

APPLICATION OF FLEXIBILITY ANALYSIS FOR COMPROMISE SOLUTION IN LARGE-SCALE LINEAR SYSTEMS

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This paper describes the use of flexibility analysis with uncertain parameters involved in linear models. Due to the presence of various judgments of value in large-scale systems, the previous formulation developed under single-objective optimization is revised by use of the minimum aspiration level, which plays an important role in multiobjective optimization. An improved algorithm to compute the flexibility index in an iterative manner is also presented. The proposed approach is applied to a post-optimal analysis of the dynamic allocation planning of an electric power system. Such consideration is shown to be of special importance in increasing reliability at the planning stage of problem-solving in uncertain systems.

Introduction

For large-scale systems it takes an extremely long time to complete planning and construction. If such lead-time becomes longer and longer, the possibility increases that the system will suffer unexpected addition and/or deviation of conditions after construction. We can find examples in recent construction of

nuclear plants, large chemical plants and so on.

Since we cannot forecast every unknown or uncertain factor at the design or planning stage, we need not only to optimize the system but also to analyze, in advance, the deviation of state and system performance against uncertainties. In most previous approaches,¹⁾ this aspect was considered after fixing decision variables at their reference values. As pointed out elsewhere,²⁾ however, this approach is inefficient due to total ignorance of the adjustability of certain

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decision variables. In addition, since these studies applied mainly to sensitivity analysis, their effectiveness was limited to rather small parameter deviations around their nominal values. Such an approach is unsatisfactory for large-scale systems, in which rather large parameter deviation is common.

Taking these points into account, this paper describes the flexibility analysis of an energy system in which decisions are made in the face of several conflicting objectives.

1. Flexibility Analysis of Linear Programs

Due to its high applicability and effectiveness, linear programming has been widely applied to strategic planning in various fields. In recent studies, we often used multiobjective linear programming³⁾ as an effective supporting tool for decision-making in large-scale systems where evaluation usually involves several conflicting objectives. We can formulate it as follows.

(p. 1)

Max $f(\mathbf{x}, \mathbf{y}) = \{\sum_j c_{ij}x_j + \sum_j c_{i,j+r}y_j, (i=1, \dots, N)\}$
subject to

$$\begin{cases} \sum_j a_{ij}x_j + \sum_j a_{i,j+r}y_j \leq b_i & (i=1, \dots, m) \\ x_j \geq 0 & (j=1, \dots, r) \\ y_j \geq 0 & (j=1, \dots, k) \end{cases}$$

Decision variables are categorized above into two kinds in terms of the foregoing statement. They are denoted by \mathbf{x} and \mathbf{y} respectively, and called⁴⁾ soft and hard variables according to whether they are easily alterable from their once-decided values after the planning stage or not. We can cite control or manipulate variables as examples of the former and design variables as examples of the latter.

Since many effective computer programs for multiobjective linear programming are now available, such programming is not so difficult even if the problem is of considerably large size. As mentioned already, however, more parameters that are uncertain are associated with the expanding boundary of the formulation. It is of special interest to work with the uncertainty analysis in such a situation.

A new approach known as flexibility analysis⁵⁻⁹⁾ is a method that can handle the rather large parameter deviation associated with the adjustability of soft variables. Through this analysis we can investigate the robustness or adaptability of the system at the reference point of the decision variables. Below, we will apply a modified approach to flexibility analysis, the effectiveness of which we have examined by taking a simple example in a chemical process.⁹⁾

With respect to uncertain parameters, we will impose the following assumptions.

(1) Coefficients of the constraints deviate around

their nominal values ($\{a_i^N\}, \mathbf{b}^N$) within the admissible region $T(\eta)$ described by

$$T(\eta) = \left\{ (\{a_{ij}\}, \mathbf{b}) \mid \begin{aligned} a_{ij}^N - \eta\alpha_{ij} &\leq a_{ij} \leq a_{ij}^N + \eta\alpha_{ij} \\ b_i^N - \eta\beta_i &\leq b_i \leq b_i^N + \eta\beta_i \end{aligned} \right\}, \\ (i=1, \dots, m, j=1, \dots, r+k). \quad (1)$$

(2) These deviations are independent of each other.

