

A MODEL FOR PREDICTING THE RELATIONSHIP BETWEEN MAXIMUM VOIDAGE IN HOMOGENEOUS FLUIDIZATION AND EMULSION PHASE VOIDAGE IN BUBBLING FLUIDIZATION

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A model was proposed to describe the expansion behavior of a gas fluidized bed of fine particles by using the elastic wave velocity. The difference between the maximum bed voidage in homogeneous fluidized beds, ε_{mb} , and the emulsion-phase voidage in bubbling beds, ε_e , could be explained by the proposed model. These voidages were calculated by comparing the voidage propagation velocity with the elastic wave velocity, the latter being represented as a function of the volume elasticity and the rigidity for uniformly expanded beds. In the emulsion phase of bubbling fluidized beds, however, the rigidity was neglected. This difference reflected the fact that ε_{mb} was larger than ε_e in any gas-solid system of fine particles. Since it was difficult to formulate quantitatively the value of the elastic wave velocity based on the theoretical approach, it was modified by experimental data in the literature.

Introduction

A criterion for the transition from homogeneous to bubbling fluidization has been sought^{8,15,27,31)} by considering the elastic wave in the emulsion phase at homogeneous fluidization. These approaches are based on the theory of Wallis.³²⁾ According to this theory, the critical condition for stability can be defined by

$$U_g = U_e \quad (1)$$

where U_g is the voidage propagation velocity and U_e

is the elastic wave velocity. When U_g is smaller than U_e , a bed is fluidized homogeneously, and when U_g becomes larger than U_e , bubbles start to form. The velocity U_e has been obtained by various approaches, while the equation obtained by Slis *et al.*³⁰⁾ has been used for U_e .

Verloop and Heertjes³¹⁾ have determined the elastic wave velocity from the elastic modulus calculated from a drag-interparticle distance relationship. Rietema²⁸⁾ has suggested that uniformly expanded beds of fine particles exhibit elastic behavior as a consequence of interparticle cohesive forces. His analysis allowed the difference in expansion behavior between cracking catalyst and cohesive polypropylene particles to be

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explained qualitatively. Kono *et al.*¹⁵⁾ have considered the emulsion phase as a plastic body rather than an elastic body. They finally expressed the elastic wave velocity in terms of the interparticle forces in the system containing group C powders.¹⁰⁾

On the other hand, Foscolo and Gibilaro⁸⁾ have formulated the elastic wave velocity by considering the interaction between a particle and fluidizing medium. Rowe and Furusawa²⁸⁾ introduced this theory in a recent review. In this theory, interparticle forces are neglected as they are of little significance. Foscolo and Gibilaro reported that good agreement was found between the proposed criterion and experimental observations for both liquid and gas fluidized systems. The difference of expansion behavior, however, due to the material of particles in gas fluidized beds cannot be explained by a model based only on the hydrodynamics.

Martin¹⁶⁾ has concluded that uniformly expanded gas fluidized beds differ mechanically from liquid fluidized beds because of the short-range interparticle forces in gas fluidized beds. Many other studies^{3,19,21,25)} have supported the significance of interparticle forces in bed behavior. Jacob and Weimer¹²⁾ have characterized the homogeneous bed expansion under ambient and elevated pressures based on the theory of Foscolo and Gibilaro.⁸⁾ Jacob and Weimer have suggested that it is important to take account of both the hydrodynamics and interparticle forces to formulate the elastic wave velocity for the system of group A powders.¹⁰⁾

In the present study, a model for describing the difference between the voidage at a minimum bubbling point, ϵ_{mb} , and the emulsion-phase voidage at bubbling fluidization, ϵ_e , was proposed by using the criterion of Wallis³²⁾ as defined by Eq. (1). For the voidage propagation velocity, the equation of Slis *et al.*³⁰⁾ was adopted. The elastic wave velocity was formulated on the basis of the hydrodynamics and interparticle forces. As shown¹⁸⁾ in Fig. 1, not only ϵ_{mb} but also ϵ_e is larger than ϵ_{mf} in fluidized beds of fine particles which are categorized as group A powders. No theoretical criterion concerning ϵ_e has been proposed, whereas ϵ_{mb} can be predicted by the criteria of the investigators noted above.^{8,15,27,31)}

The model described in this study is characterized by taking the rigidity into consideration for the elastic wave velocity in uniformly expanded beds. On the contrary, in the emulsion phase of bubbling beds, the rigidity was neglected. In consequence of this consideration, the difference between ϵ_{mb} and ϵ_e could be explained.

