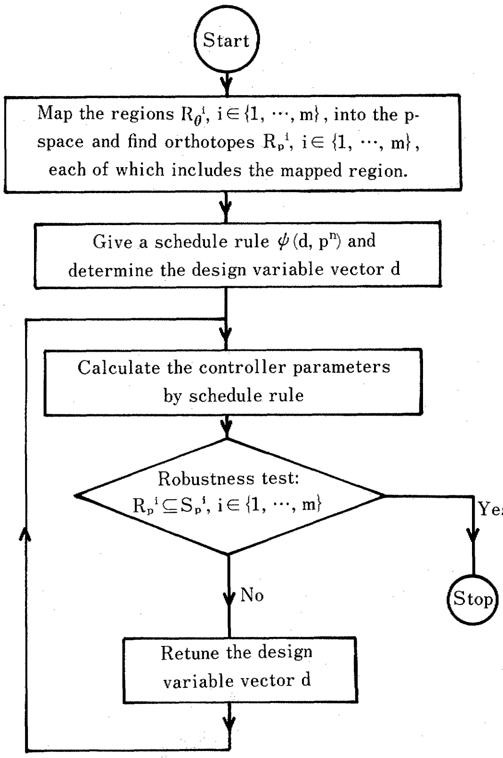

Fig. 3. Flowchart of algorithm

design variables to the shift of the inherent region of system tolerance are used for effective returning of design variables.[6)] If no design variable exists, then the schedule rule must be modified based on the basis of the characteristics of the transition of process dynamics.

A flowchart of the design algorithm is shown in **Fig. 3**.

6. Illustrative Example

In section 2, no variation in the physical parameters was considered for the design of the gain scheduling control system. In this section, it is predicted that the frequency factor k_1 deviates by $\pm 20\%$, *i.e.*,

$$-0.2k_1^n\le k_1-k_1^n\le 0.2k_1^n \qquad (23)$$

The region of required tolerance in the k_1-space is shown in **Fig. 4**. The process gain K_p in the transfer function is treated as an uncertain parameter. The mapping of the region of required tolerance in the k_1-space into the K_p-space at four typical operating points a^1, a^2, a^3, a^4 is shown in **Fig. 5**. The orthotope to include the mapped region is an interval for each fixed a.

The proposed iterative design algorithm was used to design a gain scheduling system with required tolerance. In this problem, the design variables are K_{c0} and T_{I0}. The iterative design process is summarized in **Table 2**, in which λ_i and δ_i, $i=1,2$, are weight factors for fast convergence.[6)] After four

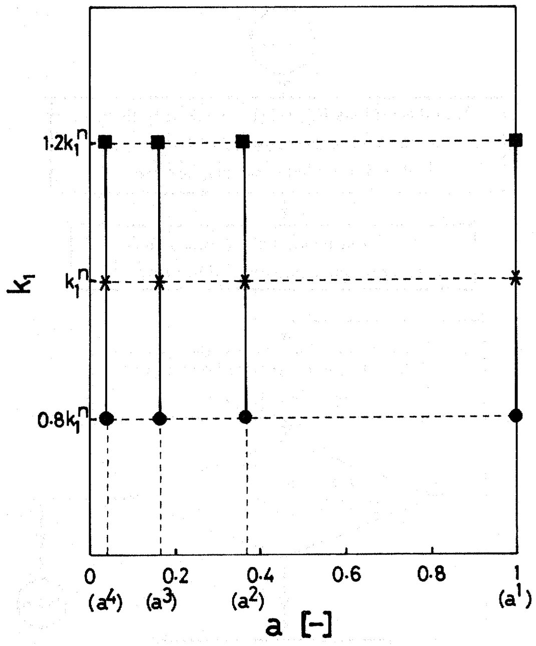

Fig. 4. Region of required tolerance

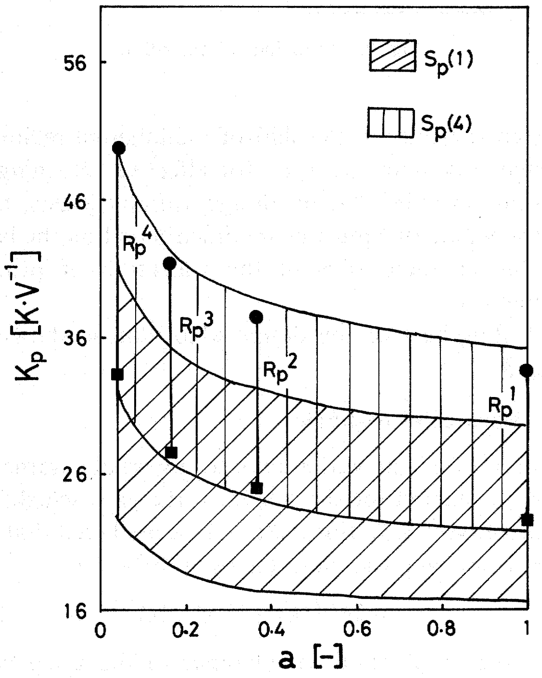

Fig. 5. Tolerance analysis of gain scheduling control system

Table 2. Iterative design of robust gain scheduling (test variable$=K_p$) ($\lambda_1=\lambda_2=\delta_1=\delta_2=1.0$)

