VOLUME OF BUBBLES FORMED FROM AN ORIFICE SUBMERGED IN HIGHLY VISCOUS NEWTONIAN AND NON-NEWTONIAN LIQUIDS

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Key Words: Bubble, Bubble Formation, Bubble Column, Highly Viscous Liquid, Non-Newtonian Liquid

Introduction

Bubble columns have recently been studied for application to fermentors and gas-phase polymerization reactors. The rheological character of culture medium including mold fungi in the fermentor and the polymer-monomer solution in the gas-phase polymerization reactor shows non-Newtonian behaviour and the apparent viscosities of medium or solution increases with the progress of the reaction. It is essential in the design of such equipment to estimate the volume of bubbles formed from an orifice submerged in liquid. Accordingly, the effects of various factors on the volume of bubbles formed from an orifice submerged in highly viscous Newtonian and non-Newtonian liquids were investigated experimentally and correlated by dimensionless groups.

1. Experimental Apparatus and Procedures

Figure 1 shows the experimental apparatus. The bubble column used has a 0.15 m square cross section and is 0.4 m high. By assuming that the bubbles formed are of the same size, a single bubble volume was calculated by dividing gas flow rate by the frequency of bubble formation as measured by phototransistor.

The operating conditions were as follows: Gas flow rate $Q_g = 0.05\text{–}10 \times 10^{-6}\,\mathrm{m^3/s}$, orifice diameter $D_o = 0.8\text{–}2.0\,\mathrm{mm}$, gas chamber volume $V_c = 34\text{–}288 \times 10^{-6}\,\mathrm{m^3}$ and liquid height $H_l = 0.30\,\mathrm{m}$.

Liquids used were glycerol aqueous solutions, glycerol and corn syrup as highly viscous Newtonian liquids, and aqueous solutions of carboxymethyl cellulose (CMC), polyvinylalcohol (PVA), and a mixture of CMC aqueous solution and glycerol as non-Newtonian liquids. The rheological characters of typical non-Newtonian liquids used were measured a by cone-plate type viscometer (Tokyo Keiki Co.). The results are shown in **Fig. 2**. It is seen that these liquids follow the power-law model

$$\tau = -m(du/dy)^n \tag{1}$$

where m and n are power-law model parameter and index. As all indexes n measured are less than 1, the non-Newtonian liquids used are pseudoplastic fluids. Their measured physical properties are shown in **Table 1**.

2. Experimental Results

The effects of V_c, D_o, m and n on the relation between bubble volume V_b and gas flow rate Q_g were investigated experimentally.

Figure 3 shows the relation between V_b and Q_g with

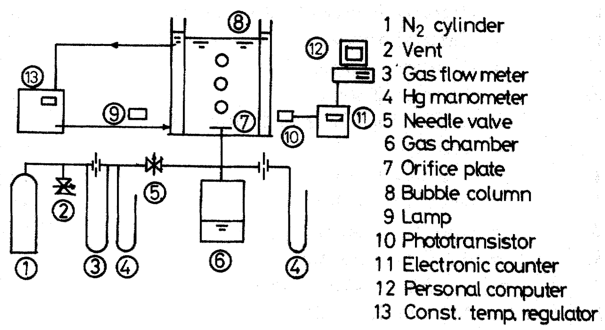

Fig. 1. Experimental apparatus

Received September 9, 1988. Correspondence concerning this article should be addressed to H. Tsuge.