1. Properties of Powders Used for Calculations

In the analysis the expansion data of group A powders under ambient conditions were used. As the

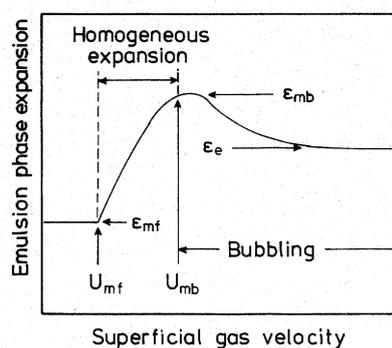


Fig. 1. Typical expansion curves for fluidized beds of group A powders

effects of the interparticle forces of group C powders and organic materials are significant, the data for these powders were not used for the calculation. Sources of the data are summarized in Table 1.

2. Model Description

2.1 Voidage propagation velocity

In a liquid fluidized bed, a sudden change in fluidizing velocity causes a discontinuity in the voidage at the bottom. The discontinuity propagates upwards through the bed. Slis *et al.*³⁰⁾ have obtained this voidage propagation velocity on the basis of the Richardson-Zaki equation:²⁶⁾

$$U_e = n(1 - \epsilon)U_t \epsilon^{n-1} \quad (2)$$

where U_t is the terminal settling velocity of unhindered particles. The same types of relationship have been previously used^{8,15,27,31)} for gas-solid fluidized beds which are uniformly expanded.

Although our basis was the same equation, as Jacob and Weimer¹²⁾ suggested, we considered the effects of interparticle forces in estimating the parameters in Eq. (2): n and U_t . The average value of n for a liquid fluidized bed is about 4.65 in the laminar flow regime.²⁶⁾ For a gas fluidized bed, the value of n is experimentally observed to be larger than 4.65. Geldart and Wong¹¹⁾ have explained that the larger values of n are due to interparticle forces, and showed that the value of n increases with decreasing particle diameter. As the expansion behavior was supposed to be influenced by other properties, we correlated the exponent n for group A powders with the dimensionless parameter N defined as²⁰⁾

$$N = \sqrt{d_p^3 g (\rho_p - \rho) / \mu} \quad (3)$$

As shown in Fig. 2, n can be expressed as a function of this parameter:

$$n = 4.65 + 90/N \quad (4)$$

In the laminar flow condition, when the sphericity factor of particles is set to be unity, U_t is given by

