COMPARISON OF BATCHWISE CENTRIFUGAL AND CONSTANT-PRESSURE FILTRATION

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Batchwise centrifugal filtration and constant-pressure filtration are compared from the viewpoint of cake structure. To correlate the filtration resistance with a pressure as a unique function, it is essential to account for the sedimentation behavior and to use the average compressive pressure.

By using the average compressive pressure as a parameter, convenient and approximate calculating methods are proposed to predict a centrifugal filtration process from the data of constant-pressure filtration. If the procedures of compression permeability tests and theoretical calculations are considered to be tedious, the predicted results can stand comparison with the experimental or precisely calculated results from the compression permeability data. An example of predicted centrifugal filtration process is shown for a practical centrifuge.

Introduction

Although many studies[1-4,15] of centrifugal filtration have been reported extensively, most of these investigations have assumed an incompressible cake and instantaneous cake formation in which the sedimentation phenomena have been neglected. Recently, the authors[6] reported a theoretical analysis of batchwise centrifugal filtration which accounted for both the compressible behavior of cake and the sedimentation processes. It was demonstrated that the average compressive pressure $\bar{p}_s$ was more reliable than a pressure difference for correlating the filtration processes of different modes.

In this study, the difference of cake structures between centrifugal and constant-pressure filtration was investigated theoretically and the validity of an average compressive pressure $\bar{p}_s$ was confirmed by comparing the experimental results. By relying on the consistency of $\bar{p}_s$, convenient methods to predict a process of centrifugal filtration from the experimental results of constant-pressure filtration are discussed.

1. Theory and Calculation

1.1 Centrifugal sedimentation

The settling rate of suspension u at radius r is obtained as[6]

$$u=\frac{r_0}{r}u_0+\frac{r\omega^2}{g}u_g \tag{1}$$

where u_0 is the superficial velocity of filtrate at medium ($r=r_0$) and u_g is the settling rate of suspension under gravity.

Because the average filtration resistance is proportional to the mass of dry solid deposited on unit area, the additional cake formation due to sedimentation of the solid particles has to be accounted for.[5,6] If the mass of cake formed is calculated from the material balance involving only the filtrate volume as is customary in ordinal filtration, it leads to serious errors in calculations. Centrifugal sedimentation with its radial flow differs from gravitational sedimentation in two important aspects. Since the acceleration and the area are both proportional to radius, the particle flux is not constant even with a homogeneous slurry. Equating the rate of change of solid flux at radius r to the rate increase of solid in the region r to $r+dr$ yields

$$d(2\pi ruC)+2\pi r\frac{\partial C}{\partial t}dr=0 \tag{2}$$

If the initial slurry concentration is uniform and u_g is a unique function of slurry concentration C, Eq. (2) reduces to

$$\frac{dC}{dt}+\frac{2\omega^2}{g}u_gC=0 \tag{3}$$

Thus the slurry concentration is independent of radius but decreases everywhere within the suspension at the same rate with respect to time,[6] whereas in one-dimensional filtration the slurry concentration remains constant.

1.2 Centrifugal filtration

The rate of cake formation in batch centrifugal filtration depends upon the combined influence of both sedimentation and filtration. Equating the flux

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at cake surface ($r=r_c$) to the rate of cake build-up, yields

$$2\pi r_c u_c C = 2\pi r_c C \frac{dr_c}{dt} + \pi \frac{d}{dt}[(1-\varepsilon_{av})(r_0^2 - r_c^2)] \quad (4)$$

where ε_{av} is the average porosity of whole cake and u_c is the settling rate of suspension at cake surface. Replacing u and r in Eq. (1) by u_c and r_c, substituting Eq. (1) and (3) into Eq. (4), and neglecting the second-order terms, Eq. (4) leads to

$$-\frac{dr_c}{dt} = \frac{s\rho_{sl}}{\{\rho_p(1-\varepsilon_{av}) - s\rho_{sl}\}} u_c + \frac{(r_0^2 - r_c^2)\rho_p}{2r_c\{\rho_p(1-\varepsilon_{av}) - s\rho_{sl}\}} \cdot \frac{d\varepsilon_{av}}{dt} \quad (5)$$

The second term of the right-hand side is so small over the range of filtration that it can be considered to be negligible. A force balance over a differential element in centrifugal cake yields[6,11]

$$\frac{dp_L}{dr} + \frac{dp_s}{dr} + (1-k_0)\frac{p_s}{r} - \{\rho\varepsilon + \rho_p(1-\varepsilon)\}r\omega^2 = 0 \quad (6)$$

where p_s is the solid compressive pressure and p_L is the hydraulic pressure at radius r in the cake, and where it is assumed that the lateral stress is proportional to the normal stress on the particles and that the momentum changes are negligible.

