EFFECT OF PARTICLE SIZE ON THE LOADING RATE AT WHICH SINGULAR FRACTURE OF A SINGLE PARTICLE OCCURS

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Introduction

In the authors' recent paper[3] in this journal, it was reported that singularity of fracture behavior of a single brittle particle has been found under impact loading of duration comparable to the particle's natural period as the result of compression tests on spherical specimens of 2.0 cm size. Crushing efficiency was especially improved. This suggests the feasibility of improving the energy efficiency of grinding machines by adjusting the rate of loading particles to the loading rate $\dot{v}_n$ at which the singularity occurs. However, $\dot{v}_n$ is expected to vary with particle size because the natural period of the single particle is presumed to vary with particle size and the strength of the particle varies with its volume.[4] It is therefore necessary to investigate the relation between the natural period t_n and the particle size d in order to develop some techniques of improving the energy efficiency. In the present study, this relation is investigated by analyzing the proper vibration of an elastic sphere by means of the finite element method.

1. Analysis

In the previous work, the natural period was measured by an acoustic method. The sound generated by collision of two spherical specimens in the air was recorded by microphone and tape recorder. The natural period was determined by analyzing these sound waves by synchroscope and camera.

In this study, the natural period is calculated by analyzing the axi-symmetric proper vibration of an elastic sphere by means of the finite element method[2] because an elastic sphere is assumed to vibrate axi-symmetrically when colliding with another sphere in the air. The axi-symmetry assumption allows us to make a two-dimensional analysis. **Figure 1** shows the two-dimensional region divided into finite elements used in the analysis. Frequencies and modes of the proper vibration were calculated by mode analysis. The natural period was calculated from the lowest frequency among them.

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2. Result

Dotted lines in Fig. 1 shows the mode of vibration having the lowest frequency. It seems very likely that, as shown in Fig. 1, the sphere is transformed into a circular ellipsoid whose axis is the z-axis when the sphere collides with another sphere and the impact force acts in z-direction. **Figure 2** shows the relation

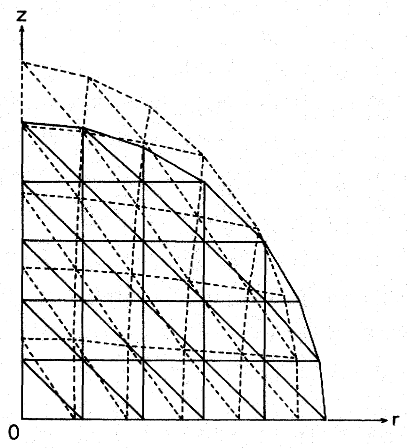

Fig. 1. Two-dimensional region divided into finite elements used for analysis and mode of vibration having the lowest frequency calculated by analysis

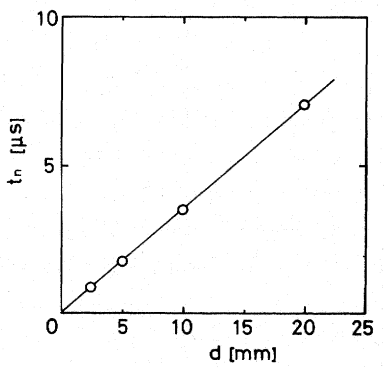

Fig. 2. Relation between natural period and diameter of an elastic sphere calculated by finite element method (borosilicate glass sphere)

between the natural period and the particle size of a borosilicate glass sphere calculated by this method. As shown in Fig. 2, the relation can be expressed by the following equation:

$$t_n = k \cdot d \tag{1}$$

where k is a coefficient. Since the natural period of the 2.0-cm sphere calculated by this method is much shorter than that measured by the acoustic method, the mode of proper vibration may be different from the axi-symmetric type. However, we assumed that the relation between the natural period and the particle size is expressed by Eq. (1).

The dependence of the strength S_s on the spherical particle's volume $V(=\pi d^3/6)$ can be expressed by the following equation:[4]

$$S_s = A \cdot V^{-1/m} = A \cdot (\pi/6)^{-1/m} d^{-3/m} \tag{2}$$

where A is a coefficient and m is Weibull's coefficient of uniformity.

S_s also depends on the loading rate $\dot{v}$ and its dependence can be expressed by the following equation:[3]

$$S_s = B(\dot{v}/\dot{v}_0)^q \tag{3}$$

where $\dot{v}_0$ is standard loading rate, B is a constant and q is exponent.

Equation (2) was obtained from the results of compression tests on single particles of various sizes under conventional rate of loading. Equation (3) was obtained from the results of compression tests on 2.0-cm spherical specimens in a wide range of loading rates. By assuming that the effect of loading rate on S_s is independent of particle size, the dependence of S_s on both particle size and loading rate is given by the following equation:

$$S_s = B(\dot{v}/\dot{v}_0)^q A \cdot (\pi/6)^{-1/m} d^{-3/m} / S_s^* \tag{4}$$

where S_s^* denotes the sphere compressive strength of the 2.0-cm specimen at conventional rate of loading as calculated by Eq. (2). Since the relation among S_s, fracture load L_f and d is given by Hiramatsu's equation,[1]

$$S_s = 2.8 L_f / \pi d^2 \tag{5}$$

the loading rate $\dot{v}_n$ defined by L_f/t_n is therefore related to the particle size as follows.

$$\dot{v}_n \propto d^{(m-3)/m(1-q)} \tag{6}$$

As shown in the previous papers,[3,4] q is very small in a wide range of loading rates and $m \simeq 3$ in a wide range of particle sizes. It is therefore found that $\dot{v}_n$ does not vary much with particle size in the size range where $m \simeq 3$.

Nomenclature

A	= coefficient in Eq. (2)	
B	= coefficient in Eq. (3)	[Pa]
d	= diameter	[m]
k	= coefficient in Eq. (1)	[s/m]
L_f	= fracture load	[N]
m	= Weibull's coefficient of uniformity	[—]
q	= exponent in Eq. (3)	[—]
S_s	= sphere compressive strength	[Pa]
S_s^*	= sphere compressive strength of 2 cm specimen under conventional rate of loading, caulcuated by using Eq. (2)	[Pa]
t_n	= natural period	[s]
$\dot{v}_n$	= loading rate at which singular fracture occurs	[N/s]
$\dot{v}_0$	= standard loading rate	[N/s]

Literature Cited

1) Hiramatsu, Y., Y. Oka and H. Kiyama: *J. Min. Met. Inst. Japn*, **81**, 1024 (1965).
2) Togawa, H.: "Analysis of Vibration by Finite Element Method," Saiensu, Tokyo, Japan (1975).
3) Yashima, S., F. Saito and H. Hashimoto: *J. Chem. Eng. Japan*, **20**, 257 (1987).
4) Yashima, S., Y. Kanda, T. Izumi and T. Shinozaki: *Kagaku Kogaku*, **36**, 1017 (1972).