Table 1. Sources of data for the analysis in this study

Reference	Key	Powder	d_p range [μm]	ρ_p range [kg m^{-3}]	Fluidizing gases
Abrahamsen and Geldart ^{1,2)}	▼	Ballotini	43–71	2400–2810	Air
	■	Alumina	29–65	1800–3970	
de Jong and Nomden ⁴⁾	⊖	FCC	28–128	823–915	Air
Donsi and Massimilla ⁵⁾	■	Alumina	90	1550	Air
	▲	Silica	90	1510	
	◆	Catalyst	90	1490	
	▼	Sodium bicarbonate	68	2200	
Donsi <i>et al.</i> ⁶⁾	⊕	FCC	41–114	880	Air
	◇	Silica	60	1510	
Dry <i>et al.</i> ⁷⁾	◇	FCC	57–71	1300	Air
Geldart and Wong ¹¹⁾	■	Alumina	23–70	2430–3970	Air Nitrogen Arction-12 Argon
	▲	Fillite	68–125	364–638	
	▼	Glass	26	1597	
	◀	Catalyst	51–120	1117–1542	
Kai <i>et al.</i> ^{13,14)}	□	Alumina	55–83	770	Argon, Oxygen Nitrogen, Air Hydrogen Carbon dioxide Methane Helium Ethane
	○	FCC	35–56	920–1750	
	△	Silica	69–132	570–1360	
	◁	Catalyst	49–59	990–1330	
Kono <i>et al.</i> ¹⁵⁾	⊖	FCC	70	1400	Air
Massimila <i>et al.</i> ¹⁷⁾	⊕	FCC	45–173	850	Air
Morooka <i>et al.</i> ²⁰⁾	⊕	FCC	39–96	930–1080	Air
	□	Alumina	69	1430	
	▼	Glass beads	28–60	2520	
Mutsers and Rietema ²¹⁾	⊙	FCC	25–128	750	Air, Neon Hydrogen Propane
	▽	Glass beads	38–61	2400	
Mutsers and Rietema ²²⁾	●	FCC	62	1414	Nitrogen Hydrogen
Piepers <i>et al.</i> ²⁵⁾	●	FCC	59	887	Argon, Nitrogen Hydrogen
Simone and Harriott ²⁹⁾	●	FCC	33–124	790–1310	Air
Weimer and Quarderer ³³⁾	⊗	Granular carbon	66–108	850	Hydrogen/Carbon monoxide

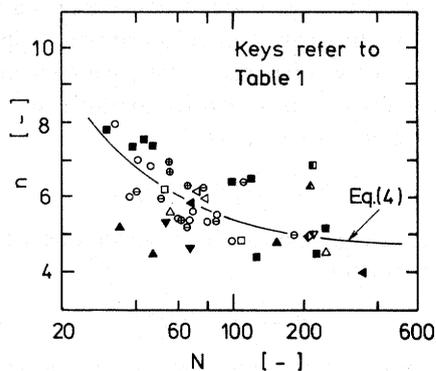


Fig. 2. Relationship between parameter N and exponent n

$$U_t = \frac{d_p^2(\rho_p - \rho)g}{18\mu} \quad (5)$$

However, the experimentally obtained terminal velocity U'_t did not agree with the theoretical values calculated from Eq. (5). **Figure 3** shows the ratio, η , of U_t to U'_t for group A powders. From this figure the relationship between n and η was determined as

$$\eta = 7.0 \times 10^{-4} n^{4.2} \quad (6)$$

Replacing U_t in Eq. (2) by Eq. (5), and considering the ratio η , we finally obtained the voidage propagation velocity as

$$U_\varepsilon = \frac{n\eta d_p^2(\rho_p - \rho)g(1 - \varepsilon)e^{n-1}}{18\mu} \quad (7)$$

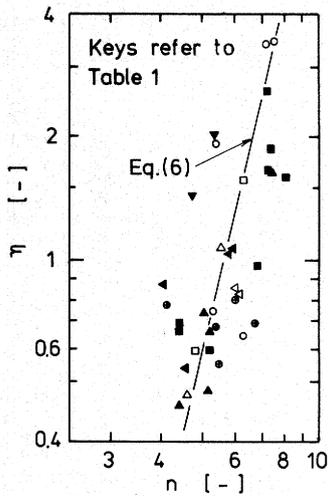


Fig. 3. Relationship between exponent n and ratio η

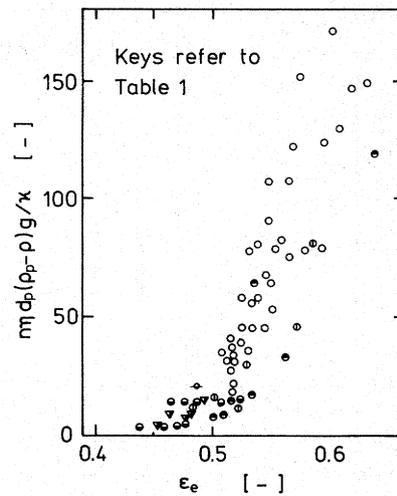


Fig. 4. Influence of voidage change on volume elasticity

2.2 Elastic wave velocity

The velocity of an elastic wave in solid materials is expressed as²³⁾

$$U_e = \left(\frac{\kappa + 4\gamma/3}{\rho_e} \right)^{0.5} \quad (8)$$

where κ is the volume elasticity, γ is the rigidity, and ρ_e is the density of an elastic body. As fluid has no shape elasticity, γ becomes zero and hence the elastic wave velocity of fluid is given by

