

l = liquid
s = solid

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DETERMINATION OF PARTICLE SHAPE FROM ELECTRICAL CONDUCTIVITY OF SUSPENSIONS

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Key Words: Particle, Shape Characterization, Electrical Conductivity, Suspension, Particle Measurement

Experimental investigations are made into the electrical conductivity of suspensions of non-conducting particles of various shapes. Theoretical relations so far available for spheroidal particle systems are tested and the Fricke equation is used for determining particle shape as the aspect ratio of an equivalent spheroid. The electrical conductivities of suspensions of disks and cylindrical particles are obtained experimentally. The effect of the shape distribution on the conductivity of suspensions is also discussed.

Introduction

Particle shape is an important factor that significantly affects the physical characteristics of granular materials. Packed beds of higher density, for example, can be made of particles with higher degree of sphericity and the flowability of particles becomes low for irregular particles.¹⁴⁾ In the surface-finishing process, the rate of grinding metals is enhanced by use of angular abrasive grains.¹⁵⁾ Since the particle shape reflects the process used for their production, one can know the history of the particles from their shape.⁷⁾

The TV image processing system^{2,5)} accurately measures particle profiles, but it is two-dimensional and usually restricted to a stable orientation, and shape characterization is time-consuming. Moreover,

it is in fact probable that the particle shape in only a particular direction is important in shape classification^{12,16)}

As indirect methods, measurement of the settling velocity leads to an equivalent shape of particles.³⁾ Gotoh *et al.*⁶⁾ related the packing density of particles to the particle shape. The residence time of particles in a system^{8,9)} may be used as a measure of particle shape, but no theoretical relation has been developed.

In the present study, an experimental investigation is made into the electrical conductivity of suspensions of non-conducting particles of various shapes. First, theoretical relations so far available for spheroidal particle systems are examined and the Fricke equation⁴⁾ is chosen to determine the aspect ratio of an equivalent spheroid. The geometrical aspect ratios of non-conducting disks and cylindrical particles are experimentally related to the electrical conductivity of

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suspensions. Next, the electrical conductivity of suspensions of binary and ternary mixtures is discussed. The effect of the distribution of particle shapes on the electrical conductivity of suspensions is also discussed, assuming the beta distribution for the aspect ratios of particles.

1. Experiments

1.1 Test cell and samples

The test cell is shown schematically in **Fig. 1**. The closed box with sides $50 \times 49 \times 23$ mm consisted of transparent methyl methacrylate sheets of 5 mm thickness, and stainless steel plate electrodes of $50 \times 49 \times 1$ mm were attached to a pair of inside walls facing each other. A side wall of the cell had a hole of 10 mm diameter through which electrolytic solution and test samples were added and removed. The electrolytic solution consisted of potassium hydroxide dissolved in distilled water (0.05 N).

Properties of test samples are listed in **Table 1**. Disks and cylindrical particles were made respectively by punching a sheet and cutting a rod into segments. The particle densities were experimentally determined. All particles could be regarded as being non-conductive.

1.2 Procedure

The electrical conductivity of suspensions was measured by the alternating current method.¹⁾ From preliminary experiments, it was confirmed that the present cell can be modeled as the serial circuit of a resistance and a capacitance for the sinusoidal voltage (< 100 mV). The conductivity is obtainable from the voltage drops in the cell and the reference resistance R_1 ($= 10.0 \Omega$) shown in **Fig. 2**, where the frequency is set at 10 kHz so that one can neglect the influence of the capacitance and the flow of suspension.

To determine the particle shape from the electrical conductivity of suspensions, the particles should be dispersed uniformly in the suspension. Since mechanical dispersion is preferable in practical applications, rotating and rolling devices were devised in the present study. For rotating dispersion, the cell held by two side disks was rotated on rollers as shown in **Fig. 1**. For rolling dispersion, it was rotated clockwise and counterclockwise by a pulse motor controlled by a microcomputer. The relative conductivity is defined as follows.

$$\Delta K = (K_C - K_S) / K_S \quad (1)$$

in which K_C and K_S are the electrical conductivities respectively of the continuous phase and the suspension. It was observed that the maximum ΔK corresponds to the uniform dispersion. Hence, the rotational frequency of the rotating equipment and the rolling frequency and amplitude of the rolling equipment were adjusted so as to make ΔK maximum for