The basic flow equation can be written for a differential element at radius r as

$$dp_L = \cdot \rho r\omega^2 \cdot dr - \mu\alpha\rho_p(1-\varepsilon)\cdot u_r \cdot dr \quad (7)$$

by assuming the momentum change of liquid to be negligible. In Eq. (7), u_r is the apparent relative velocity of liquid to solid.[9] The local value of porosity ε and the local value of specific resistance α of cake are functions of p_s. The average specific resistance α_{av} is defined for overall flow rate $Q=2\pi r_0 u_0 h$ and total mass of dry solid W. Integrating Eq. (7) from r_c to r_0 and incorporating the filter medium resistance R_m yield

$$u_0 = \frac{Q}{A_0} = \frac{\Delta p}{\mu(\alpha_{av} W/A_e + R_m)} \quad (8)$$

where A_e is the effective filtration area[8] defined by

$$A_e = 2\pi \cdot \left(\frac{r_{av} \cdot r_{lm}}{r_0}\right) h = \frac{\pi(r_0^2 - r_c^2)h}{r_0 \ln(r_0/r_c)} \quad (9)$$

and Δp is the total liquid driving pressure, given by

$$\Delta p = \frac{\omega^2}{2} \cdot \{\rho(r_s^2 - r_l^2) + \rho_{sl}(r_c^2 - r_s^2) + \rho(r_0^2 - r_c^2)\} = p_{Lc} + \rho\omega^2(r_0^2 - r_c^2)/2 \quad (10)$$

where ρ_{sl} is the apparent density of the slurry and the thickness of filter medium is considered to be negligible.

1.3 Theoretical calculation

The theoretical equations described above were applied to data of compression permeability and gravitational settling tests to predict the centrifugal filtration process. When a compressive pressure p_s-distribution and a value of u_0 are assumed for a given centrifugal cake, characteristic values of ε and α are obtained as a function of r by using the compression permeability data. The settling rate, the changing slurry concentration and the rate of cake formation can be calculated from Eqs. (1), (3) and (5), respectively. The stress and flow equations, Eqs. (6) and (7), can be combined to get a new p_s-distribution, so that the trial-and-error procedure leads to the predicted filtration process as a function of time. As the relationship between u_r and r is obtained on the basis of the flow equation through a cylindrical filter cake,[9] the liquid pressure distribution p_L can be obtained by integrating Eq. (7) over the range from r_c to r in cake. The average specific resistance is obtained from Eq. (8).

The theoretical calculation for constant-pressure filtration is the same as the procedure for centrifugal filtration. The theoretical background of the calculations is the so-called modern filtration theory.[10,13,14]

2. Experimental Methods and Materials

The experimental apparatus for centrifugal filtration is illustrated in **Fig. 1**. The filter basket (0.15 m I.D. and 0.083 m height) was covered with a transparent plate on the top surface and the filter medium (Toyo, No. 4) was supported by a stainless steel sieve having 1.68 mm (JIS) openings. Slurry of fixed volume (500 ml) was fed at a stroke, and the radii r_l and r_s were measured by stroboscopic photography.

The slurries used for this investigation were composed of limestone-water, limestone-30% glycerol in water, and coal-water. **Table 1** shows the physical properties of these slurries. The local porosity ε and specific resistance α were obtained as a function of p_s