$$U_e = \left(\frac{\kappa}{\rho_e} \right)^{0.5} \quad (9)$$

Foscolo and Gibilaro⁸⁾ introduced the elastic wave velocity based on Eq. (9). The experimental results of Mutsers and Rietema,²¹⁾ however, indicated the presence of rigidity in uniformly fluidized beds of fine particles. They found that the bed of fine particles could be tilted around an axis perpendicular to the plane of the bed over a certain angle without the bed surface sliding off. As suggested by Kono *et al.*,¹⁵⁾ the emulsion phase behaves as a plastic body rather than an elastic body. We supposed, therefore, that the emulsion phase could be treated as an elastic body when a stress was within a yield value: at homogeneous fluidization. At bubbling fluidization, as the stress beyond a yield value acts, the emulsion phase behaves like liquid and therefore it is supposed that the phase shows no rigidity.

Judging from the consideration noted above, Eq. (8) was used for calculating the elastic wave velocity in uniformly expanded beds. On the other hand, Eq. (9) was used in the emulsion phase at bubbling fluidization.

2.3 Theoretical description of κ and γ

The volume elasticity was derived⁸⁾ by considering the force balance between the dynamic drag and the effective weight on a particle:

$$\kappa = \frac{2}{3} n \eta d_p (\rho_p - \rho) g (1 - \varepsilon)^2 \quad (10)$$

In the equation of Foscolo and Gibilaro,⁸⁾ the parameter η was not included. But we take it into consideration because the effect of interparticle forces could be represented¹²⁾ by n and η .

Assuming that the rigidity is due to interparticle forces and that they are inversely proportional to particle diameter, the following equation is derived:²⁷⁾

$$\gamma = K f(\varepsilon) d_p^{-1} \quad (11)$$

where K is a function of the nature of the solid material. The value of K increases with the interparticle forces.

2.4 Modification of κ and γ

The theoretical descriptions of κ and γ were modified based on experimental data. The value of κ could be calculated from the emulsion phase voidage (see **Appendix 1**). **Figure 4** shows the relationship between the obtained values of κ and voidage. The experimental results for FCC in the literature were mainly used for the calculation. The value of κ was divided by the parameters other than the voidage function in Eq. (10). This figure suggests that κ could not be correlated with the voidage function $(1 - \varepsilon)^2$ and that κ became very large at a voidage around 0.42. In a gas-solid fluidized bed, when voidage became smaller than ε_{mf} the effect of resistance force transmitted from contacting particles was considered to be larger than that of hydrodynamic force. This effect caused an increase in volume elasticity at the voidage around 0.42.

We modified Eq. (10) by considering the voidage term $(\varepsilon - 0.42)$ and finally determined the following equation from experimental data:

$$\kappa = 1.06 \times 10^{-3} n \eta d_p (\rho_p - \rho) g (1 - \varepsilon)^2 / (\varepsilon - 0.42)^{2.2} \quad (12)$$

