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SHORT CYCLE TIME APPROXIMATION AND GRAPHICAL SOLUTION OF PRESSURE SWING ADSORPTION

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A method of graphical solution of ordinary differential equations, derived on the basis of a short cycle time approximation, is proposed to evaluate the performance of pressure swing adsorption. The longitudinal distribution of the time-averaged concentration $\bar{C}_A$ is obtained by graphical integration of the reciprocal of the effective driving force, which is found graphically by the construction of the operating line (which relates concentrations between the adsorption and desorption steps) and the tie lines (which relate bulk and surface values). The concentration swing ΔC_A is solved as a function of $\bar{C}_A$ by the isoclinal line method, in which the current coordinates are simply constructed to determine the direction of a tangent on a $\Delta C_A - \bar{C}_A$ diagram. Simplified construction procedures are discussed for special cases of surface diffusion control, film diffusion control and linear isotherm. The usefulness of the proposed method is comfirmed in an illustrative solution.

Introduction

A short cycle time approximation was proposed in previous papers[1,4] to evaluate the performance of pressure swing adsorption (PSA) and it was in part supported by experiment.[2,3] In the method, a set of partial differential equations of material balance is expanded into power series of a small value of cycle time t_c or throughput ratio $1/n_A$. Since the time averaged concentration $\bar{C}_A$ is given by a quadrature and the concentration swing ΔC_A is given by the solution of an ordinary linear differential equation, mathematical difficulties have been reduced greatly for not only the linear adsorption isotherm[1] but also the nonlinear case.[4]

The purpose of the present paper is to propose a

method of graphical solution in the same mathematical frame as before.[4] The graphical integration and the isoclinal line method, respectively, are employed to solve the time averaged concentration $\bar{C}_A$ and concentration swing ΔC_A since simplified equations of material balance are amenable to these methods. Graphical solution may be suitable especially to the adsorption isotherm given graphically.

Graphical solution in general is simple and plain, as seen widely in textbooks of unit operations. Mathematical background or a calculating device is not required in particular. The process to reach the solution can be visualised by simple construction of figures. Therefore, graphical solution is still a powerful tool for the performance evaluation of PSA in spite of its less accurate results and the easy availability of computer facilities.

1. Basic Concept

The present analysis is based on the following conditions and assumptions as in the previous papers:[1,4] 1) dual column adsorbers, 2) instantaneous blowdown and repressurising, 3) single component adsorption of dilute adsorbate, 4) neglect of accumulation term in gas phase, 5) plug flow of gas with constant superficial velocities, 6) constant mass transfer coefficient, 7) series process of gas film diffusion and surface diffusion in particle phase and 8) isothermal operation.

In the previous paper,[4] dimensionless concentration in adsorption step C_A was expanded in power series around a small value of the throughput ratio $1/n_A$ in the form of

$$C_A = C_A^{(0)} + (v_A/n_A)S_A(\tau - 1/2) + O(1/n_A^2) \quad (1)$$

from which the time-averaged concentration $\bar{C}_A$ and the concentration swing ΔC_A are given by, respectively,

$$\bar{C}_A = \int_0^1 C_A d\tau = C_A^{(0)} + O(1/n_A^2) \quad (2)$$

$$\Delta C_A = C_A|_{\tau=1} - C_A|_{\tau=0} = (\tau_A/n_A)S_A + O(1/n_A^3) \quad (3)$$

Concentration $C_A^{(0)}$ and swing parameter S_A, respectively, are obtained by solving a quadrature, Eq. (T-1) in **Table 1**, and a linear differential equation, Eq. (T-13) in **Table 2**, as developed in the previous paper.[4] Other relevant equations for solving $C_A^{(0)}$ and S_A are cited in Tables 1 and 2 from the previous paper.[4]

It should be noted that the present method can be applied to a small value of throughput ratio $1/n_A$. The limit of application of the present method is reasoned from the analysis in the case of linear isotherm[1] that v_A/n_A is less than 0.5.