Table 1. Physical properties of liquids

Liquid	Conc. [wt%]	$\rho \times 10^{-3}$ [kg/m^3]	m [Pas^n]	n [—]	$\sigma \times 10^3$ [N/m]	Mo [—]
Newtonian						
Glycerol	94	1.248	0.44	1	62.9	1.18
	100	1.261	1.35	1	62.0	1.09×10^2
Corn syrup	100	1.394	5.05	1	68.6	1.42×10^4
Non-Newtonian						
CMC (A)	0.8	1.002	0.348	0.744	71.0	6.12×10^{-3}
	0.9	1.002	0.401	0.758	70.8	1.35×10^{-2}
	1.1	1.003	0.768	0.729	70.8	1.12×10^{-1}
	1.3	1.003	1.35	0.663	70.7	3.66×10^{-1}
	1.5	1.004	3.12	0.628	69.6	6.03
	1.6	1.005	3.22	0.680	65.0	4.34
	1.7	1.006	5.67	0.576	61.7	3.87×10^1
CMC (B)	1.2	1.006	10.2	0.428	68.7	2.73×10^1
	1.4	1.015	17.6	0.412	68.4	1.79×10^2
	1.6	1.038	24.7	0.378	64.4	4.48×10^2
CMC (C)	2.8	1.012	1.35	0.731	65.1	1.31
CMC (D)	1.2	1.005	0.134	0.793	69.1	3.51×10^{-4}
PVA	14	1.035	15.8	0.899	34.4	3.32×10^6
	15	1.036	20.0	0.852	33.6	2.56×10^6
Mixture of glycerol and CMC (B) aq.		1.181	1.35	0.614	64.8	1.64×10^{-1}

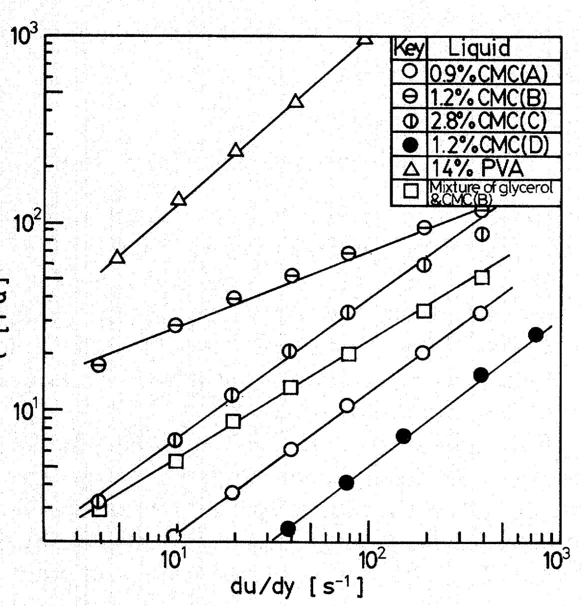

Fig. 2. Rheological character

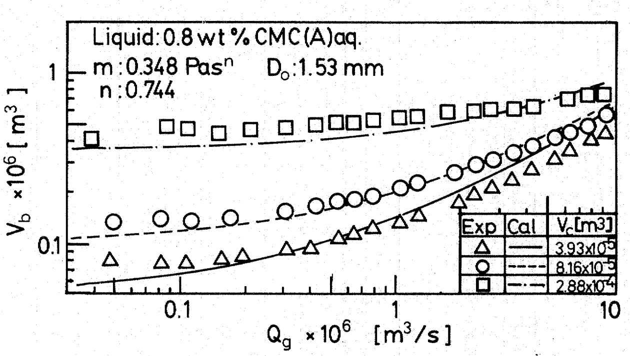

Fig. 3. Effect of V_c on relation between V_b and Q_g

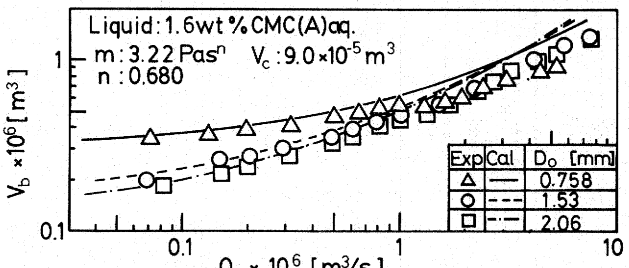

Fig. 4. Effect of D_o on relation between V_b and Q_g

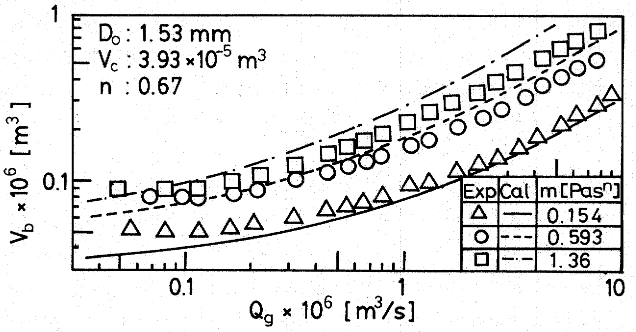

Fig. 5. Effect of m on relation between V_b and Q_g

gas chamber volume V_c as a parameter. V_b increases with increasing V_c and the effect of Q_g on V_b decreases with increasing V_c.