Then the flexibility index \mathcal{F} of the system designed at $\hat{\mathbf{x}}$ is defined as³⁾

(p. 2) For $\hat{\mathbf{x}}$

$\mathcal{F} = \text{Max } \eta$ subject to

$$\begin{cases} \text{Max}_{(\{a_{ij}\}, \mathbf{b}) \in T(\eta)} \text{Min}_y \text{Max}_{i \in \{1, \dots, m\}} \sum_j a_{ij}\hat{x}_j + \sum_j a_{i,j+r}y_j \leq b_i \\ y \geq 0 \end{cases}$$

The flexibility index teaches us by what degree we can keep the system feasible against the estimated interval of parameter deviation.

Let z_j^+ and z_j^- be additive corrections from the reference \hat{y}_j on the positive and negative sides respectively. For linear systems, we have shown⁹⁾ that the infinite programming problem above refers simply to the bi-linear programming problem such that

(p.3) For $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$

Max η subject to

$$\begin{cases} \eta \sum_j a_{i,j+r}(z_j^+ - z_j^-) + \sum_j a_{i,j+r}(\hat{y}_j^+ - \hat{y}_j^-) \\ \quad + (\sum_j a_{ij}\hat{x}_j + \sum_j a_{i,j+r}\hat{y}_j + \beta_i)\eta \\ \leq b_i - \sum_j a_{ij}\hat{x}_j - \sum_j a_{i,j+r}\hat{y}_j, & (i=1, \dots, m) \\ z_j^+ \leq Z_j^+ \\ z_j^- \leq Z_j^- & (j=1, \dots, k) \\ \eta, z_j^+, z_j^- \geq 0 \end{cases}$$

where capital Z denotes the upper limits of the respective correction.

For the solution above we improved the bisectional algorithm⁹⁾ so that we need check the feasibility of the linear program fewer times than before. The essence of the idea is as follows. (See also Fig. 1)

Given a certain value for η , we can express (p. 3) by a set of linear equations alone. Hence the solution of (p. 3) refers to a one-dimensional search to find the maximum value of η so long as the resulting linear inequalities defined for the given η remain feasible. In doing this, we regarded the objective value at the first step in the two-step algorithm of the linear program as an infeasibility index. Whenever the infeasibility index is zero, we can guarantee the feasibility of the system.

Then we found a new searching point η_i through extrapolation from two points $(\eta_{i-2}, \varepsilon_{i-2})$ and $(\eta_{i-1}, \varepsilon_{i-1})$ such that

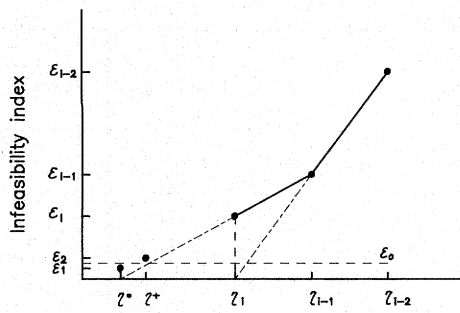


Fig. 1. Concept of searching procedure

$$\eta_i = \eta_{i-1} - \varepsilon_{i-1}(\eta_{i-1} - \eta_{i-2})/(\varepsilon_{i-1} - \varepsilon_{i-2}) \quad (2)$$

We repeat this procedure until the infeasibility index becomes ε_1 at η^* and ε_2 at η^+ for those satisfying $\varepsilon_1 \leq \varepsilon_0 \leq \varepsilon_2$ and $\eta^+ = \eta^* + \varepsilon_3$. Here each ε is a small positive value, and in particular, ε_0 and ε_3 denote zero and accuracy criteria respectively.

2. Application to Electric Power System

The growing portion of electricity in total energy demand is making it more important to establish a reliable and flexible electric power system. To study efficient energy utilization, we considered⁽¹⁰⁾ the dynamic allocation planning of power plant capacity formulated as a multiobjective mixed-integer linear program (MOMILP). Under several scenarios reflecting possible energy situations, we then solved the problem by three different solution strategies. The appendix shows a brief summary of the study.