Iteration Number j	K_{cO} [V·K^{-1}]	T_{IO} [s]	Length from farthest point to upper bound of $S_p(j)$	Length from farthest point to lower bound of $S_p(j)$
1	0.384	36.888	−2.132	4.294
2	0.330	36.888	−0.652	0.908
3	0.319	36.888	−0.282	0.059
4	0.316	36.888	−0.173	−0.189

iterations, the following result was obtained.

$$K_{cO}=0.316\mathrm{V}\cdot\mathrm{K}^{-1},\quad T_{IO}=36.888\,\mathrm{s}$$

The inherent region of system tolerance for an arbitrary deactivating factor is defined as follows:

$$S_p(j)=\{K_p\,|\,f[K_p,a,\boldsymbol{d}(j)]\in\boldsymbol{F},0.04\leq a\leq 1\} \tag{24}$$

Where j is the number of redesign. $S_p(1)$ for the initial design and $S_p(4)$ for the final design are shown in Fig. 5. The regions of required tolerance R_p^i, $i\in\{1,2,3,4\}$, are completely contained in the inherent region of system tolerance $S_p(4)$ for the final design. This means that the admissible control performance is always obtained in spite of parameter variations in the specified region of the physical parameter k_1.

7. Conclusion

An iterative design method was proposed for gain scheduling control system with required tolerance. This method gives a gain scheduling control system that is robust enough for the predicted variations of the physical parameters. As a result, model-process mismatch can be accepted to some extent for the gain scheduling control system. A reactor system with catalyst decay is controlled well by the schedule rule designed by this method.

Appendix 1: Nonlinear Process Dynamics

The carbon dioxide-methane reforming reaction is carried out with a shift reaction in a catalytic reactor.

$$\text{Main reaction:}\quad CO_2+CH_4=2CO+2H_2 \tag{R-1}$$

$$\text{Shift reaction:}\quad CO_2+H_2=CO+H_2O \tag{R-2}$$

The following nonlinear ordinary differential equations (NODEs) are obtained from the energy balances for heating wire, air gap and catalyst bed.[3)]

$$H_1\frac{dT_w}{dt}=-U_{A1}(T_W-T_a)-U_{A3}(T_w^4-T^4)+K_u u^2 \tag{A-1}$$

$$H_2\frac{dT_a}{dt}=U_{A4}(T-T_a)+U_{A1}(T_w-T_a)-U_{A2}(T_a-T_e) \tag{A-2}$$

$$H_3\frac{dT}{dt}=H_r+U_{A3}(T_w^4-T^4)-U_{A4}(T-T_a)+vC_p(T_i-T) \tag{A-3}$$

where heat generation by both the reactions is represented as follows:

$$H_r=[(-\Delta H_1)aX_1+(-\Delta H_2)aX_2]F_{CO_2} \tag{A-4}$$

The conversions of CO_2 in the reactions (R-1) and (R-2), X_1 and X_2, on the fresh catalyst are calculated by using the rate equations of both reactions. The reaction rates r_1 and r_2 are represented as follows:[3)]

$$r_1=\frac{\bar{k}_1(P_{CO_2}P_{CH_4}-P_{CO}^2P_{H_2}^2/K_1)}{P_{CO_2}+b_1P_{CH_4}+b_2P_{CO}^2P_{H_2}+b_3P_{H_2}}$$
$$r_2=\frac{\bar{k}_2(P_{CO_2}P_{H_2}-P_{H_2O}P_{CO}/K_2)}{P_{CO_2}+b_4P_{H_2}} \tag{A-5}$$

where $\bar{k}_1$ and $\bar{k}_2$ are rate constants of reactions (R-1) and (R-2) respectively. These rate constants are represented by the Arrhenius relationship with respect to temperature T.

$$\bar{k}_1 = k_1 \cdot \exp(E_1/T) \tag{A-7}$$

$$\bar{k}_2 = k_2 \cdot \exp(E_2/T) \tag{A-8}$$

where k_1 and k_2 are frequency factors.