Table 1. Equations for solving the zeroth-order concentration $C_A^{(0)}$

$$X = \int_{C_A^{(0)}}^{1} (1/J_A^{(0)}) dC_A^{(0)} \quad \text{(T-1)}$$

in which

$$J_A^{(0)} = (Q_{As}^{(0)} - Q_{Ds}^{(0)})/(1/N_{AS} + 1/\gamma N_{DS}) \quad \text{(T-2)}$$

(general case)

$$= \text{Eq. (T-2) with } Q_{As}^{(0)} = G(C_A^{(0)}) \text{ and } Q_{Ds} = G(C_D^{(0)}) \quad \text{(T-3)}$$

(surface diffusion control)

$$= (C_A^{(0)} - C_D^{(0)})/(1/N_{AF} + 1/\gamma N_{DF}) \quad \text{(T-4)}$$

(film diffusion control)

$$= (C_A^{(0)} - C_D^{(0)})/(1/N_{ASF} + 1/\gamma N_{DSF}) \quad \text{(T-5)}$$

(linear isotherm)

Values of $C_{As}^{(0)}$, $C_{Ds}^{(0)}$ and $C_D^{(0)}$ are determined by solving the following set of equations simultaneously.
Operating line (p-line)

$$C_D^{(0)} = \beta C_{A1}^{(0)} + (1/\gamma)(C_A^{(0)} - C_{A1}^{(0)}) \quad \text{(T-6)}$$

Short cycle time condition

$$Q_A^{(0)} = Q_D^{(0)} \quad \text{(T-7)}$$

Tie lines (t-lines)

$$(Q_{As}^{(0)} - Q_A^{(0)})/(C_{As}^{(0)} - C_A^{(0)}) = -N_{AF}/N_{AS} \quad \text{(T-8)}$$

$$(Q_{Ds}^{(0)} - Q_D^{(0)})/(C_{Ds}^{(0)} - C_D^{(0)}) = -N_{DF}/N_{DS} \quad \text{(T-9)}$$

$$(Q_{As}^{(0)} - Q_A^{(0)})/(Q_D^{(0)} - Q_{Ds}^{(0)}) = \gamma N_{DS}/N_{AS} \quad \text{(T-10)}$$

Adsorption equilibrium (e-curve)

$$Q_{As}^{(0)} = G(C_{As}^{(0)}) \quad \text{(T-11)}$$

$$Q_{Ds}^{(0)} = G(C_{Ds}^{(0)}) \quad \text{(T-12)}$$

Table 2. Equations for solving the swing parameter S_A

$$\frac{dS_A}{dC_A^{(0)}} = \frac{1}{H_A(C_A^{(0)})}\{S_A - h_A(C_A^{(0)})\} \quad \text{(T-13)}$$

with $S_A = 0$ at $C_A^{(0)} = 1$ (T-14)
in which

$$H_A(C_A^{(0)}) = (C_A^{(0)} - C_{As}^{(0)}) + (Q_{As}^{(0)} - Q_A^{(0)})/(G'(C_{As}^{(0)})) \quad \text{(T-15)}$$

$$h_A(C_A^{(0)}) = (Q_{As}^{(0)} - Q_A^{(0)})/G'(C_{As}^{(0)}) \quad \text{(T-16)}$$

(general case)

$$= \text{Eq. (T-16) with } Q_{As}^{(0)} = G(C_A^{(0)}) \text{ and } C_{As}^{(0)} = C_A^{(0)} \quad \text{(T-17)}$$

(surface diffusion control)

$$= (C_A^{(0)} - Q_A^{(0)})/(G'(C_A^{(0)})) \quad \text{(T-18)}$$

(film diffusion control)

$$= (C_A^{(0)} - Q_A^{(0)}) \quad \text{(T-19)}$$

(linear isotherm)

2. Solution of Zeroth-Order Concentration C_A

In the present method of solution, the time-averaged concentration at the exit of product stream in the adsorption step $C_A^{(0)}$ is specified first and then the required value of number of mass transfer units N_A is determined. The concentration distribution is found by solving the values of X corresponding to

given values of $C_A^{(0)}$ for the above determined value of N_A.

2.1 General case

Gas-phase concentration C and amount adsorbed Q, respectively, should be read along the abscissa and the ordinate for graphical presentation. A normalized equilibrium curve (e-curve) and a diagonal line (d-line) are first drawn there as shown in **Fig. 1**.

When $C_A^{(0)}$ and $C_D^{(0)}$, respectively, are read along the abscissa and the ordinate, Eq. (T-6) is a straight line (designated as p-line) which passes through a predetermined point $P(C_{A1}^{(0)}, \beta C_{A1}^{(0)})$ with a slope of $1/\gamma$. The value of $C_D^{(0)}$ corresponding to a given value of $C_A^{(0)}$ can be obtained graphically on the abscissa by turning the p-line and the d-line along the route $A \to B \to C \to D$ as shown in Fig. 1. Eq. (T-6) may be called an operating line in that it gives the relationship of concentrations between adsorption and desorption steps.