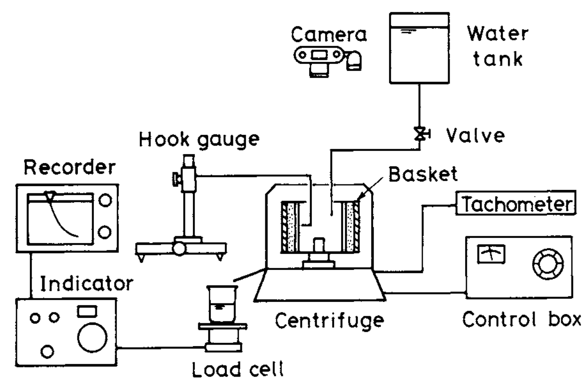

Fig. 1. Experimental apparatus

Table 1. Materials used

Solid materials	Density ρ_p [kg/m³]	Particle size $d_{p,50}$ [μm]	Compression permeability tests: Porosity ε [—]	Compression permeability tests: Specific resistance α [m/kg]
Limestone	2710	4.1	$\begin{cases}\varepsilon=0.78 & (p_s\leqq 23.5\,\mathrm{Pa})\\ \varepsilon=0.934\cdot p_s^{-0.057} & (p_s>23.5\,\mathrm{Pa})\end{cases}$	$\begin{cases}\alpha=1.72\times 10^{10} & (p_s\leqq 23.5\,\mathrm{Pa})\\ \alpha=5.46\times 10^{9}\cdot p_s^{0.364} & (p_s>23.5\,\mathrm{Pa})\end{cases}$
Coal	1440	12.4	$\varepsilon=0.68-9.89\times 10^{-4}\cdot p_s^{0.378}$	$\alpha=1.10\times 10^{11}+7.17\times 10^{8}\cdot p_s^{0.499}$

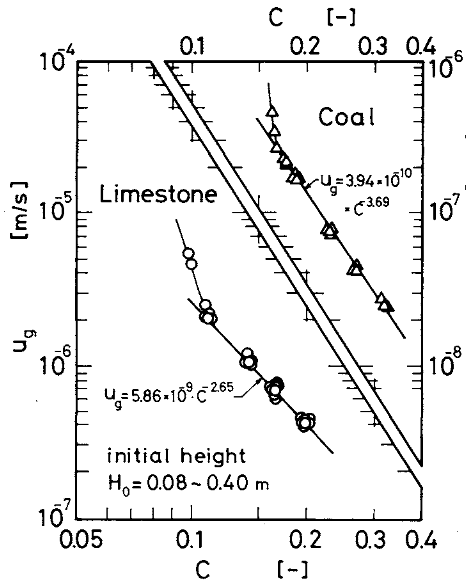

Fig. 2. Settling rates *vs.* volume concentration

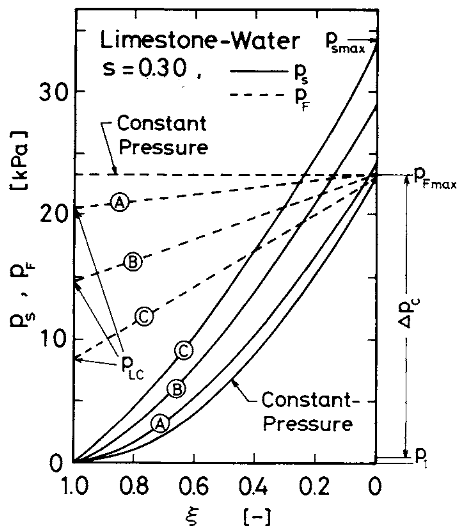

Fig. 3. Various p_s- and p_F-distributions in cake at the same Δp_c

by using both the compression permeability cell and the settling method.[12] The gravitational sedimentation velocity u_g was determined experimentally by using an acrylic plastic cylinder (0.056 m I.D.),[7] and it is plotted as a function of the slurry concentration C as shown in **Fig. 2**.

Constant-pressure filtrations with the same slurries were performed in order to compare the characteristic values of the filter cakes.[5]