Noticing the changeable and indefinite situation in energy problems, it is of special importance to examine the tolerance of the compromise policy against the uncertain parameters in the model. For this purpose, we conducted the flexibility analysis by applying the approach presented in the foregoing chapter after slight modification.

First, the decision variables were divided into hard and soft variables. We viewed the expansion size of each power plant as the hard variable because it is hard to change from the once-decided value in the planning horizon. Regarding the soft variables, we classified them further into two groups for convenience in performing their adjustment. We expressed this distinction by index sets I_1 and I_2 .

The first class, such as the operating rate of the oil-fired plant, are alterable around the compromise values within their permissible extent such that

$$\underline{y}_j \leq \hat{y}_j + z_j^+ - z_j^- \leq \bar{y}_j, \quad j \in I_1 \quad (3)$$

This is the same case as in (p. 3), noticing that

$$\begin{cases} z_j^+ \leq \bar{y}_j - \hat{y}_j \\ z_j^- \leq \hat{y}_j - \underline{y}_j \end{cases} \quad j \in I_1 \quad (4)$$

In contrast, regarding interchangeable electricity

Table 1. Distinction between variables

<i>Hard variables</i>	
Plant capacity	
<i>Soft variables</i>	
I_1 :	Operating rate of oil-fired plant
I_2 :	Amount of electricity interchanged among regions
	Amount of electricity purchased
	Amount of each waste generated
	Amount of each fuel consumed
	Reserve margin for electricity demand

and some other variables, when the sum total per item is bounded we can adjust each amount freely within its upper limit. For such variables, we removed any restrictions and permitted arbitrary alteration. This is the same use as in (p. 2). The distinction among such decision variables is summarized in Table 1.

Using multiobjective optimization, we attained the solution after articulation among the conflicting objectives. We are satisfied only with the relative relation of achievability of each objective. It is never satisfactory if maintaining the system's feasibility will change such relation drastically. To accept the compromise solution as flexible, therefore, we must maintain a minimally required performance of each objective. It is convenient to use the minimum aspiration level which played a basic role in the compromise. Hence as another major point of the modification we defined the flexibility of the compromise solution under multiobjectives as follows; the system should always keep the performance at least above the minimum aspiration level, not merely the feasibility. This requires the following additional condition in the formulation.*

$$\begin{aligned} & \sum_j c_{ij} \hat{x}_j + \sum_{j \in I_1} c_{ij} (\hat{y}_j + z_j^+ - z_j^-) + \sum_{j \in I_2} c_{ij} y_j \\ & \geq \underline{f}_i \quad (i = 1, \dots, N) \end{aligned} \quad (5)$$

where \underline{f}_i denotes the minimum aspiration level set forth for each objective at the stage of compromise.

Given this development, (p. 3) is rewritten as follows.

(p. 3') For \hat{x} and $\hat{y} \in I_1$

Max η subject to

$$\begin{cases} \eta \{ \sum_{j \in I_1} \alpha_{i,j+r} (z_j^+ - z_j^-) + \sum_{j \in I_2} \alpha_{i,j+r} y_j \} \\ + \sum_{j \in I_1} a_{i,j+r} (z_j^+ - z_j^-) + \sum_{j \in I_2} a_{i,j+r} y_j \\ + (\sum_j \alpha_{ij} \hat{x}_j + \sum_{j \in I_1} \alpha_{i,j+r} \hat{y}_j + \beta_i) \eta \\ \leq b_i - \sum_j a_{ij} \hat{x}_j - \sum_{j \in I_1} a_{i,j+r} \hat{y}_j, \quad (i = 1, \dots, m) \end{cases}$$

* Supposing that each objective should be maximized.