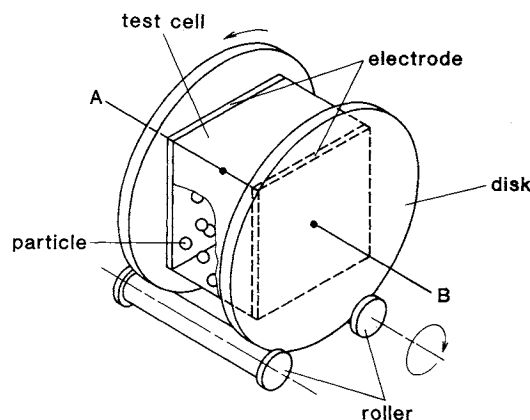


Fig. 1. Test cell

Table 1. Properties of test samples

Particle	Material	Density [g/cm ³]	<i>D</i> [mm]	<i>L</i> [mm]	<i>T</i> [mm]	<i>R</i> [—]
Disk 1	p	1.04	5.5	5.5	0.50	0.091
Disk 2	p	1.04	3.3	3.3	1.0	0.30
Disk 3	r	1.55	3.3	3.3	2.1	0.636
Disk 4	m	1.19	2.14	2.14	1.63	0.762
Disk 5	m	1.19	2.12	2.12	1.99	0.939
Sphere	n	1.13	3.2	3.2	3.2	1.00
Needle	n	1.13	0.30	3.57	0.30	0.084
Cylinder 1	m	1.19	2.01	4.26	2.01	0.472
Cylinder 2	m	1.19	2.14	3.42	2.14	0.625
Cylinder 3	m	1.19	2.12	2.62	2.12	0.808
Cylinder 4	m	1.19	2.11	2.28	2.11	0.933

Note: *D*=intermediate diameter; *L*=length; *T*=thickness; *R*=aspect ratio which is equivalent to *T/D* for oblate particles and to *T/L* for prolate particles. Material: m=methyl methacrylate; n=nylon; p=polystyrene; r=rubber.

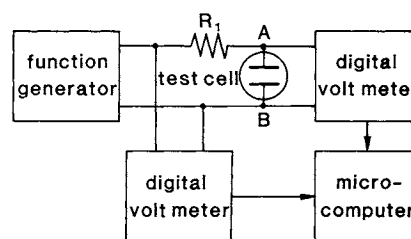


Fig. 2. Schematic diagram of measuring system

each run. As mentioned later, however, shaking the test cell by hand is the best way to obtain uniform dispersion.

Prior to shape measurement, particles were weighed by an electric balance and put into the cell filled with the electrolytic solution with great care to keep it free from air bubbles. The electrical conductivity of the suspension was measured for one minute during which the particle dispersion was observed to be steady and the temperature remained unchanged. It was sampled every half-second and 100 data were averaged for each run, using a microcomputer.

2. Results and Discussion

2.1 Electrical conductivity of suspension of equal particles

For the electrical conductivity of suspensions of non-conducting spheroids homogeneously and isotropically dispersed in a conducting medium, Fricke,⁴⁾ Meredith *et al.*¹¹⁾ and Meredith¹⁰⁾ respectively developed the following equations.

$$\Delta K = pV_d / (1 - V_d) \quad (2)$$

$$\Delta K = [2 + (p - 1)V_d][2(1 - V_d) + pV_d] / [2(2 - V_d)(1 - V_d) - 1] \quad (3)$$

$$\Delta K = (1 - V_d)^{-p} - 1 \quad (4)$$

where

$$p \equiv (3M + 2) / [3M(2 - M)] \quad (5)$$

V_d is the particle volume fraction and M is the shape parameter of the particles. For the oblate spheroid with half-axes a and b ($a < b$),⁴⁾

$$M = \frac{y - (1/2)\sin 2y}{\sin^3 y} \cos y, \quad \cos y = a/b \quad (6)$$

For the prolate spheroid ($a > b$),

$$M = \frac{1}{\sin^2 y'} - \frac{\cos^2 y'}{2 \sin^3 y'} \ln \left(\frac{1 + \sin y'}{1 - \sin y'} \right), \quad \cos y' = b/a \quad (7)$$

Equations (6) and (7) are depicted by broken curves in Fig. 6.