Appendix 2: Changes in Transfer Function Parameters

Since H_3 is much smaller than H_1 and H_2, Eq. (A-3) can be reduced to an algebraic equation. Linearizing Eqs. (A-1) and (A-2) at a set of values of T^s, T_w^s, T_a^s, u^s, and taking the Laplace transformation, we obtain the transfer function between manipulated variable $u(s)$ and reactor temperature $T(s)$. The dead time τ_4 was added from the experimental data, *i.e.*,

$$G(s) = \frac{K_p(\tau_3 s+1)e^{-\tau_4 s}}{(\tau_1 s+1)(\tau_2 s+1)} \tag{A-9}$$

where K_p is the process gain, and τ_1, τ_2 and τ_3 are the time constants. These parameters are functions of the deactivating factor a and the steady-state conditions. For example, the static process gain K_p is represented as follows:

$$K_p = \frac{2K_u u^s[U_{A1}K_{Oa} + (U_{A4}K_{Oa} + C_2)K_{ow}]}{C_1 C_2 - C_3} \tag{A-10}$$

where

$$C_1 = U_{A1} + 4U_{A3}((T_w^s)^3 - (T^s)^3 K_{ow})$$

$$C_2 = U_{A4} + U_{A1} + U_{A2} - U_{A4}K_{oa}$$

$$C_3 = (U_{A1} + U_{A4}K_{ow})(U_{A1} + 4U_{A3}(T^s)^3 K_{oa})$$

$$K_{ow} = \frac{4U_{A4}(T_w^s)^3}{4U_{A3}(T^s)^3 + U_{A4} - (\partial H_r/\partial T)|_s + vC_p}$$

$$K_{oa} = \frac{U_4}{4U_3(T^s)^3 + U_4 - (\partial H_r/\partial T)|_s + vC_p}$$

$(\partial H_r/\partial T)|_s$ means evaluation at the corresponding steady state. This relationship is written simply as follows:

$$K_p = \tilde{\phi}(T^s, T_w^s, T_a^s, u^s, a) \tag{A-11}$$

When the reactor is operated to maintain constant conversion as the catalyst fouls,[2] the manipulated variable u^s is dependent on the deactivating factor a. The state variables T^s, T_w^s and T_a^s depend on a and u^s. Therefore, all the variables T^s, T_w^s, T_a^s, u^s are functions of the deactivating factor a.

On the other hand, it is assumed that the frequency factor k_1 in the rate equation of reaction (R-1) shows statistical variation. Then the gain K_p depends on k_1 through H_r. Therefore, K_p is represented by a function of a and the parameter k_1.

$$K_p = \phi(k_1, a) \tag{A-12}$$

Nomenclature

a	= deactivating factor	[—]
b_1	= kinetic adsorption parameter	[—]
$\boldsymbol{C}$	= transfer function of controller	[—]
C_p	= specific heat capacity of feed gas	[J·mol^{-1}·K^{-1}]
$\boldsymbol{d}$	= design variable vector in schedule rule	[—]
$\boldsymbol{F}$	= admissible region of f	[—]
F_{CO_2}	= feed flowrate of CO_2	[mol·s^{-1}]
$\boldsymbol{f}$	= vector of system performance	[—]
$\boldsymbol{G}$	= transfer function of plant	[—]
H_1	= heat capactiy	[J·K^{-1}]
K_c	= gain of PI controller	[V·K^{-1}]
K_i	= equilibrium constant	[—]
K_p	= gain of reactor system	[K·V^{-1}]
K_u	= heater parameter	[W·V^{-1}]
k_i	= frequency factor	[mol·(g·cat.)$^{-1}$·s^{-1}]
P_j	= modal fraction of species j	[—]
$\boldsymbol{p}$	= vector of uncertain parameters in G	[—]
$\boldsymbol{q}$	= vector of controller parameters	[—]
R	= required region of tolerance	[—]
S	= inherent region of tolerance	[—]
s	= Laplace transformation variable	[—]
T_I	= integral time of PI controller	[s]
T_a, T_w, T, T_i, T_e	= temperatures at air gap, heating wire, catalyst bed, feed and environment	[K]
U_{A1}, U_{A2}, U_{A4}	= heat transfer parameters	[W·K^{-1}]
U_{A3}	= heat transfer parameter	[W·K^{-4}]
u	= manipulating variable	[—]
v	= molal flow rate of feed	[mol·s^{-1}]
X_i	= conversion calculated from rate equation for reaction i	[—]
$\boldsymbol{x}$	= state vector	[—]
$\boldsymbol{y}$	= controlled variable vector	[—]
α	= auxiliary variable	[—]
α^i	= value of α at i-th operating condition	[—]
$\boldsymbol{\theta}$	= physical parameter vector	[—]
τ_1, τ_2, τ_3	= time constants	[s]
τ_4	= dead time	[s]
$\boldsymbol{\phi}$	= function for estimation of $\boldsymbol{p}$	[—]
$\boldsymbol{\psi}$	= schedule rule function	[—]
$(-\Delta H_i)$	= heat of reaction i	[J·mol^{-1}]

⟨Subscripts⟩

$_0$	= initial value of controller parameter
$_p$	= region in $\boldsymbol{p}$-space
$_-$	= lower limit value
$_\theta$	= region in $\boldsymbol{\theta}$-space

⟨Superscripts⟩

^{s}	= pseudo-steady state value
^{n}	= nominal value
$^-$	= upper limit

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