Figure 5 shows a comparison of the values of κ

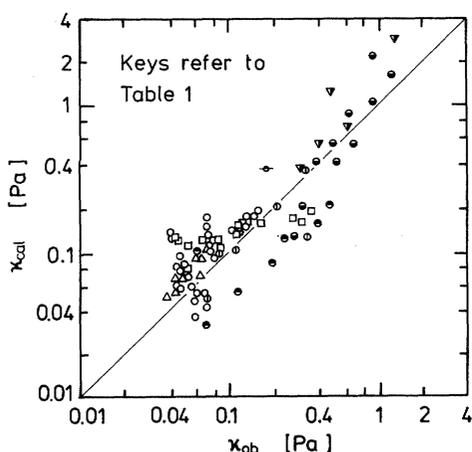


Fig. 5. Comparison of observed and calculated values of volume elasticity

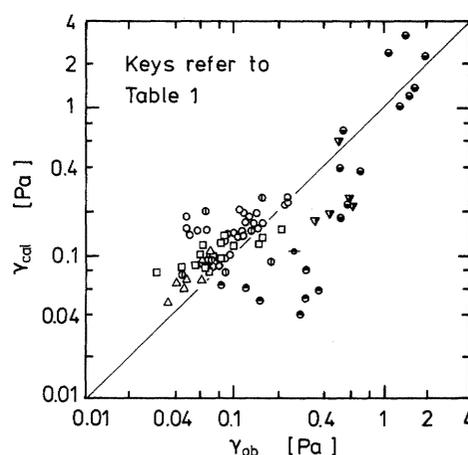


Fig. 6. Comparison of observed and calculated values of rigidity

calculated from Eq. (12) with those obtained from experimental data. The values of κ for cracking catalysts calculated by Mutsers and Rietema²¹⁾ based on a theory including interparticle forces and wall friction are from 0.01 to 2 Pa. Figure 5 shows that the same order of magnitude was obtained in the present study.

By using the values of ε_{mb} , γ could be obtained (see Appendix 2). The coefficient K in Eq. (11) was determined by the obtained value of γ and the following correlation was derived:

$$\gamma = 2.2 \times 10^{-3} (\varepsilon - 0.42)^{-1.7} d_p^{-1} \quad (13)$$

The voidage function $(\varepsilon - 0.42)$ was also used in this case.

Figure 6 shows a comparison between the values of γ calculated from Eq. (13) and those obtained from experimental data. As shown in Fig. 6, the rigidity value obtained were from 0.04 to 2 Pa. These orders of magnitude correspond to those of 0.05–0.1 wt% agar gels.²³⁾ This indicates that the values obtained would be reasonable.

3. Prediction of ε_{mb} and ε_e

The voidages ε_{mb} and ε_e can be determined at the critical condition: $U_c = U_{c,c}$. By combining Eqs. (7) and (8), ε_{mb} can be predicted from the following equation:

$$N_c = \frac{0.343(\rho_p - \rho)}{n\eta\rho_p(1 - \varepsilon_{mb})(\varepsilon_{mb} - 0.42)^{2.2}\varepsilon_{mb}^{2n-2}} \times \left(1 + \frac{2.8 \times 10^{-4}(\varepsilon_{mb} - 0.42)^{0.5}}{n\eta d_p^2(\rho_p - \rho)g(1 - \varepsilon_{mb})^2} \right)^{0.5} \quad (14)$$

where N is the dimensionless parameter defined by Eq. (3). The voidage ε_e is obtained by the combination of Eqs. (7) and (9) as

$$N = \left(\frac{0.343(\rho_p - \rho)}{n\eta\rho_p(1 - \varepsilon_e)(\varepsilon_e - 0.42)\varepsilon_e^{2n-2}} \right)^{0.5} \quad (15)$$

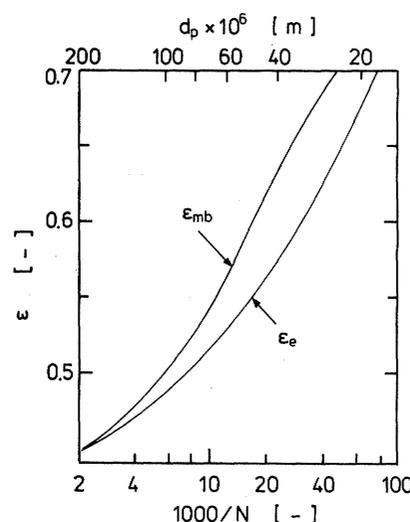


Fig. 7. Variation of calculated values of ε_{mb} and ε_e with parameter N

In Eqs. (14) and (15), n and η are calculated from Eqs. (4) and (6), respectively.