3. Results and Discussion

3.1 p_s-distribution and α_{av}

Calculated results of pressure distributions in filter cakes are shown in **Fig. 3**. ξ expresses the relative position in the cake, and it is x/L for one-dimensional constant-pressure filter cake and $(r_0^2-r^2)/(r_0^2-r_c^2)$ for centrifugal filter cake. In the case of constant-pressure filtration, the maximum value of compressive pressure p_s, which determines the state of the cake structure, equals the liquid driving pressure across a cake, $\Delta p_c=\Delta p-p_1$. However, in the case of centrifugal filtration p_{smax} does not equal Δp_c $(=p_{F\max}-p_1)$, and p_F is the liquid driving pressure in the cake. It is obtained by $p_{Lc}+\rho\omega^2(r^2-r_c^2)/2$. The lines marked Ⓐ, Ⓑ and Ⓒ show the p_F and p_s distributions in centrifugal cakes. The Δp_c values are approximately the same as in constant-pressure filtration. Because p_{Lc} and p_{smax} may vary considerably according to the incorporated relation of increasing cake thickness and decreasing liquid layer above the cake, many variations of the p_s vs. r relationship could occur for a single value of Δp_c. The differing p_s curves lead to different cake structures and, as a consequence, α_{av} will not be the same even at the same value of Δp_c. Also, they will be not in accordance with α_{av} of constant-pressure filtration of the same Δp_c. Calculated α_{av}[10] values corresponding to the p_s-distributions in Fig. 3 are shown in **Table 2**.

Table 2. Flow resistance and characteristic pressures

Δp_c [kPa]	$\bar{p}_s$ [kPa]	α_{av} [m/kg]
Constant-Pressure		
23.2	7.98	1.32×10^{11}
Centrifugal		
Ⓐ 22.4	9.25	1.39×10^{11}
Ⓑ 23.2	11.98	1.54×10^{11}
Ⓒ 23.0	15.46	1.70×10^{11}

Calculated time courses of characteristic pressures are illustrated in **Fig. 4** as a function of filtrate

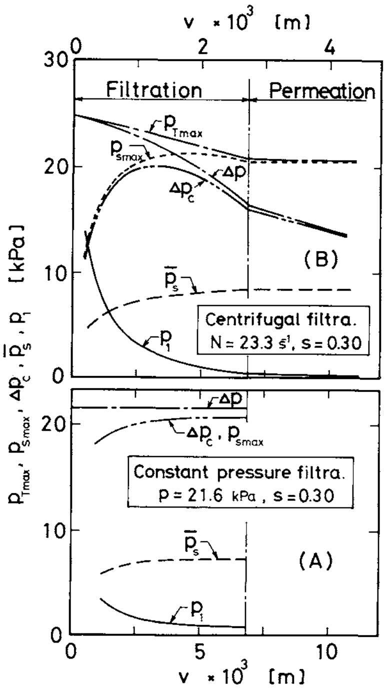

Fig. 4. Changing course of characteristic pressures with volume of filtrate; centrifugal (B) and constant-pressure filtration (A)

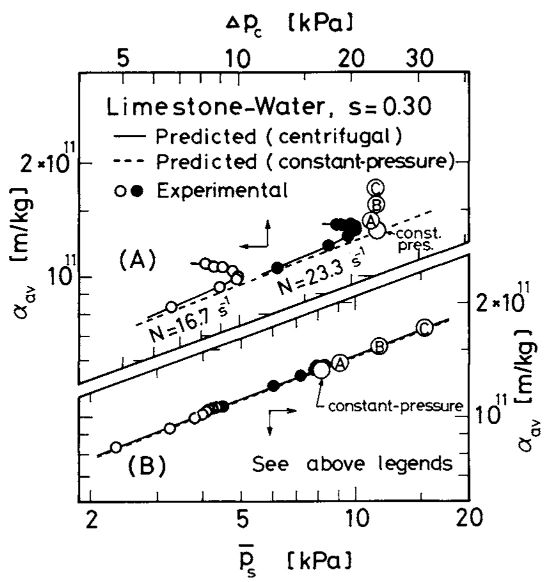

Fig. 5. Experimental and predicted results of α_{av}; for Δp_c (A) and for $\bar{p}_s$ (B)

volume, and the experimental and predicted values of α_{av} are plotted in **Fig. 5(A)** as a function of the liquid driving pressure Δp_c. At first, Δp_c increase because p_1 decreases with time more rapidly than does Δp, so that α_{av} also increases. After Δp_c attains its maximum value, it begins to decrease, but α_{av} continues to increase gradually because of the increasing p_s value and the irreversible behavior of the cake structure. The α_{av} values marked Ⓐ, Ⓑ, Ⓒ and constant-pressure in Fig. 5 correspond to the same pressure distributions in Fig. 3.

