

DONNAN DIALYSIS CONCENTRATION OF CUPRIC IONS

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Key Words: Membrane Separation, Ion Exchange, Donnan Dialysis, Mass Transfer, Metal Recovery, Cupric Ion, Osmotic Water

The concentration of cupric ion was studied with a flat-plate dialyzer having a single cation exchange membrane. The exchange equilibrium constants were expressed as a function of the ionic strength of the bulk solution. The flux of osmotic water was linearly proportional to the difference of ionic strength between the feed and the strip solutions. The maximum values of the enrichment ratio obtained experimentally became smaller than those calculated from the relationship of Donnan equilibrium. The experimental changes of enrichment ratio with time, however, agreed well with the values calculated by using the measured values of physical properties of the membrane, self-diffusion coefficients, distribution coefficients of ions, liquid film resistances and the flux of osmotic water.

Introduction

Donnan dialysis, one of the most useful membrane processes, is based on the Donnan equilibrium principle.⁴⁾ In spite of its simple operation, Donnan dialysis is a complex process accompanied by concentration polarization, the generation of diffusion potential and so on. Applications of this process such as separation and enrichment of valuable ions and removal of heavy metal ions from waste water have been studied by many researchers.^{2,3,5-7,9-12,14,15)} The transfer rate of ions across the membrane was often simplified and arranged by the first-order rate equation regarding the concentration of the concerned ion.^{12,15)} Mass transfer correlations in liquid films were presented for a plate dialyzer⁹⁾ and a shell-and-tube dialyzer with a tubular membrane.¹¹⁾ However, the effects of mass transfer rate in the liquid film, the flux of osmotic water and the distribution coefficient in the membrane on the overall transport rate of ions have not been clarified so far.

The objective of this paper is to determine the several factors mentioned above for concentrating the cupric ion with a flat-plate dialyzer having a cation exchange membrane and also to clarify theoretically the changes of enrichment ratio with time.

1. Theory

The following assumptions are made to introduce fundamental equations.

(1) The amounts of ions accumulated in the membrane are negligibly small compared with total amounts of ions in feed and strip solutions.

(2) The amount of co-ion included in the mem-

brane is negligibly small compared with the ion exchange capacity of the membrane.*

(3) The concentration changes of ions in a dialyzer are negligibly small compared with the concentrations of respective ions at the dialyzer inlet.**

Figure 1 shows a schematic diagram of Donnan dialysis operated in a recirculating batch mode. The concentration changes of ions in feed and strip solutions with time are given by the following equations (Appendix 1).

$$-d(V_A C_{1A})/dt = SJ_1 \quad (1)$$

$$dC_{2A}/dt = 2(-dC_{1A}/dt) \quad (2)$$

$$d(V_B C_{1B})/dt = SJ_1 \quad (3)$$

$$-d(V_B C_{2B})/dt = 2SJ_1 \quad (4)$$

where V indicates the total volume of the solution and S the dialysis area of the membrane.

The flux J_1 of species 1 through the membrane is expressed by the Nernst-Planck equation and an electroneutrality constraint (Appendix 2).

$$J_1 = \frac{\bar{D}_1}{\delta} \left\{ \frac{\bar{C}_{1A} - \bar{C}_{1B}}{2m-1} + \frac{Q(m-1)}{(2m-1)^2} \ln \frac{(2m-1)\bar{C}_{1A} + Q/2}{(2m-1)\bar{C}_{1B} + Q/2} \right\} \quad (5)$$

where symbols with the superscript bar refer to the membrane phase.

The cation exchange equilibrium constant K is defined

* The concentration of co-ion included in the membrane was respectively $0.080 \text{ kmol} \cdot \text{m}^{-3}$ and $0.175 \text{ kmol} \cdot \text{m}^{-3}$ when the HCl concentration was $1 \text{ kmol} \cdot \text{m}^{-3}$ and $2 \text{ kmol} \cdot \text{m}^{-3}$.

** For example, at $g_L = 2.5 \times 10^{-5} \text{ m}^3 \cdot \text{s}^{-1}$, the initial value of J_1 was obtained to be $1.8 \times 10^{-4} \text{ mol} \cdot \text{m}^{-2} \cdot \text{s}^{-1}$. The concentration change of the cupric ion in a dialyzer, calculated from Eq. (A-2), was $2.88 \times 10^{-4} \text{ kmol} \cdot \text{m}^{-3}$.