The values of $C_{As}^{(0)}$, $C_{Ds}^{(0)}$, $Q_A^{(0)}$, $Q_D^{(0)}$, $Q_{As}^{(0)}$ and $Q_{Ds}^{(0)}$ can be determined for a given value of $C_A^{(0)}$ by solving Eqs. (T-7)–(T-12) graphically as follows. First assume the value of $Q_A^{(0)}(=Q_D^{(0)})$ and take points $E(C_A^{(0)}, Q_A^{(0)})$ and $H(C_D^{(0)}, Q_D^{(0)})$, which represent the bulk condition, on the C–Q diagram as shown in Fig. 1. Eqs. (T-8) and (T-9), respectively, show that points $F(C_{As}^{(0)}, Q_{As}^{(0)})$ and $I(C_{Ds}^{(0)}, Q_{Ds}^{(0)})$, which represent the surface condition, fall on straight lines (t-lines) passing through points E and H with the slope of $-N_{AF}/N_{AS}$ and $-N_{DF}/N_{DS}$. Thus the t-lines EF and HI may be called the tie lines in that they relate the bulk and the surface values. Unless Eq. (T-10) or the relation

$$FG/GK = \gamma N_{DS}/N_{AS} \tag{4}$$

is satisfied, the above procedure must be repeated for a newly assumed value of $Q_A^{(0)}$ until sufficient convergence is attained. Finally, the effective driving force $(Q_{As}^{(0)} - Q_{Ds}^{(0)}) = FK$ is determined for a given value of $C_A^{(0)}$.

Axial distance from the feed inlet X corresponding to $C_A^{(0)}$ can be obtained by graphical integration of Eq. (T-1) coupled with Eq. (T-2), in which the effective number of mass transfer units $1/(1/N_{As} + 1/\beta N_{Ds})$ is obtained by graphically integrating Eq. (T-1) to the lower limit of $C_{A1}^{(0)}$ at $X = 1$.

2.2 Some special cases

In some special cases, the above procedure is simplified since mass transfer rate $J_A^{(0)}$ or the effective driving force in Eq. (T-3), (T-4) or (T-5) can be determined directly from the p-line and the e-curve without iteration.

1) **Surface diffusion control** The surface concentrations $C_{As}^{(0)}$ and $C_{Ds}^{(0)}$, respectively, become equal to the bulk concentrations $C_A^{(0)}$ and $C_D^{(0)}$. In graphical terms, tie lines are vertical lines starting at points A
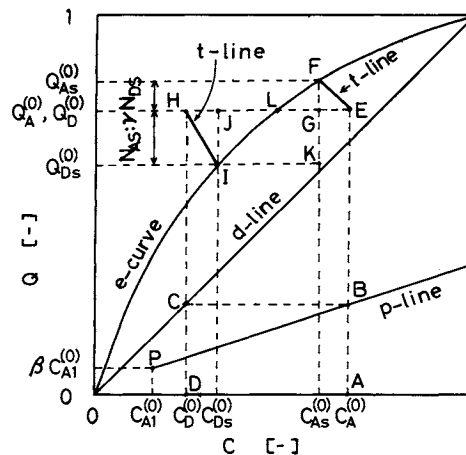

Fig. 1. Graphical method to determine effective driving force (general case).

and D as shown in **Fig. 2(a)** and distance FK or effective driving force in Eq. (T-3), $(Q_{As}^{(0)} - Q_{Ds}^{(0)})$, is found straightforwardly. The values of $Q_A^{(0)}(=Q_D^{(0)})$ is then given by point G which divides FK in the ratio of $\gamma N_{DS}:N_{AS}$.

2) **Film diffusion control** Amounts adsorbed at the surface $Q_{As}^{(0)}$ and $Q_{Ds}^{(0)}$ become equal to those in bulk $Q_A^{(0)}$ and $Q_D^{(0)}$, and in graphical presentation the tie lines become horizontal lines which link at point L as shown in **Fig. 2(b)**. The effective driving force $(C_A^{(0)} - C_D^{(0)})$ in Eq. (T-4) is given simply by EH $(=BC)$. The surface concentration $C_{As}^{(0)}$ or $C_{Ds}^{(0)}$ is given by point F or I, falling on the e-curve, which divides EH $(=BC)$ in the ratio of $\gamma N_{DF}:N_{AF}$. Point G falls on the same point as F or I.