Experimental results for the ΔK versus V_d relations are depicted in Figs. 3–5 respectively for disk 1, spheres and needles. The solid curves depict Eqs. (2), (3) and (4) where the shape parameter M is calculated from Eq. (6) or (7) using the aspect ratio of each particle. In the dilute region, the experiments by the hand-shaking method are in good agreement with the solid curves. However, there is a large discrepancy between them, especially for the disks dispersed by the rotating method. This is so because the particles are observed to orientate parallel to the rotating axis of the cell, giving rise to an increase in K_s or to a decrease in ΔK . The rolling dispersion method seems to be superior to the rotating method. The influence of the dispersing methods is negligible for spheres. Since the experiments deviate in general from the theoretical prediction as the particle volume fraction increases, particle shape should be determined in dilute suspensions.

From the measured ΔK for a known V_d , one can determine the shape parameter M of the equivalent spheroid. Figure 6 depicts the relation between the shape parameter M and the geometrical aspect ratios $(a/b)_g$ and $(b/a)_g$ respectively of disks and cylindrical particles. Plots close to $M = 2/3$ are for spheres. The

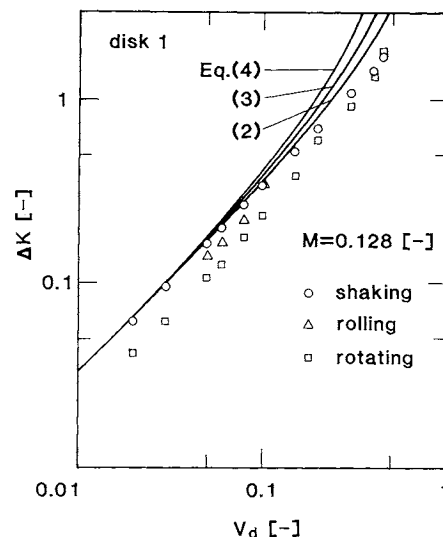


Fig. 3. Relation between V_d and ΔK for suspensions of disk 1

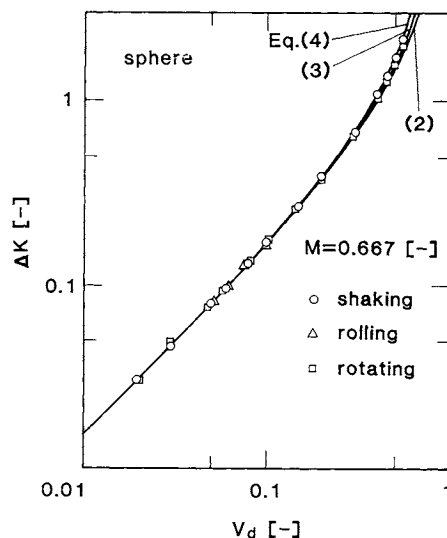


Fig. 4. Relation between V_d and ΔK for suspensions of spheres

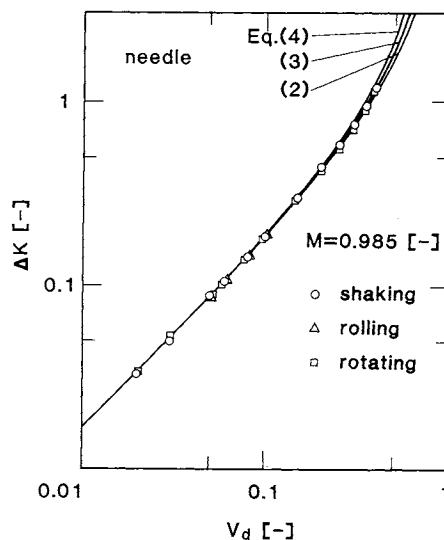


Fig. 5. Relation between V_d and ΔK for suspensions of needles

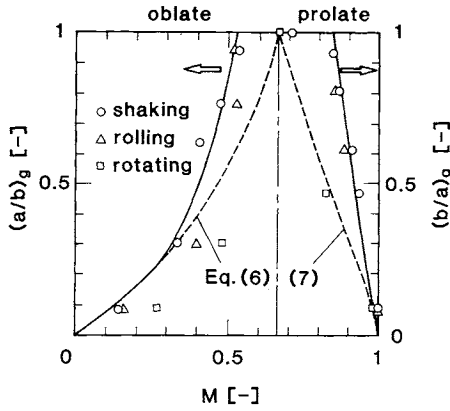


Fig. 6. Shape parameter M vs. $(a/b)_g$ or $(b/a)_g$

values of M are determined from Eq. (2) by making it fit the experiments in the range of $V_d = 0.05-0.1$. As can be seen from Fig. 6, the geometrical aspect ratios of disks and cylinders differ from those of the corresponding spheroid, shown by the broken curve. To determine the aspect ratio of disks and cylinders from the electrical conductivity measurement, one should therefore use the solid curve in Fig. 6.