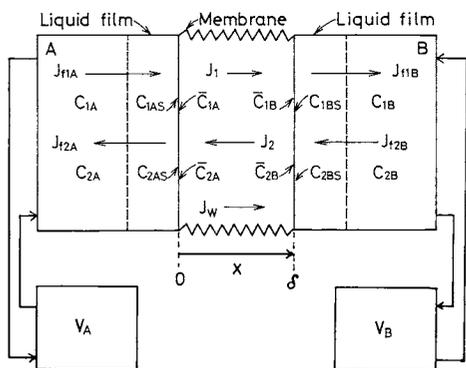


Fig. 1. Schematic diagram of Donnan dialysis operated in recirculating batch mode.

as follows.

$$K = C_1 \bar{C}_2^2 / (\bar{C}_1 C_2^2) \quad (6)$$

Since the co-ion is assumed to be excluded from the membrane, the concentrations of hydrogen ions at the interfaces of the membrane, \bar{C}_{2A} and \bar{C}_{2B} , are given by the following equations.

$$\bar{C}_{2A} = Q - 2\bar{C}_{1A} \quad (7)$$

$$\bar{C}_{2B} = Q - 2\bar{C}_{1B} \quad (8)$$

The concentrations of cupric ions at the interfaces of the membrane, \bar{C}_{1A} and \bar{C}_{1B} , are derived from Eq. (6) with Eqs. (7) and (8).

$$\bar{C}_{1A} = \frac{Q}{2} \left[1 + \frac{K_A C_{2AS}^2}{4C_{1AS}Q} - \left\{ \left(1 + \frac{K_A C_{2AS}^2}{4C_{1AS}Q} \right)^2 - 1 \right\}^{1/2} \right] \quad (9)$$

$$\bar{C}_{1B} = \frac{Q}{2} \left[1 + \frac{K_B C_{2BS}^2}{4C_{1BS}Q} - \left\{ \left(1 + \frac{K_B C_{2BS}^2}{4C_{1BS}Q} \right)^2 - 1 \right\}^{1/2} \right] \quad (10)$$

The fluxes of ions through the liquid films are as follows.

$$J_{f1A} = k_{f1A}(C_{1A} - C_{1AS}) \quad (11)$$

$$J_{f2A} = k_{f2A}(C_{2A} - C_{2AS}) \quad (12)$$

$$J_{f1B} = k_{f1B}(C_{1BS} - C_{1B}) \quad (13)$$

$$J_{f2B} = k_{f2B}(C_{2BS} - C_{2B}) \quad (14)$$

The following relations are obtained in the pseudo-steady state operation.

$$J_1 = J_{f1A} = -J_{f2A}/2 = J_{f1B} = -J_{f2B}/2 \quad (15)$$

The volume changes of each solution with time are related to the flux of osmotic water.

$$dV_A/dt = -J_w S / \rho_w \quad (16)$$

$$dV_B/dt = J_w S / \rho_w \quad (17)$$

The experimental relationships of K and J_w as functions of the ionic strength were used for the

calculation. With the assumed values of C_{1AS} , C_{2AS} , C_{1BS} and C_{2BS} , \bar{C}_{1A} and \bar{C}_{1B} were calculated from Eqs. (9) and (10), respectively. J_1 was calculated from Eq. (5) with \bar{C}_{1A} and \bar{C}_{1B} . J_{f1A} , J_{f2A} , J_{f1B} and J_{f2B} were respectively calculated from Eqs. (11)–(14). The calculation was repeated to good agreement of Eq. (15). With the determined value of J_1 , Eqs. (1)–(4), (16) and (17) were solved numerically by using the Runge-Kutta method and the changes of the concentrations of ions with time were obtained.

It was confirmed that the concentrations of ions at equilibrium calculated by the above procedure without regarding osmotic water agreed with those calculated from the relationship of Donnan equilibrium.

2. Experimental

2.1 Equilibrium experiment

The membrane used in all experiments was a strongly acidic cation exchange membrane Neosepta C66-5T (Tokuyama Soda Co.). Various physical properties of the membrane, water content, dimensional change due to swelling and the exchange capacity were determined by ordinary procedures.⁷⁾ Two membrane pieces (2 cm × 2 cm) were immersed in the solution ($1 \times 10^{-4} \text{ m}^{-3}$), whose ionic strength was changed by various concentrations of hydrochloric acid and cupric chloride. The solution temperature was kept at 298 K. The samples were allowed to equilibrate for at least 24 hours. The sample pieces were then separated from the solution and washed in deionized water, and the surfaces of the pieces were wiped with a filter paper. The equilibrating membrane was thoroughly soaked with $1 \text{ kmol} \cdot \text{m}^{-3}$ HCl solution ($5 \times 10^{-6} \text{ m}^3$) to strip cupric ions. After three hours, the membrane was soaked with new solution. This procedure was repeated three more times. The total amount of cupric ions in the equilibrating membrane was calculated as the sum of cupric ions in the strip solutions.