Table 2. Example of numerical result

Result of flexibility analysis								Compromise solution ¹⁰⁾			
With aspiration level				Without aspiration level							
Amount of electricity interchanged [10 ³ Gwh/y] (from A to B ^a , to C ^b , from B to A ^c , from C to A ^d)											
0.0 ^a	8.616 ^b	0.0 ^c	0.0 ^d	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0 ¹
4.392	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.903	0.0 ²
0.0	7.963	0.0	0.0	0.0	0.0	3.498	0.0	0.0	0.0	3.498	0.0 ³
0.0	0.0	4.223	0.0	0.0	0.0	3.467	0.0	0.0	0.0	2.200	0.0 ⁴
Reserve margin for electricity demand [—]											
0.080				0.0				0.128			
Amount of electricity purchased [10 ³ Gwh/y] (Chugoku ^a , Shikoku ^b , Kyushu ^c)											
0.0 ^a	0.0 ^b	13.09 ^c		0.0	0.0	0.0		0.0	0.0	0.0 ¹	
31.46	0.0	0.0		0.0	0.0	0.0		0.208	0.0	1.520 ²	
38.04	0.0	0.0		0.495	0.0	3.526		4.484	0.0	7.821 ³	
19.61	1.547	24.95		18.79	0.0	23.28		22.58	0.0	25.96 ⁴	
Amount of waste (Coal dust [10 ⁶ ton/y] ^a , Low level rad-waste [10 ³ m ³ /y] ^b , High level rad-waste [m ³ /y] ^c)											
0.142 ^a	0.623 ^b	12.47 ^c		0.154	0.673	13.46		0.136	0.594	11.88 ¹	
0.455	0.920	18.39		0.491	0.992	19.85		0.434	0.876	17.52 ²	
0.404	1.364	27.28		0.436	1.471	29.44		0.385	1.299	25.98 ³	
0.348	1.573	31.47		0.375	1.698	33.96		0.331	1.499	29.97 ⁴	
Amount of fuel consumed (Coal [10 ⁶ ton/y] ^a , Oil [10 ⁶ kl/y] ^b , LNG [10 ⁶ ton/y] ^c , Uranium [ton/y] ^d)											
0.765 ^a	12.89 ^b	2.294 ^c	3.524 ^d	0.826	24.90	2.475	3.803	0.729	23.32	2.185	3.356 ¹
2.447	18.71	4.264	5.198	2.641	26.91	4.602	5.609	2.331	23.75	4.061	4.950 ²
2.171	17.58	5.770	7.709	2.343	24.76	6.226	8.319	2.068	21.85	5.495	7.342 ³
1.869	20.20	6.263	8.894	2.017	21.80	6.758	9.597	1.780	19.24	5.964	8.470 ⁴
Additive correction, z ⁻ [Gw] (Chugoku ^a , Shikoku ^b , Kyusyu ^c)								Working rate of oil-fired plant [Gw]			
4.814 ^a	2.833 ^b	4.535 ^c		0.974	0.508	0.0		4.814	2.833	4.661 ¹	
4.900	1.564	0.082		0.0	0.0	0.0		4.900	3.058	5.100 ²	
4.490	0.811	0.334		0.0	0.0	0.0		4.490	2.862	4.630 ³	
0.0	0.0	0.0		0.0	0.0	0.0		3.870	2.720	4.020 ⁴	

1: 1983–1987 (1985); 2: 1988–1992 (1990); 3: 1993–1997 (1995); 4: 1998–2002 (2000).
A: Chugoku; B: Shikoku; C: Kyusyu districts.

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$$\begin{cases} z_j^+ \leq Z_j^+ \\ z_j^- \leq Z_j^- \end{cases} \quad j \in I_1$$

$$\sum_{j \in I_1} c_{ij}(z_j^+ - z_j^-) + \sum_{j \in I_2} c_{ij}y_j \geq f_i - \sum_j c_{ij}\hat{x}_j - \sum_{j \in I_1} c_{ij}\hat{y}_j \quad (i=1, \dots, N)$$

$$\eta, y \in I_2, z^+, z^- \geq 0$$

We can solve the problem above by applying the algorithm presented in the preceding section.