Figure 7 shows the relationship between N and the voidage for particles 1000 kg m^{-3} in density when beds were fluidized by air under ambient conditions. From this figure the relationship between the voidages and particle diameter can be seen.

As γ is strongly dependent on the interparticle forces, γ is probably dominated by the intrinsic nature of materials, humidity of fluidizing gas, temperature, pressure, etc. However, the variation of γ may be small in systems in which conditions are similar. Consequently, we compared the predictions with the experimental data obtained for inorganic materials by various investigators. Figures 8 and 9 show the comparison for ε_{mb} and ε_e , respectively. When κ and γ were determined in an earlier section of this study, only the results of experiments in which both ε_e and ε_{mb} were measured were used. In Figs. 8 and 9, the data from experiments in which only one of them was

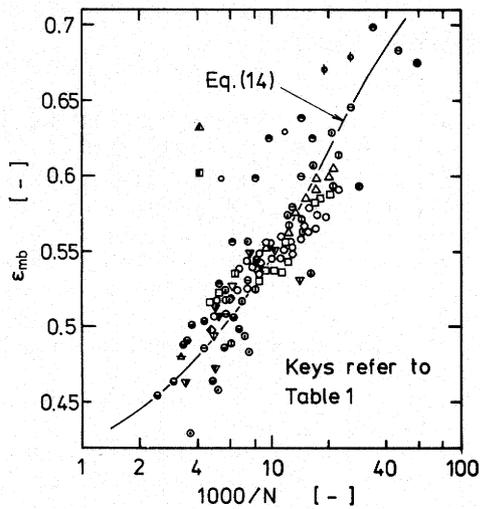


Fig. 8. Comparison of experimental data with predictions for ϵ_{mb}

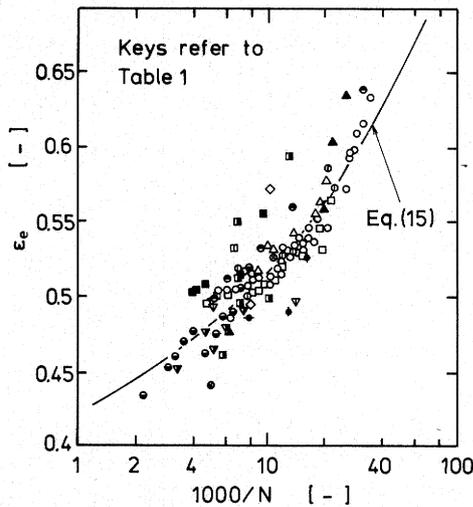


Fig. 9. Comparison of experimental data with predictions for ϵ_e

measured are added.

4. Discussion

Large dispersions of experimental voidage for given values of the parameter N are seen in Figs. 8 and 9. Several reasons can be considered for these dispersions. Investigators differed as to definition of average particle diameter. The values of n and η could not be predicted accurately for all powders. The voidage ϵ_{mb} was sensitive to experimental conditions such as bed height and type of distributors.²⁹⁾ The leading cause of the large dispersion is that each investigator has used a very different value of ϵ_{mf} , which is the basic variable for calculating ϵ_{mb} and ϵ_e . Even for FCC particles which show similar physical properties, the value of ϵ_{mf} varies from 0.4²¹⁾ to 0.58.¹⁷⁾ This difference is directly reflected in the large dispersions. Since it is very difficult to accurately estimate the pore volume of porous particles such as FCC, differences

in the value of ϵ_{mf} occurred. Nozaki²⁴⁾ also mentioned this point and used the expansion ratio, $(1 - \epsilon_{mf}) / (1 - \epsilon_e)$, to diminish the influence of difference in ϵ_{mf} .