2.2 Electrical conductivity of suspension of multishape particles

The shape of particles is not uniform but has a distribution in general. It is therefore important to elucidate the influence of that distribution on shape measurement.

Consider the electrical conductivity of a suspension in which particles having n kinds of shapes are uniformly dispersed with a random orientation. If the particle volume fraction is low, say less than 0.1, electrical conductivity K_{S1} of the suspension of particle 1 is expressed by the Fricke equation as follows.

$$K_{S1} = K_C \left[1 + \frac{p_1 V_{d1}}{1 - V_{d1}} \right], \quad p_1 \equiv \frac{3M_1 + 2}{3M_1(2 - M_1)} \quad (8)$$

in which M_1 and V_{d1} respectively denote the shape parameter and the particle volume fraction. Regarding the suspension above as a continuum of conductivity K_{S1} , the Fricke equation is applied again for the electrical conductivity K_{S2} of suspension of particle 1 and particle 2 as follows.

$$K_{S2} = K_{S1} \left[1 + \frac{p_2 V_{d2}}{1 - (V_{d1} + V_{d2})} \right], \quad p_2 \equiv \frac{3M_2 + 2}{3M_2(2 - M_2)} \quad (9)$$

in which V_{d2} is the volume fraction of particle 2. Hence, the relative conductivity $\Delta K(2)$ of the binary suspension becomes

$$\Delta K(2) = \left[1 + \frac{p_1 V_{d1}}{1 - V_{d1}} \right] \left[1 + \frac{p_2 V_{d2}}{1 - (V_{d1} + V_{d2})} \right] - 1 \quad (10)$$

Similarly, for the suspension of particles of n kinds of shapes, one can obtain the relative conductivity

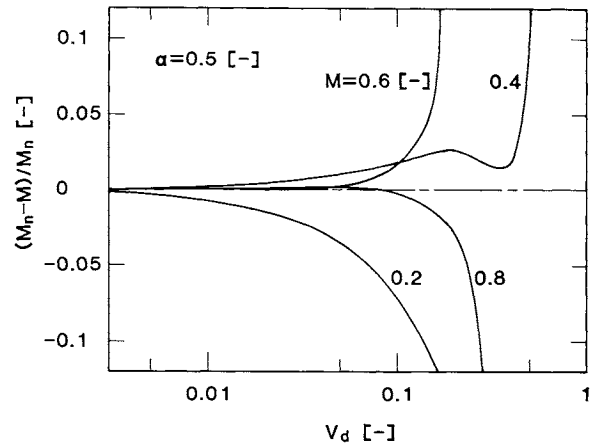


Fig. 7. Relation between V_d of spheroidal particles and differential error $(M_n - M)/M_n$ caused by manner of particle feed

$\Delta K(n)$.

$$\Delta K(n) = \prod_{i=1}^n \left[1 + \frac{p_i V_{di}}{1 - V_d(i)} \right] - 1 \quad (11)$$

$$V_d(i) = \sum_{j=1}^i V_{dj}, \quad p_i \equiv \frac{3M_i + 2}{3(2 - M_i)M_i} \quad (12)$$

where M_i and V_{di} respectively denote the shape parameter and the volume fraction of particle i and the total particle volume fraction becomes $V_d = V_d(i = n)$. Equation (11) is reduced to Eq. (2) when $n=1$, $M_i = M$ and $V_{di} = V_d$; to Eq. (3) when $n=2$, $M_i = M$ and $V_{di} = V_d/2$; and to Eq. (4) when $n \rightarrow \infty$, $M_i = M$ and $V_{di} \rightarrow 0$. Even if for equal particles, this leads to the irrational situation where the relative conductivity changes according to whether the particles are fed into the suspension all at the same time or little by little separately. To avoid the contradiction, Eq. (11) is modified as follows.