Since $C_D^{(0)}$ is a linear function of $C_A^{(0)}$, as given by Eq. (T-6), the integration of Eq. (T-1) with Eq. (T-4) can be carried out analytically as well, resulting in the equation

$$X = \left(\frac{1}{N_{AF}} + \frac{1}{\gamma N_{DF}}\right) \frac{1 - C_A^{(0)}}{(C_A^{(0)} - C_D^{(0)})_{lm}} \tag{5}$$

in which subscript lm refers to a logarithmic mean over $C_A^{(0)}$ and 1. It should be noted that the value of $C_A^{(0)}$, and thus the average concentration $\bar{C}_A$, is independent of the type of adsorption isotherm.

3) **Linear isotherm** The e-curve coincides with the d-line as shown in **Fig. 2(c)** and the effective driving force in Eq. (T-5), $(C_A^{(0)} - C_D^{(0)})$, is simply found to be EH $(=BC)$. The value of $Q_A^{(0)}$ $(=Q_D^{(0)})$ and point E are determined by point L on the d-line which divides EH $(=BC)$ in the ratio of $\gamma N_{DSF}:N_{ASF}$. Points F and I representing the values at surface are finally determined by drawing t-lines starting at points E and H as shown in Fig. 2 (c).

Analytical integration of Eq. (T-1) with $J_A^{(0)}$ given by Eq. (T-5) leads to Eq. (5) with $(1/N_{AF} + 1/\gamma N_{DF})$ replaced by $(1/N_{ASF} + 1/\gamma N_{DSF})$, which is identical with that given in the previous paper.[1]
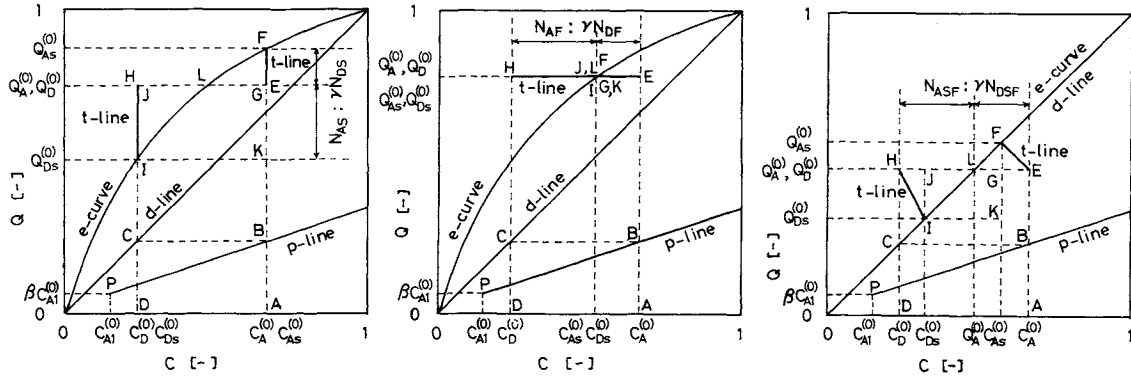
Fig. 2. Graphical method to determine effective driving force (special cases): (a) surface diffusion control; (b) film diffusion control; (c) linear isotherm.

3. Solution of Swing Parameter S_A

3.1 General case

Concentration $C_A^{(0)}$ and swing parameter S_A, respectively, should be read along the abscissa and the ordinate to solve Eq. (T-13) by the isoclinal line method.[5] When the slope of a tangential line of a $S_A - C_A^{(0)}$ curve is known at a given point $S(C_A^{(0)}, S_A)$, the value of S_A at the next point is determined successively. The equation of tangent at point S ($C_A^{(0)}$, S_A) is given in terms of the current coordinates (ξ, η) by the equation

$$\eta = \left(\frac{dS_A}{dC_A^{(0)}}\right)(\xi - C_A^{(0)}) + S_A$$

$$= h_A(C_A^{(0)}) + \frac{S_A - h_A(C_A^{(0)})}{H_A(C_A^{(0)})} [\xi - \{C_A^{(0)} - H(C_A^{(0)})\}]$$