Figure 4 shows the effect of D_o. V_b increases with decreasing D_o at low gas flow rate* and the effect of D_o on V_b decreases with increasing Q_g. **Figures 5** and **6** show the effect of m and n. V_b increases with increasing m or decreasing n.

* However, V_b decreases with further decrease of D_o at low gas flow rate. In the present research, these experimental data were not included in the following correlation.

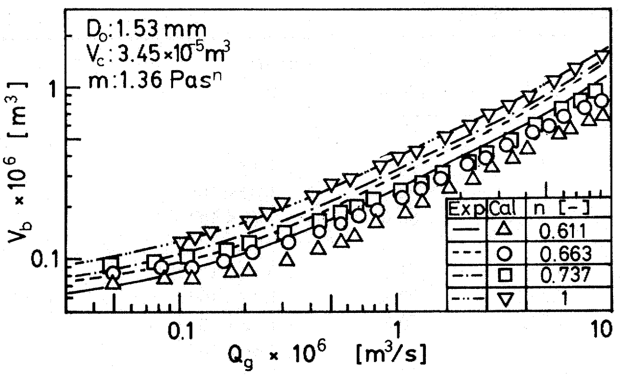

Fig. 6. Effect of n on relation between V_b and Q_g

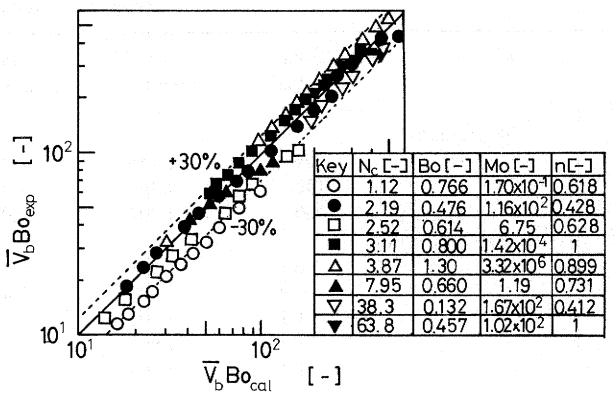

Fig. 7. Correlation between experimental and calculated results

3. Dimensionless Correlation of Bubble Volume

The bubble volume formed from an orifice in non-Newtonian liquids following the power-law model under constant flow condition is described as follows:

$$V_b=\phi(Q_g, D_o, \rho, m, n, \sigma, g) \tag{2}$$

By dimensional analysis,

$$\bar{V}_b=\phi(Fr, Bo, Mo, n) \tag{3}$$

where dimensionless bubble voluem $\bar{V}_b=V_b/D_o^3$, Froude number $Fr=u_o^2/D_o g$, Bond number $Bo=\rho\times D_o^2 g/\sigma$, and Morton number $Mo=g^{3n-2}m^4/\rho^{2-n}\sigma^{2+n}$. In the case of Newtonian liquids, that is, $m=\mu$ and $n=1$, the Morton number reduces to $g\mu^4/\rho\sigma^3$, which was used for the correlation of bubble volume in highly viscous Newtonian liquids.[2]

For intermediate and constant pressure conditions, the capacitance number $N_c=4V_c\rho g/\pi D_o^2 P_h$ including V_c was used[1] so that N_c is introduced in the dimensionless equation as follows:

$$\bar{V}_b=\phi(Fr, Bo, Mo, n, N_c) \tag{4}$$

Referring to the previous work[2] for Newtonian liquids, it might be adequate to use $\bar{V}_b Bo$ as the dimensionless bubble volume.