The concentrations of cupric and hydrogen ions were determined by a chelatometric titration with EDTA and by a titration of standard solution of NaOH, respectively.

2.2 Determination of self-diffusion coefficients in membrane

The dialyzer and procedure described in 2.5 were also used here. The total volumes of feed and strip solutions were each 0.001 m^3 . For determining \bar{D}_1 , the feed solution was $1 \text{ kmol} \cdot \text{m}^{-3}$ HCl solution containing $1.57 \times 10^{-3} \text{ kmol} \cdot \text{m}^{-3}$ cupric ion and the strip solution was $1 \text{ kmol} \cdot \text{m}^{-3}$ HCl solution, while for \bar{D}_2 the feed solution was $0.1 \text{ kmol} \cdot \text{m}^{-3}$ CuCl_2 solution containing $10^{-3} \text{ kmol} \cdot \text{m}^{-3}$ HCl and the strip solution was $0.1 \text{ kmol} \cdot \text{m}^{-3}$ CuCl_2 solution.

2.3 Measurement of mass transfer coefficient in liquid film

The mass transfer coefficient in the liquid film was determined by the limiting current method.^{8,13)} In place of an end plate and a membrane, details of which are described in Fig. 3, two copper plates were installed in the dialyzer. The electrolyte contained 100 ppm Cu^{2+} and $0.5 \text{ kmol} \cdot \text{m}^{-3} \text{ H}_2\text{SO}_4$ flowed into the channel where a constant voltage was applied by a potentiostat.

2.4 Flux of osmotic water

Figure 2 shows a schematic diagram of the apparatus for measuring the flux of osmotic water. The dialysis area of the membrane was $2.5 \times 10^{-3} \text{ m}^2$. At the dialyzer outlet, capillary tubes of 1 mm, 2 mm or 3 mm i.d. were installed on the same horizontal level. After the tubes and the channel were filled with the feed and strip solutions containing HCl solutions, the flow was stopped and the cocks of tube outlets were opened to air. The solution in the capillary tube moved from the solution of low ionic strength to that of high ionic strength. The flux of osmotic water was calculated by using the rate of the solution moving in the capillary tube.

2.5 Enrichment experiment

Figure 3 shows the detail of a Donnan dialyzer. The dialyzer consisted of two end plates, a single membrane of 0.04 m^2 ($20 \text{ cm} \times 20 \text{ cm}$) dialysis area and spacers. For a limited case to enhance the mass transfer rate in the liquid film, polypropylene nets were inserted in the spacers. The net (z-31), made of strings 1.8 mm wide and 2.0 mm thick, had hexagonal openings at 9-mm intervals and a maximum thickness of 3 mm, which was the same as the thickness of the spacer.

Figure 4 shows a diagram of whole experimental apparatus. The feed and strip solutions flowed countercurrently in the dialyzer at the same rates and recycled to respective tanks. Total volumes of the feed solution were 0.005, 0.01, 0.02 or 0.04 m^3 . Total volume of the strip solution was 0.001 m^3 . The feed and strip solutions used were $1.57 \times 10^{-2} \text{ kmol} \cdot \text{m}^{-3} \text{ CuCl}_2$ (1000 ppm Cu^{2+}) solution and HCl solution of $0.08\text{--}1.85 \text{ kmol} \cdot \text{m}^{-3}$, respectively. The liquid flow rate was $2.5 \times 10^{-5} \text{ m}^3 \cdot \text{s}^{-1}$ except for experiments to clarify the effects of liquid flow rate on enrichment rates.

3. Results and Discussion

3.1 Equilibrium constants

The physical properties of the membrane C66-5T are listed in Table 1.

Figure 5 shows the relation between the distribution coefficient λ_1 and the concentration C_1 of the cupric ion in the bulk solution at equilibrium. λ_1 decreased with increasing HCl concentration and C_1 . When C_1

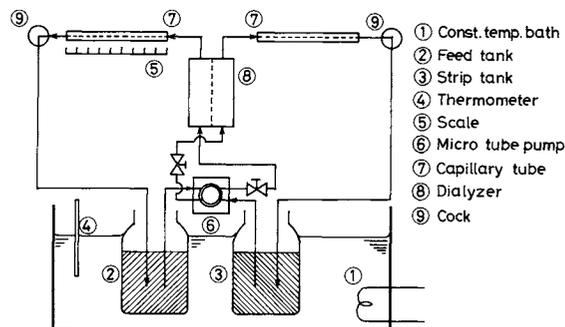


Fig. 2. Schematic diagram of apparatus for measuring flux of osmotic water.