(6)

by substituting Eq. (T-13) for $dS_A/dC_A^{(0)}$. Thus, the tangential line must pass through the point $R(\xi_0, \eta_0)$ of which the coordinates are given by

$$\xi_0 = C_A^{(0)} - H_A(C_A^{(0)})$$ 　(7)

$$= C_{As}^{(0)} - (Q_{As}^{(0)} - Q_A^{(0)})/G'(C_{As}^{(0)})$$ 　(8)

$$\eta_0 = h_A(C_A^{(0)})$$ 　(9)

$$= (Q_{As}^{(0)} - Q_A^{(0)})/G'(C_{As}^{(0)})$$ 　(10)

When a point of intersect of a horizontal line passing through point E and a tangential line of the e-curve at point F is designated as M on the diagram in **Fig. 3**, the term $(Q_{As}^{(0)} - Q_A^{(0)})/G'(C_{As}^{(0)})$ in Eqs. (8) and (10) is given by a line segment MG. In this procedure, points E, F and G have been determined previously in Section 2. Thus the point $R(\xi_0, \eta_0)$ is given by

$$R(\xi_0, \eta_0) = (C_{As}^{(0)} - MG, MG)$$ 　(11)

as shown graphically in Fig. 3.

Since the slope at point S is directed to point R, the next point of solution S can be obtained as a function
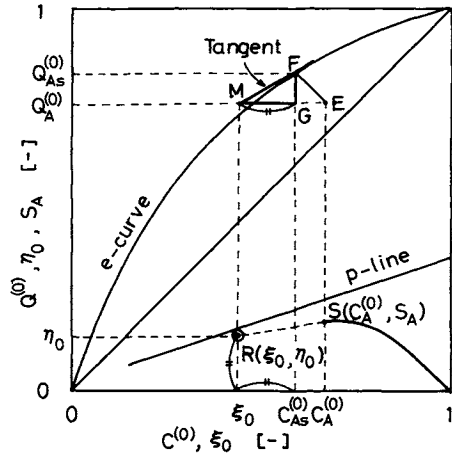

Fig. 3. Graphical method to determine point $R(\xi_0, \eta_0)$ (general case).

of $C_A^{(0)}$ by the successive procedure starting at the point S_0 (1, 0). Since $C_A^{(0)}$ is a function of X as determined in Section 2, the axial distribution of S_A is obtained.

3.2 Some special cases

In some special cases, the locus of point $R(\xi_0, \eta_0)$ can be determined more straightforwardly since points E, F and G have been found simply in Section 2.

1) **Surface diffusion control** Concentration $C_{As}^{(0)}$ is equal to $C_A^{(0)}$, and points G and E are identical. Point R is given by the equation

$$R(\xi_0, \eta_0) = R(C_A^{(0)} - MG, MG)$$ 　(12)

and graphical construction is shown in **Fig. 4(a)**.

2) **Film diffusion control** Adsorbed amount $Q_{As}^{(0)}$ is equal to $Q_A^{(0)}$ and points F and G are identical. Referring to **Fig. 4(b)**, point E' is designated as a point below point F by a distance FE, and M' as a point of intersect of a tangential line at F and a horizontal line passing through E'. Thus, point R is given by
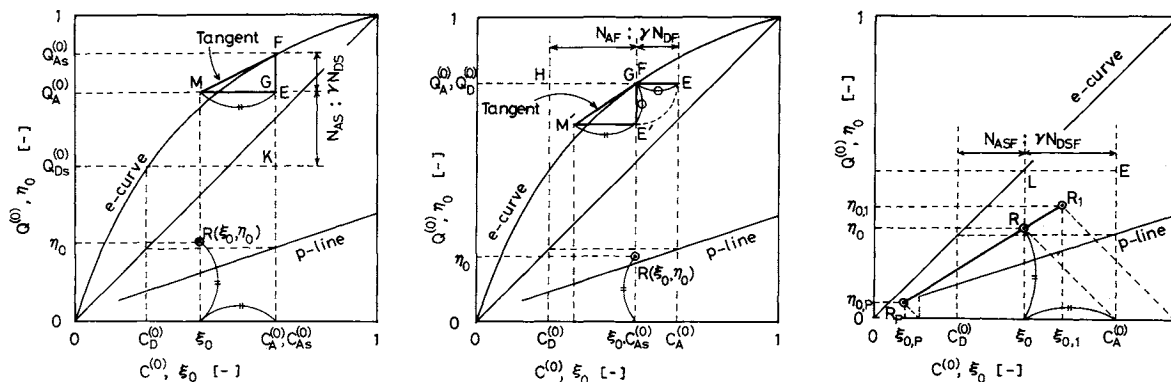
Fig. 4. Graphical method to determine point R (ξ_0, η_0) (special cases): (a) surface diffusion control; (b) film diffusion control; (c) linear isotherm.

$$R(\xi_0, \eta_0) = R(C_{As}^{(0)}, M'E') \qquad (13)$$

as shown in Fig. 4 (b).