$$\Delta K(n) = \prod_{i=1}^n \left[1 + \frac{p_i V_{di}}{1 - \{V_{di} + \alpha V_d(i-1)\}} \right] - 1 \quad (13)$$

where $V_d(0) = 0$ and $\alpha (< 1)$ is a constant to be determined so as to make Eq. (13) independent of the manner of particle feed. Consider that equal particles of total volume fraction V_d and shape parameter M_n are fed into a continuous medium little by little with each amount of $V_{di} = V_d/n$ where $n = 1000$. $\Delta K(n)$ is calculated from Eq. (13) for various V_d , using an arbitrary value of α . In the dilute region, say $V_d \lesssim 0.1$, $\Delta K(n)$ should be equal to Eq. (2), which holds for the case where the particles are fed into the continuous medium all at the same time. Accordingly, one can compare M_n with M in relation to V_d . By iterating the above procedure for different values of α , one can determine $\alpha = 0.5$ where $M \approx M_n$ holds for the widest range of V_d . Figure 7 shows the calculated results for spheroidal particles. As can be seen from the figure, $|(M_n - M)/M_n| \leq 0.05$ for $V_d \leq 0.07$.

The shape parameters determined from the electrical conductivity measurement for suspensions of binary mixtures (sphere+disk 2) and ternary mixtures (disk 1+disk 2+disk 4) are plotted respectively in Figs. 8 and 9 against the mixture ratio of each component. The experimental data are obtained by hand-shaking dispersion. Solid curves are obtained from Eq. (13) with $\alpha=0.5$. It should be noted that the shape parameter evaluated from the electrical conductivity differs from the corresponding volume average, as shown by the broken line in Fig. 8. The experiment agrees well with the theoretical result. Hence, one can determine the mixture ratio of binary shaped particles from the electrical conductivity measurement if the shape parameter of each component and the total particle volume fraction of the mixture are known.

In what follows, the influence of the shape distribution on the electrical conductivity of suspensions is discussed. As shown by Nakajima *et al.*,¹³⁾ the aspect ratio defined by $x = (\text{thickness})/(\text{width})$ has a beta distribution. It is herein expressed for oblate spheroidal particles as follows.

$$f(x) = \frac{x^{v-1}(1-x)^{w-1}}{B(v, w)} \quad (0 \leq x \leq 1)$$

$$= 0 \quad (x < 0 \text{ or } x > 1) \quad (14)$$

and the average and the standard deviation respectively become

$$\bar{x} = v/(v+w) \quad (15)$$

$$\sigma(x) = \sqrt{vw/[(v+w)^2(v+w+1)]} \quad (16)$$

where $x = (a/b)_g$, v and w ($v, w > 0$) are the parameters, and $B(v, w)$ is the beta function. Varying v and w yields various distributions. Figure 10 shows the relationships between the standard deviation $\sigma(x)$ of the geometrical aspect ratio x and the bulk-mean aspect ratio $(a/b)_e$ estimated from the electrical conductivity of a suspension of oblate spheroidal particles. \bar{x} and $\sigma(x)$ are chosen for the range of $v, w > 1$ where $f(x)$ becomes bell-shaped. $(a/b)_e$ is calculated by setting Eq. (2) = Eq. (13) and using Eq. (6), where the total particle volume fraction $V_d = 0.05$, $V_{di} = V_d/n$, $n = 1000$, and $\alpha = 0.5$. As one can see from Eq. (2), $\Delta K = \infty$ at $p = \infty$ or $M = 0$, which corresponds to $(a/b)_e = 0$ and $(a/b)_g = 0$. Hence the lower part of the $(a/b)_g$ distribution increases ΔK significantly, giving rise to a decrease in M and hence a decrease in $(a/b)_e$. As one can see from Fig. 10, therefore, wider shape distributions cause a larger difference between \bar{x} and $(a/b)_e$, especially for $\sigma(x) > 0.1$.