At relatively low gas flow rate, the bubble volume is nearly constant irrespective of gas flow rate, as shown in Fig. 3. These constant values are a function of Mo so that the following equation for dimensionless bubble volume at low gas flow rate, $\bar{V}_b Bo_1$, which is similar to the equation obtained for highly viscous Newtonian liquids[2], was obtained.

$$\bar{V}_b Bo_1=\pi N_c Mo^{0.07} \tag{5}$$

On the other hand, at high gas flow rate the bubble volume increases with increasing gas flow rate. The dimensionless bubble volume at high gas flow rate, $\bar{V}_b Bo_2$, is obtained by subtracting $\bar{V}_b Bo_1$ from the dimensionless bubble volume obtained experimentally, $\bar{V}_b Bo_{exp}$. By referring to Eq. (4), $\bar{V}_b Bo_2$ is correlated as

$$\bar{V}_b Bo_2=2^{3(2-n)}Fr^{3/8}Mo^{3/16}Bo^{(10-3n)/16} \tag{6}$$

The summation of $\bar{V}_b Bo_1$ and $\bar{V}_b Bo_2$ is considered to be the estimated value of dimensionless bubble volume, $\bar{V}_b Bo_{cal}$, so that $\bar{V}_b Bo_{cal}$ is defined as

$$\bar{V}_b Bo_{cal}=\bar{V}_b Bo_1+\bar{V}_b Bo_2 \tag{7}$$

By substituting Eqs. (5) and (6) into Eq. (7),

$$\bar{V}_b Bo_{cal}=\pi N_c Mo^{0.07} + 2^{3(2-n)}Fr^{3/8}Mo^{3/16}Bo^{(10-3n)/16} \tag{8}$$

Equation (8) is applicable in the range of $10^{-3}<Mo<1.7\times10^6$, $0.38<n<1$, $10^{-3}<Fr<1.8\times10^5$, $0.13<Bo<1.3$, and $1<N_c<64$. Lines in Figs. 3–6 show the results calculated by Eq. (8). $\bar{V}_b Bo_{exp}$ obtained from experimental results and $\bar{V}_b Bo_{cal}$ calculated by Eq. (8) are correlated within an accuracy $\pm30\%$ as shown in **Fig. 7**.

Few studies have been made of bubble formation in non-Newtonian liquids, so that the experimental results in highly viscous Newtonian liquids by the other researchers were compared with $\bar{V}_b Bo_{cal}$ by Eq. (8). $\bar{V}_b Bo_{exp}$ is better correlated, within an accuracy $\pm30\%$, by Eq. (8) than by the correlation equation presented in the previous work[2] for the same reference data.

Conclusion

The volumes of bubbles formed in highly viscous Newtonian and pseudoplastic liquids were experimentally obtained under various conditions and were correlated within an accuracy of $\pm30\%$ by a dimensionless equation.

Acknowledgement

The authors are indebted to Mr. K. Tanaka and Misses S. Kuroki and M. Sato for their assistance in the experiments.

Nomenclature

Bo	= Bond number, $\rho D_o^2 g/\sigma$	[—]

D_o	=	orifice diameter	[m]
Fr	=	Froude number, u_o^2/D_og	[—]
g	=	gravitational acceleration	[m/s^2]
m	=	power-law model parameter	[Pa·s^n]
Mo	=	modified Morton number, $g^{3n-2}m^4/\rho^{2-n}\sigma^{2+n}$	[—]
n	=	power-law model index	[—]
N_c	=	capacitance number, $4V_c\rho g/\pi D_o^2P_h$	[—]
P_h	=	pressure at orifice plate	[Pa]
Q_g	=	average gas flow rate	[m^3/s]
u_o	=	gas superficial velocity through orifice	[m/s]
V_b	=	bubble volume	[m^3]
$\bar{V}_b$	=	dimensionless bubble volume, V_b/D_o^3	[—]
V_c	=	gas chamber volume	[m^3]
μ	=	viscosity of Newtonian liquids	[Pa·s]
ρ	=	density	[kg/m^3]
σ	=	surface tension	[N/m]
τ	=	shear stress	[Pa]

⟨Subscript⟩

cal	=	value calculated from the correlation equation
exp	=	experimental data

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