3. Numerical Results and Discussion

For the several compromise solutions obtained previously¹⁰⁾ we carried out a flexibility analysis assuming uncertainties for a few parameters involved in the model. The deviation of those parameters causes an increase in demand for electricity under degradation of the operating rate of power plants and tightens restriction on the amount of electricity purchase. It affects about 300 elements which corresponds about 40% of the non-zero elements of the

coefficient matrix.

Since independent deviation gives a more conservative result,^{11–12)} that is, a lower flexibility index compared with dependent deviation, for simplicity we assumed independent deviation in both column and row directions of the coefficient matrix.

For comparison, we also performed a flexibility analysis that required only the feasibility under deviation like the original formulation, (p. 3). Apparently this case will have a greater flexibility index than the modified one because we restrict the system more loosely.

Table 2 shows an example of numerical results for the planning obtained from the screened method under the standard scenario. (denoted STD herein-after). Though no considerable modification is necessary just to keep the plan feasible, certain structural changes in electricity supply are needed to keep the minimum aspiration level. Increase in the amount of purchase and interchange and decrease in the operating rate of oil-fired plants are major revisions among

them. Noting that the compromise plan was the result of tradeoff between the environmental impact and the reserve margin regarding demand requirement, this is because the environmental impact became the active condition in both cases in contrast to being only the margin condition in the latter case.

To examine how the proposed approach can be effective from both computation and application aspects, we conducted the analysis for several compromise solutions obtained under various scenarios.

Looking through the results summarized in **Table 3**, we know that the observation above is basically independent of the scenario applied. In turn, this fact tells us that intimate interconnection and flexible operation of the subsystems are essential to achieving a flexible system.

The results in Table 3 also show that, except for the cases in STD (Empirical) and PR4, we can revise the planning accordingly against deviation of about 4–5 percent of nominal values of the uncertain parameters while keeping system performance above the minimum aspiration level. On the other hand, these values increased by more than 10 percent when only feasibility is required.

In the case of PR4, because the system had no flexibility against the parameter deviation it is better to reconsider the compromise solution, adding an evaluation from this point of view.

Since these results show how to adjust the applied plan in the adverse case of the deviation, we can know by what degree and how to deal with the situation through operation. Such consideration, carried out at the prior stage, is unavoidable in developing dynamic planning in practical application.

From computation results shown also in Table 3, the algorithm usually ended only by several iterations of solution of the first stage of the linear program. This was from about one-half to one-third compared with the previous bisectional search. In addition, since it took only a few seconds of total CPU time in all cases, we can expect to deal with much larger systems by the proposed approach without suffering serious computational difficulties.

Conclusion

To improve the adaptability of the compromise solution obtained from the multiobjective optimization, in this study a flexibility analysis was conducted with uncertain parameters involved in the model. Concerning the multiple objectives, in particular, the formulation developed previously was revised by the aid of the minimum aspiration level of each objective.

The algorithm to accelerate convergence in numerical computation was also improved. Finally the proposed approach was applied to the dynamic allo-

Table 3. Results of computation of flexibility index*

Scenario (method)	Minimum asp. level	Flexibility index	Total iterations	CPU time** (msec)
STD (Screened)	with	0.501	6	4653
	without	1.33	6	3699
STD (Overall)	with	0.436	6	3959
	without	1.27	6	3959
STD (Empirical)	with	0.0	3	1581
	without	1.01	5	3366
PR2 (Screened)	with	0.412	6	4287
	without	1.24	5	3107
PR3 (Screened)	with	0.380	6	4116
	without	1.24	7	4393
PR4 (Screened)	with	0.0	3	1480
	without	0.0	6	3395

STD: Standard case of scenario

Compared with STD scenario,

PR2: with higher excursion rate in construction cost of nuclear plants

PR3: assuming that nuclear is less weighed as the alternative option for oil-fired

PR4: assuming much less than PR3 scenario

* Supposing that the parameters deviate by $\pm 10\%$ around their nominal values

** Using FACOM M780 at data processing center in Kyoto University

cation planning problem of an electric power system. It was confirmed that flexibility analysis can provide much useful information for the management of uncertain systems.