3.2 Average compressive pressure $\bar{p}_s$

It has been thought that α_{av} depends only on the pressure drop across the filter cake, which equals the maximum cake compressive pressure. However, the present work demonstrated that there may be various values of $p_{s\max}$ for a given Δp_c in centrifugal filtration. To compare filtrations of another type with centrifugal filtration or to convert the results from one to another, it is necessary and essential to find a more reasonable and unique pressure that correlates α_{av} favorably. The compressive pressure p_s at any point in the cake is weighted by the volume of cake, and its average value is defined as

$$\bar{p}_s = \int_0^1 p_s d\xi \qquad (11)$$

In Fig. 5B, the data of Fig. 5A are replotted to show α_{av} with $\bar{p}_s$. They are fairly coincident with a unique relationship of α_{av} vs. $\bar{p}_s$ for both centrifugal and constant-pressure filtration. These relations corresponding to Fig. 3 are shown numerically in Table 2.

4. Convenient Prediction Methods

Centrifugal filtration processes can be acculately estimated by numerical calculations based on the theory mentioned above. However, this requires reliable compression permeability data and somewhat complex calculations. For practical and approximate calculation, a simpler method to predict the processes is desirable. It might be more convenient if the constant-pressure filtration data are available and can be utilized.

As can be seen in Fig. 3, at the initial stage of centrifugal filtration Ⓐ, the p_s-distribution is similar to that in constant-pressure filtration of moderately compressible cake, which has been approximated as a complement of the sine curve ($p_s = p_{s\max}(1-\sin\xi)$) for one-dimensional filtration. In this case, Eq. (11) is calculated as $\bar{p}_s/p_{s\max} = 0.36$, while it must be remains 0.5 for non-compressible cake. As the centrifugal process is approaching the end of filtration Ⓒ, the p_s-distribution becomes more linear, which means that the ratio approaches 0.5. Thus in regard to moderately compressible cake, it can be inferred that the ratio of $\bar{p}_s/p_{s\max}$ in centrifugal filter cake may change from 0.36 to 0.5. This ratio from the theoretical calculations are plotted for moderate compressible cake (limestone, $n=0.36$) and poorly compressible cake (coal, $n=0.20$), taking the abscissa of $(p_{Lc}-p_1)/p_{s\max}$ in **Fig. 6**. If a straight line is assumed for convenience as shown by the dotted line, the $\bar{p}_s$ value can be approximately obtained from the characteristic pressures for any time of a filtration period. Accordingly, by the aid of average compressive pressure $\bar{p}_s$, the structure of centrifugal filter cake for any

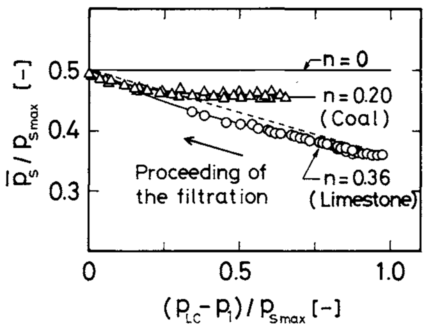

Fig. 6. Variations of $\bar{p}_s/p_{smax}$ with ratio of characteristic pressures

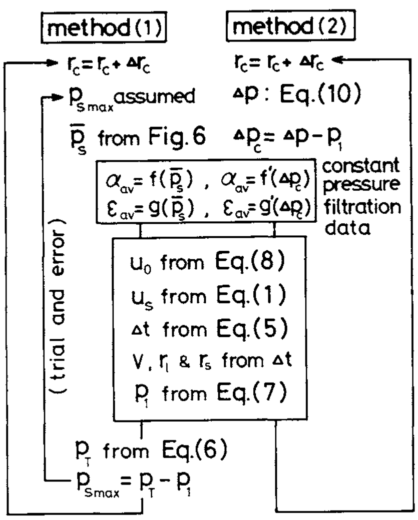

Fig. 7. Schematic flowchart for convenient or approximate calculations

situation can be predicted from the data of constant-pressure filtration. As a consequence, a series of calculations gives a centrifugal filtration process.