3) __Linear isotherm__ The gradient $G'(C_{As}^{(0)})$ is unity and $C_{As}^{(0)}$ is equal to $Q_{As}^{(0)}$ in this case. Since $Q_A^{(0)}$ has been given by point L on the d-line, point R is given by

$$R(\xi_0, \eta_0) = R(Q_A^{(0)}, EL) \qquad (14)$$

as shown in **Fig. 4(c)**.

The term $C_D^{(0)}$ is a linear function of $C_A^{(0)}$ and thus ξ_0 and η_0 given by Eqs. (8) and (10) are also linear with $C_A^{(0)}$ and each other. Therefore, the locus of point R is a straight line $R_1 R_P$ which ties R_1 for the feed inlet ($C_A^{(0)} = 1$) and R_P for the product exit ($C_A^{(0)} = C_{A1}^{(0)}$), and point R for a particular value of $C_A^{(0)}$ is given by the interpolation of $R_1 R_P$ as shown in Fig. 4(c). An analytical solution for Eq. (T-13) also can be obtained in this case, as developed in the previous paper.[1]

4. Illustrative Solution

An example of a solution is given in this section to show the detailed procedure. Operating conditions to be considered are as follows: adsorption isotherm obeying the Freundlich type of $G(C) = C^{1/1.6}$; product gas concentration at the exit $C_{A1} = 0.01$; pressure ratio $\beta = 0.1$; ratio of superficial velocity $\gamma = 1.5$; ratio of number of mass transfer units $N_{AS}/N_{DS} = 1.5$ and surface diffusion control. It is necessary to determine 1) required number of transfer units N_{AS}, and 2) longitudinal distribution of the time-averaged concentration $\bar{C}_A$ and the concentration swing ΔC_A (or the swing parameter S_A). Eq. (T-1) for $C_A^{(0)}$ and Eq. (T-13) for S_A were solved numerically for the above condition in the previous paper[4] and the present result is compared with the former. Since the concentration of the exit product is as low as 0.01 of the feed concentration, the coordinates are enlarged by 10 times in a range of C or Q less than 0.1 as shown in **Figs. 5** and **6**.
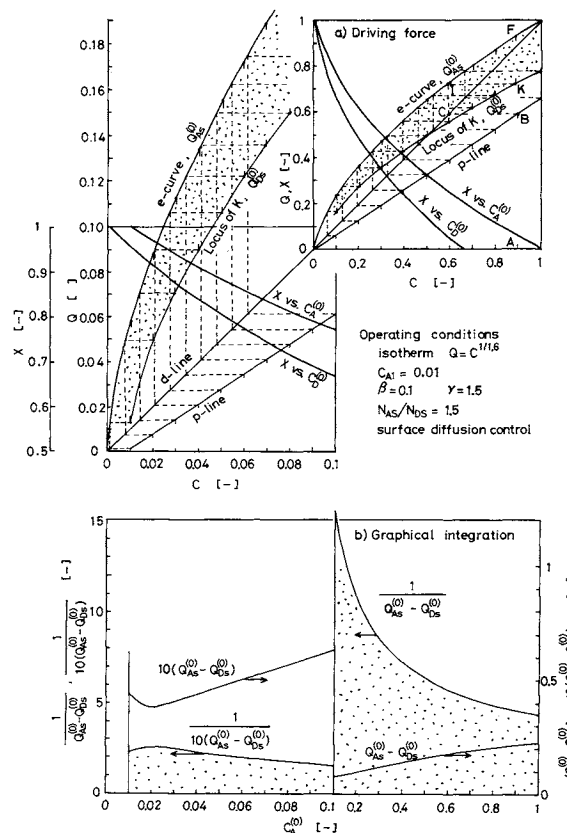

Fig. 5. Illustrative solution of longitudinal distribution of concentration $C_A^{(0)}$.