In this study, interparticle forces indirectly affect the volume elasticity by varying the values of n and η , while the rigidity is directly influenced by interparticle forces. Agbim *et al.*³⁾ reported that the bubbling point was delayed and beds expanded uniformly before bubble formation when a permanently magnetized steel shot was fluidized. Foscolo *et al.*⁹⁾ have explained this stabilization by considering the magnetic forces in their previously proposed model.⁸⁾ In the present model, interparticle forces are considered in calculating the rigidity. The value of the rigidity increases with the interparticle forces, and this results in an increase in the voidage.

The voidage ϵ_{mb} and ϵ_e for resinous powders such as polypropylene and polyvinyl chloride were larger than those for inorganic materials.²⁰⁻²²⁾ This can be also attributed to the significant effect of the interparticle forces. Rietema²⁷⁾ has described the elastic wave velocity based only on the effect of interparticle forces. According to his model, the cohesive force for polypropylene particles was about 5000 times larger than that for FCC. Rietema also pointed out that this value was too large. In the analysis by the present model, however, the factor in Eq. (11) was 3 to 30 times larger for the organic materials than for FCC. It appears that our values are rather more reasonable than that obtained by Rietema.

Conclusions

The difference between the voidage at a minimum bubbling point and the emulsion phase voidage was explained by a model that considers the volume elasticity and rigidity in estimating the elastic wave velocity. Both volume elasticity and rigidity were taken into account for uniformly expanded beds. On the other hand, the rigidity was neglected for the emulsion-phase voidage at bubbling fluidization, because it is considered that a fully fluidized bed has no shape elasticity.

The other characteristic of the present model is that not only hydrodynamic forces but also interparticle forces were taken into consideration. Although the coefficients and voidage function of the volume elasticity and rigidity were determined from experimental data, the predicted voidages agreed with the experimental values obtained by other investigators.

Appendix 1. Volume elasticity

The value of κ is obtained by substituting Eqs. (7) and (9) into the critical condition: Eq. (1), when the measured values of the emulsion phase voidage are used for Eq. (7):

$$\kappa = \left(\frac{m d_p^2 (\rho_p - \rho) g (1 - \varepsilon_e) \varepsilon_e^{n-1}}{18 \mu} \right)^2 \rho_e \quad (\text{A1})$$

Appendix 2. Rigidity

Rearranging Eq. (8), we obtain

$$\gamma = \frac{3}{4} (U_e^2 \rho_p (1 - \varepsilon) - \kappa) \quad (\text{A2})$$

By using the experimentally obtained values of ε_{mb} for Eqs. (7) and (12), and substituting them for U_e and κ respectively, in Eq. (A2), the value of γ can be obtained.

Nomenclature

d_p	= average particle diameter	[m]
g	= gravitational acceleration	[m s ⁻²]
K	= function of solid material in Eq. (11)	[N m ⁻¹]
N	= dimensionless parameter = $\sqrt{d_p^3 g (\rho_p - \rho) / \mu}$	[—]
n	= exponent in Richardson-Zaki equation	[—]
U_e	= elastic wave velocity	[m s ⁻¹]
U_f	= superficial gas velocity	[m s ⁻¹]
U_{mb}	= minimum bubbling velocity	[m s ⁻¹]
U_{mf}	= minimum fluidization velocity	[m s ⁻¹]
U_t	= theoretical terminal velocity	[m s ⁻¹]
U_t'	= experimental terminal velocity	[m s ⁻¹]
U_e	= voidage propagation velocity	[m s ⁻¹]
γ	= rigidity	[Pa]
ε	= bed voidage	[—]
ε_e	= emulsion phase voidage	[—]
ε_{mb}	= bed voidage at minimum bubbling point	[—]
η	= ratio of terminal velocities = U_t / U_t'	[—]
κ	= volume elasticity	[Pa]
μ	= viscosity of gas	[Pa s]
ρ	= density of gas	[kg m ⁻³]
ρ_b	= bed density	[kg m ⁻³]
ρ_e	= density of elastic body	[kg m ⁻³]
ρ_p	= particle density	[kg m ⁻³]

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