Two methods for the calculation are shown schematically in **Fig. 7**. Prior to the calculation, the data of constant-pressure filtration, ε_{av} and α_{av} vs. Δp_c, are converted to ε_{av} and α_{av} vs. $\bar{p}_s$ by multiplying Δp_c by 0.36 as mentioned above. In method (1), the r_c value will be determined by adding an arbitrary increment of Δr_c to the previous r_c value, and $\bar{p}_s$ by using the relation of Fig. 6 for an assumed p_{smax}. Calculating u_0, u_s (u value at $r=r_s$), Δt may lead to obtaining V, r_l, r_s, t ($t=t+\Delta t$) and p_1. The calculations are repeated until satisfactory agreement is obtained between assumed and calculated values of p_{smax}. In method (2), ε_{av} and α_{av} are obtained directly from Δp_c, so that the method does not need the trial-and-error procedure. However, the result yields a step in accuracy because the Δp_c may not be a characteristic pressure in centrifugal filtration, as explained in 3.1. The predicted results are compared with the experimental data in **Fig. 8**. Method (1), represented by dot-dash line, shows good agreement with the experimental results and with the most precise theoretical

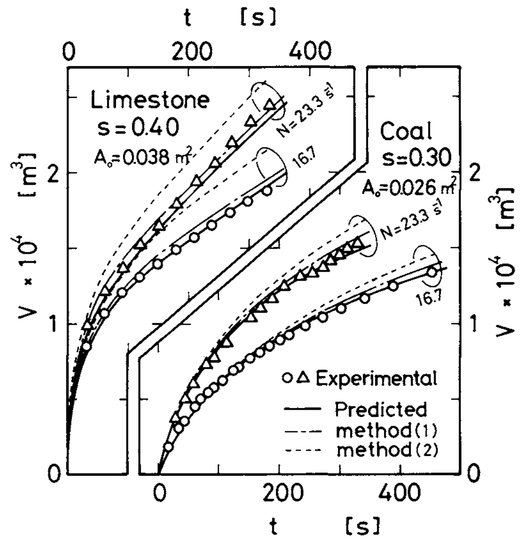

Fig. 8. Comparison of filtering courses calculated by three prediction methods and experimental

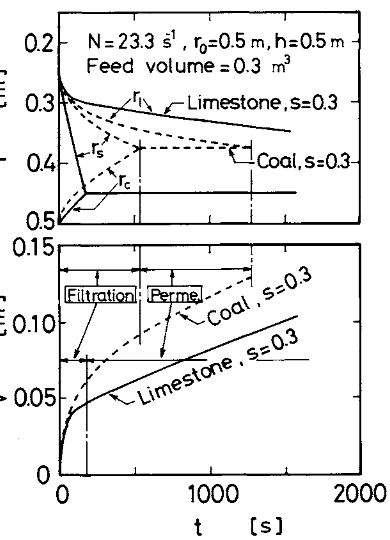

Fig. 9. Predicted filtering course for practical centrifuge by method (1)

calculations based on the compression permeability data (solid line), whereas method (2), represented by broken line, shows a deviation of about 10%.

Figure 9 shows an example of a predicted filtration process with a practical centrifuge having a diameter of 1 m and speed N of 23.3 1/s. If only the data of constant-pressure filtration are available, the convenient prediction methods make it easy and precise in some measure to calculate the filtration process of a practical centrifuge for a given slurry.

Conclusions

It was possible to estimate the exact centrifugal filtration process by using compression permeability data. The changing slurry concentration and the mass deposited due to sedimentation had to be accounted for. A reliable and consistent specific resistance could be obtained, but the value in centrifugal filtration was larger in itself than that of constant-pressure filtration at the same liquid driving force. The average value of compressive pressure in the cake led to good correlations between flow resistance and pressure in either type of filtration.

The ratio of average to maximum values of cake compressive pressure was examined for centrifugal filter cake. A linear relationship was assumed, from about 0.36 at the beginning to 0.5 near the end of centrifugal filtration, while it remained 0.36 for constant-pressure filtration. This relation made it possible to predict the centrifugal filtration process from the flow resistance of constant-pressure filtration. The calculation was easier and more practical than a strictly precise calculation based on the modern filtration theory and the compression permeability data.