4.1 Time-averaged concentration $\bar{C}_A$

Time-averaged concentration $\bar{C}_A$ is equal to the zeroth-order term $C_A^{(0)}$ according to Eq. (2). The Freundlich isotherm of adsorption (e-curve) and a diagonal line (d-line) are drawn first as shown in Fig. 5(a). The operating line (p-line) in this case is

$$C_D^{(0)} = 0.001 + 0.667 \,(C_A^{(0)} - 0.01) \qquad (15)$$

according to Eq. (T-6). Point K, which indicates the value of $Q_{Ds}^{(0)}$ corresponding to $C_A^{(0)}$, is determined by drawing a polygon along $A \rightarrow B \rightarrow C \rightarrow I \rightarrow K$ as shown in Fig. 2(a). The curve of $Q_{Ds}^{(0)}$ given in Fig. 5(a) is a
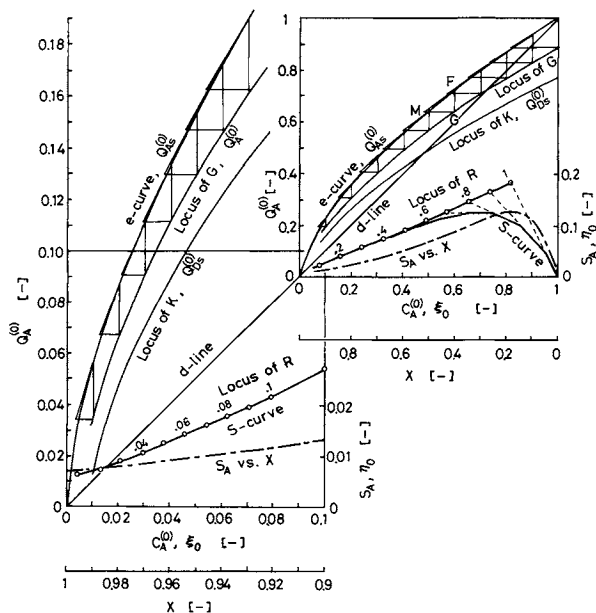
Fig. 6. Illustrative solution of longitudinal distribution of swing parameter S_A.

locus of points K determined thus for various values of $C_A^{(0)}$ with an increment of 0.1 ($C_A^{(0)}=0.1$–1.0) or 0.01 ($C_A^{(0)}=0.01$–0.1). The vertical distance of the dotted region FK corresponds to the effective driving force ($Q_{As}^{(0)}-Q_{Ds}^{(0)}$), of which the value and its reciprocal are replotted in Fig. 5(b). The total area of the dotted region of Fig. 5(b) gives the value of $N_{AS}/2$ according to Eq. (T-1). The value of N_{AS} thus determined is $N_{AS}=16.3$ and is very close to the value of $N_{AS}=16.7$ obtained by the numerical integration of Eq. (T-1) in the previous paper.[4] The distance from the feed inlet X for a given value of $C_A^{(0)}$ is obtained by the ratio of the partial area between $C_A^{(0)}$ and 1 to the total area of the dotted region of Fig. 5(b). The longitudinal distribution of $C_A^{(0)}$ obtained in this way is given in Fig. 5(a) and agrees well with the previous result.[4] Distribution of $C_D^{(0)}$ is obtained by converting $C_A^{(0)}$ by way of p-line.

4.2 Swing parameter S_A

Time variation of concentration C_A and the concentration swing ΔC_A is discussed in terms of the swing parameter S_A. The e-curve ($Q_A^{(0)}$) and the locus of K-points ($Q_{As}^{(0)}$) shown in Fig. 5(a) are reproduced in Fig. 6. Point G corresponding to $Q_A^{(0)}$ is the midpoint of the line segment FK since $N_{AS}=\gamma N_{DS}$ in this case. The locus of point G is drawn then. A right triangle $\triangle FMG$ of which the hypotenuse is the tangential line at point F is constructed for each value of $C_A^{(0)}$. The length of the base MG is used to take point R according to Eq. (12) or the procedure shown in Fig. 4(a). Points R corresponding to various values of $C_A^{(0)}$ and their locus are shown in Fig. 6, in which the scale of η_0 is doubled for clarity. The fractional number beside point R refers to the corresponding value of

$C_A^{(0)}$, and the tangential line of the curve $S(C_A^{(0)}, S_A)$ should pass through the corresponding point R.