Conclusion

A method for determining particle shape from the electrical conductivity of suspensions is proposed. For

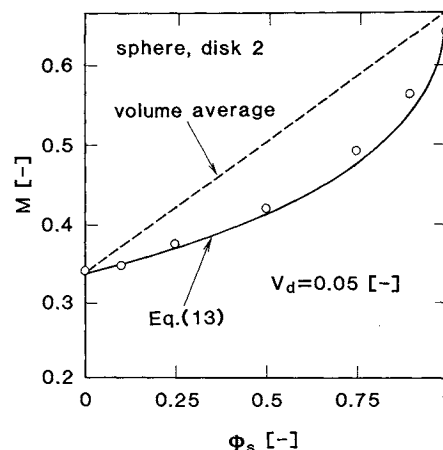


Fig. 8. Shape parameter M determined from electrical conductivity of suspensions of binary mixtures (sphere+disk 2). ϕ_s = mixture ratio of spheres

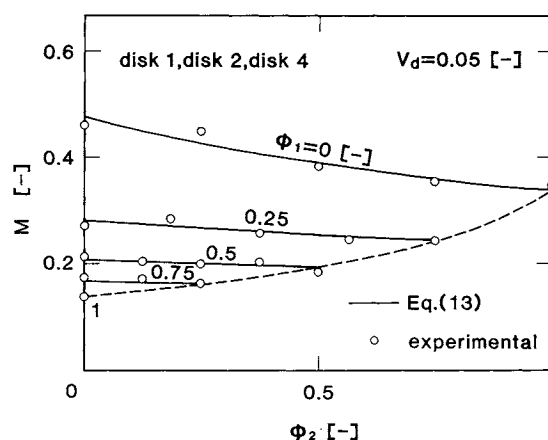


Fig. 9. Shape parameter M determined from electrical conductivity of suspensions of ternary mixtures (disk 1+disk 2+disk 4). ϕ_1 = mixture ratio of disk 1, ϕ_2 = mixture ratio of disk 2

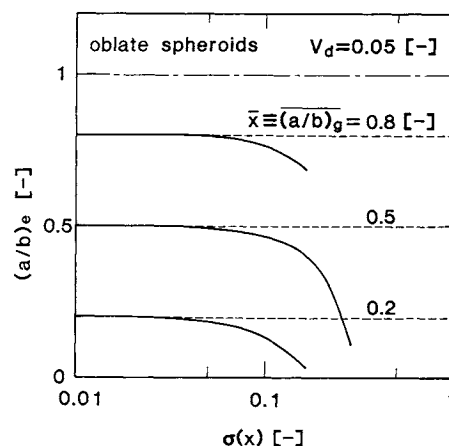


Fig. 10. Relationships between $\sigma(x)$ and $(a/b)_e$. $x \equiv (a/b)_g$.

dilute suspensions of equal particles, the particle shape can be determined as the aspect ratio of the equivalent spheroid, provided that the particles are dispersed uniformly with a random orientation. The difference between the geometrical aspect ratio and

that of the equivalent spheroid is shown experimentally for disks and cylindrical particles. Equation (13) is derived for the electrical conductivity of suspensions of multishape particles and successfully applied to suspensions of ternary mixtures. As a result, the mixture ratio of binary systems is measurable by the electrical conductivity method. The difference between the average geometrical shape and the bulk-mean shape determined from the electrical conductivity measurement becomes large for broader shape distributions.

Nomenclature

$B(v, w)$	= beta function where v and w are parameters	[—]
$f(x)$	= beta distribution function where $x = (a/b)_g$	[—]
K_c	= electrical conductivity of continuous phase	$[\Omega^{-1} \text{ cm}^{-1}]$
K_{Si}	= electrical conductivity of suspension made by i times particle feed	$[\Omega^{-1} \text{ cm}^{-1}]$
ΔK	= relative conductivity of suspension of equal particles	[—]
$\Delta K(i)$	= relative conductivity of suspension made by i times particle feed	[—]
M	= bulk-mean shape parameter determined from electrical conductivity	[—]
M_i	= shape parameter of particle i	[—]
n	= number of components	[—]
p_i	= function of M_i , Eq. (5)	[—]
V_d	= total particle volume fraction	[—]
V_{di}	= volume fraction of particle i	[—]
$V_d(i)$	= cumulative volume fraction from particle 1 to particle i	[—]
\bar{x}	= average of geometrical aspect ratio of spheroid	[—]
α	= correction factor (< 1), Eq. (13)	[—]
$\sigma(x)$	= standard deviation of geometrical aspect ratio of spheroid where $x = (a/b)_g$	[—]

$(a/b)_g$	= geometrical aspect ratio of disk or oblate spheroid	[—]
$(b/a)_g$	= geometrical aspect ratio of cylinder or prolate spheroid	[—]
$(a/b)_e$	= bulk-mean aspect ratio of oblate spheroid determined from electrical conductivity	[—]

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