Nomenclature

A_e	= effective filtration area defined by Eq. (9)	[m^2]
A_0	= filtration area at filter medium	[m^2]
C	= volume fraction of solids in slurry	[—]
g	= gravitational acceleration	[m/s^2]
h	= height of cylindrical filter surface	[m]
k_0	= ratio of lateral stress to normal stress	[—]
L	= thickness of filter cake	[m]
N	= rotational speed	[s^{-1}]
n	= compressibility coefficient of cake	[—]
p_F	= liquid driving pressure developed by centrifugal action	[Pa]
p_L	= hydraulic pressure	[Pa]
p_{Lc}	= hydraulic pressure at cake surface	[Pa]
p_s	= solid compressive pressure	[Pa]
$p_{s\max}$	= solid compressive pressure at medium	[Pa]
$\bar{p}_s$	= average compressive pressure derived by Eq. (11)	[Pa]
p_T	= total pressure $=(p_L+p_s)$	[Pa]
p_1	= hydraulic pressure at medium	[Pa]
Δp	= total liquid driving pressure	[Pa]
Δp_c	= liquid driving pressure at medium	[Pa]
Q	= rate of filtrate flow through cake	[m^3/s]
R_m	= resistance of filter medium	[1/m]
r	= radius	[m]
r_c	= radius of cake surface	[m]
r_l	= radius of a clear liquid surface	[m]
r_s	= radius of slurry-supernatant interface	[m]
r_0	= radius of medium surface of centrifuge	[m]
s	= mass fraction of solids in slurry	[—]
t	= time	[s]
u_c	= settling rate of suspension at cake surface in a filtering centrifuge	[m/s]
u_g	= settling rate of suspension supernatant interface in a gravitational field	[m/s]
u_s	= settling rate of suspension supernatant interface in a filtering centrifuge	[m/s]
u_0	= superficial velocity of filtrate at medium	[m/s]
V	= total volume of filtrate	[m^3]
v	= volume of filtrate per unit area of filter medium	[m]
W	= total true mass of solid in centrifugal cake	[kg]
x	= distance from filter medium	[m]
α	= local value of specific resistance of cake	[m/kg]
α_{av}	= average specific resistance of cake based on W	[m/kg]
ε	= local value of porosity in cake	[—]
ε_{av}	= average porosity of whole cake	[—]
μ	= viscosity of filtrate	[Pa·s]
ξ	= relative position in the filter cake	[—]
ρ	= density of liquid	[kg/m^3]
ρ_p	= true density of solids	[kg/m^3]
ρ_{sl}	= apparent density of slurry	[kg/m^3]
ω	= angular velocity	[rad/s]

Literature Cited

1) Grace, H. P.: *Chem. Eng. Progr.*, **49**, 303, 367, 427 (1953).
2) Hultsch, G. and H. Wilkesmann: "Solid-liquid Separation Equipment Scale-Up," p. 493, Upland Press, Croydon (1977).
3) Oyama, Y. and S. Sumikawa: *Kagaku Kōgaku*, **18**, 593 (1954).
4) Rushton, A. and M. Spear: *Filtration & Separation*, **12**, 254 (1975).
5) Sambuichi, M., H. Nakakura and K. Osasa: *Memoirs of the Faculty of Engineering, Yamaguchi University*, **33**, 65 (1982).
6) Sambuichi, M., H. Nakakura, K. Osasa and F. M. Tiller: *AIChE J.*, **33**, 109 (1987).
7) Sambuichi, M., H. Nakakura and K. Osasa: *Nippon Kogyo Kaishi*, **103**, 109 (1987).
8) Shirato, M. and K. Kobayashi: *Memoirs of the Faculty of Eng., Nagoya University*, **19**, 280 (1967).
9) Shirato, M., K. Kobayashi and M. Tanimura: *Kagaku Kōgaku*, **37**, 76 (1973).
10) Shirato, M., M. Sambuichi, H. Kato and T. Aragaki: *AIChE J.*, **15**, 405 (1969).
11) Shirato, M. and T. Aragaki: *Kagaku Kōgaku*, **33**, 205 (1969).
12) Shirato, M., T. Murase, E. Iritani and N. Hayashi: *Filtration & Separation*, **20**, 404 (1983).
13) Tiller, F. M. and M. Shirato: *AIChE J.*, **10**, 61 (1964).
14) Tiller, F. M., S. Haynes and W. M. Lu: *AIChE J.*, **18**, 13 (1972).
15) Valleroy, V. V. and J. O. Maloney: *AIChE J.*, **6**, 382 (1960).

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