It was also shown that the proposed algorithm is attractive from the computational aspect, and almost free from difficulties associated with problem size and number of uncertain parameters.

These facts lead to the conclusion that the proposed approach is promising as an effective supporting tool for decision-making in large-scale systems.

Appendix. Outline of the dynamic capacity allocation models

We defined the capacity allocation of an electric power system as dynamic expansion planning among interrelated regions. It determines the optimal "kind, timing, and capacity" of power plants and some interrelated amounts among regions while meeting various conditions in a planning horizon.

They involved demand requirements, restrictions on interchanging and purchasing amounts of electricity, allowable investment cost, and maximum increment of each plant in every region at every period. Regarding fuel consumption and waste disposal, we applied upper bounded constraints at every period. Furthermore, a total capacity of each plant permitted in every siting region was imposed. A few solution-specific conditions were involved additionally.

We set up the following four objectives for evaluation. To measure the economic factor, we used the total cost discounted to the present value. It was composed of construction and operation costs of plant, fuel cost, transmission and purchased costs of electricity and waste disposal cost.

To evaluate the environmental factor, we introduced a hazard index calculated from effluent wastes at normal operation of each plant. We took into account SO_x , NO_x , coal dust and low-level radioactive waste in both aqueous and gaseous forms.

Concerning societal aspects, we evaluated administrative reliability and political feasibility through stable energy supply and resource security acquisition respectively.

We used published statistical data, economic and physical models, and regression analysis to estimate the parameters.

Under the prescription shown in Table A-1, we formulated the problem as a multiobjective mixed-integer linear program (MOMILP) and solved by means of three models, called screened, overall and empirical models. After removing some appropriate integer variables at the prior stage, we defined the screened model as the MOMILP and solved by a modified version¹³⁾ of RESTEM.³⁾ On the other hand, the overall model was given without such pre-screening. Moreover, we defined the empirical model as a linear program by fixing the integer variables at prescribed values.

Depending on both the scenario applied and the model, problem size varied around 130 inequalities, 82 continuous variables and 25 integers.

As a general conclusion of the numerical study, LNG-fired and nuclear options were favored as alternatives to oil-fired power generation in contrast to rather sophisticated acceptance of the coal option. This did not depend basically on the scenarios characterized as higher and lower demands, higher excursion rate in construction cost of nuclear plants and so on. The reason is that, given the present statistical input data, we can view the nuclear plant in normal operation as cheap and clean, and the LNG-fired plant as clean and economically comparable to the oil-fired plant. (For the recent situation, selection of data can be elaborated if necessary.) Since this was not the case for the coal-fired plant, it suggests a necessity for developing more efficient processes for treating coal plant waste economically.

The result also showed that with respect to the total capacities of nuclear plants installed both by 1990 and by 2000, if lower target values are established in the political aspect then a square trade-off among the objectives can be made.

Thus inspecting the results by comparison with commonly known or fairly predictable facts, the dependability of the model has been confirmed.

Nomenclature

a_i	= coefficient vector
b	= r.h.s. coefficient vector
c_i	= coefficient vector of objective function
f	= vector objective function
f_i	= minimum aspiration level of i -th objective
\tilde{f}	= flexibility index
$I_i (i=1, 2)$	= index set denoting class of soft variables
N	= total number of objectives

Table A-1. Prescription of the model

Kind of plant: Hydro, Coal-, Oil-, LNG-fired, Nuclear
 Planning horizon: 1983–2002 (5-year intervals)
 Target region: 3 regions in western Japan corresponding to the service areas of Chugoku, Shikoku, and Kyusyu Electric Power Cos.

T	= hyperpolyhedron for parameter deviation
x	= hard variable vector
y	= soft variable vector
z, Z	= adjusting vector and its upper bound
α_{ij}	= boundary value of deviation of a_{ij}
β_i	= boundary value of deviation of b_i
ε	= small positive value
η	= scale of flexibility

<Symbol and Superscript>

$\bar{}$	= upper/lower bound
	= reference value
$+/-$	= positive/negative side
N	= nominal value

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