The curve $S(C_A^{(0)}, S_A)$ is constructed as follows. Starting at $S_1(1, 0)$ we first draw a line segment in the direction to point R_1 until $C_A^{(0)}=0.95$ and then turn the line in the direction to $R_{0.9}$ until $C_A^{(0)}=0.85$. By repeating this procedure, the swing parameter S_A is given as a function of $C_A^{(0)}$ by a polygonal curve (S-curve) as shown in Fig. 6. In this example the S-curve gradually approaches the locus of R and they coincide if $C_A^{(0)}<0.4$. Since $C_A^{(0)}$ is a function of the position X as shown in Fig. 5, the S-curve can be converted to the longitudinal distribution of the swing parameter S_A. The result is shown by a chain line in Fig. 6 and is in good agreement with that of the previous paper[4] obtained by numerical analysis of Eq. (T-13).

Conclusion

According to a short cycle time approximation, the time-averaged concentration $C_A^{(0)}$ is represented by a quadrature and the swing parameter S_A is given by the solution of a linear ordinary differential equation with $C_A^{(0)}$ as an independent variable. These relationships were found, in the present paper, to be amenable to graphical solution by graphical integration and the isoclinal line method in the following procedure.

The operating line for a specified value of exit concentration $C_A^{(0)}$ is drawn first on a diagram of the adsorption isotherm to relate concentrations between adsorption and desorption steps, and then two tie lines are drawn to relate concentrations and adsorbed amounts between particle surface and bulk state. By this construction, the effective driving force can be read as the length of a line segment on the diagram as shown in Fig. 1. The number of mass transfer units N_{AS} required to attain the specified value of $C_{A1}^{(0)}$ and the longitudinal distribution of concentration $C_A^{(0)}$ can be obtained by integrating graphically the reciprocal of the effective driving force thus obtained.

The isoclinal line method is successfully used to get the swing parameter S_A. The current coordinates R can be determined from a segment of a tangential line of adsorption isotherm as shown in Fig. 3. The distribution of S_A is given by a polygonal graph of which the line segment is directed to the corresponding point R.

Special cases of surface diffusion control, gas film diffusion control and linear isotherm were discussed respectively as well as the general case. The usefulness of the present method was verified in an illustrative solution.

$C^{(0)}$	=	zeroth-order concentration in expansion	[—]
$\bar{C}$	=	time-averaged concentration	[—]
ΔC_A	=	concentration swing, defined by Eq. (3)	[—]
c_{A0}	=	feed gas concentration	[mol/m^3]
$G(C)$	=	dimensionless adsorption isotherm normalized by c_{A0} and q_0^*	[—]
$G'(C)$	=	gradient of $G(C)$, $=dG(C)/dC$	[—]
$H_A(C_A^{(0)}), h_A(C_A^{(0)})=$		functions of $C_A^{(0)}$, defined by Eqs. (T-15) and (T-16)	[—]
J	=	dimensionless mass transfer rate normalized by uc_{A0}/L	[—]
k_Fa, k_Sa	=	volumetric mass transfer coefficients in gas film and in solid phase, respectively [m/s], [kg/m$^2\cdot$s]	
L	=	column length	[m]
N_F, N_S	=	number of mass transfer units, $=k_FaL/u$ and $k_SLq_0^*/uc_{A0}$, respectively	[—]
N_{SF}	=	overall number of mass transfer units $=1/(1/N_F+1/N_S)$	[—]
n	=	capacity ratio $=\rho_sLq_0^*/uc_{A0}t_c$	[—]
O	=	order symbol	
p	=	pressure in column	[Pa]
Q	=	dimensionless amount adsorbed normalized by q_0^*	[—]
q_0^*	=	amount adsorbed in equilibrium with c_{A0}	[mol/kg]
S_A	=	swing parameter in adsorption step, defined by Eq. (1)	[—]
t_c	=	half cycle time	[s]
u	=	superficial gas velocity	[m/s]
X	=	dimensionless distance normalized by L	[—]

β	=	p_D/p_A	[—]
γ	=	$u_D/u_A=n_A/n_D$	[—]
η, ξ	=	current coordinates	[—]
ν_A	=	parameter to normalize S_A, $=N_{AS}$ (general case and surface diffusion control), N_{AF} (film diffusion control) or N_{ASF} (linear isotherm)	[—]
ρ_s	=	bulk density of particle bed	[kg/m^3]
τ	=	dimensionless time normalized by t_c	[—]

⟨Subscripts⟩

A, D	=	adsorption step and desorption step, respectively
F	=	gas film diffusion
S	=	surface diffusion
s	=	particle surface
0, 1	=	$X=0$ (feed inlet) and $X=1$ (product exit), respectively

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