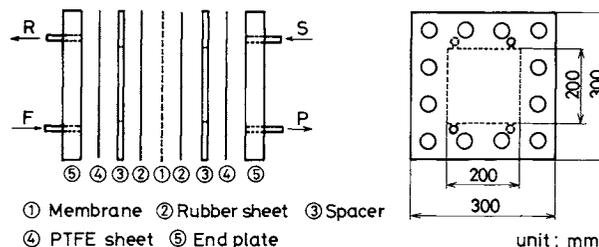


Fig. 3. Detail of Donnan dialyzer.

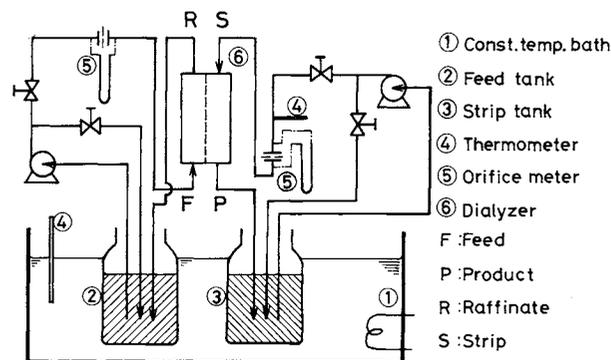


Fig. 4. Schematic diagram of whole experimental apparatus.

Table 1. Physical properties of Neosepta C66-5T

Thickness (H-type)	0.143 mm
(Cu-type)	0.137 mm
Ion exchange capacity	2.39 mol · kg ⁻¹ -dry Mem. 1.94 kmol · m ⁻³ -wet Mem.
Fixed ion concentration	6.32 mol · kg ⁻¹ · H ₂ O
Water content	0.378 kg · H ₂ O · kg ⁻¹ -dry Mem.
Density (H-type)	1130 kg · m ⁻³
(Cu-type)	1110 kg · m ⁻³

was relatively low, \bar{C}_2 approached Q and λ_1 indicated the constant value of λ_{01} which was given by the following equation.

$$\lambda_{01} = Q^2 / (KC_2^2) \quad (18)$$

The broken lines in Fig. 5 were calculated from Eq. (18) with the values of K , which were arranged as a function of the ionic strength of the bulk solution

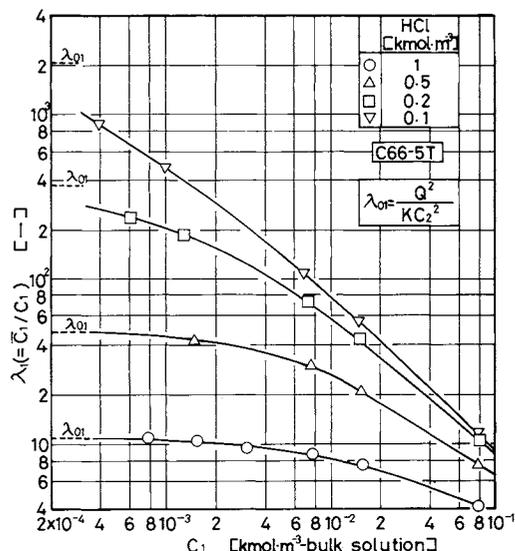


Fig. 5. Distribution coefficient of cupric ion.

shown in Fig. 6. The following empirical equation was obtained for the solid line in Fig. 6.

$$-\log K = 4.72I^{1/2}10^{-1.08I^{1/2}} \quad (19)$$

where the unit of I was $\text{kmol} \cdot \text{m}^{-3}$.

The relation between K and I was similar to the result for the membrane CMV by Kojima *et al.*⁷⁾

3.2 Estimation of mass transfer coefficients in liquid films

The mass transfer coefficient k_f in the liquid film was calculated from the following equation.¹³⁾

$$k_f = i_L / (2FC) \quad (20)$$

where i_L was the limiting current density, F the Faraday's constant and C the cupric ion concentration. The values obtained were arranged by the j_d -factor.⁸⁾

$$j_d = Sh / (Re \cdot Sc^{1/3}) \quad (21)$$

A diffusivity of cupric ion of $6.29 \times 10^{-10} \text{m}^2 \cdot \text{s}^{-1}$,¹⁾ a viscosity of $9.74 \times 10^{-4} \text{kg} \cdot \text{m} \cdot \text{s}^{-1}$ and a density of $1.034 \times 10^3 \text{kg} \cdot \text{m}^{-3}$ were used for this calculation. Figure 7 shows the correlation between j_d -factor and Re number.

$$j_d = 0.132Re^{-0.240} \quad \text{for no net} \quad (22)$$

$$j_d = 0.664Re^{-0.408} \quad \text{for net z-31} \quad (23)$$

The value of j_d for inserting nets was higher than that for no nets since nets acted as a turbulence promoter and enhanced mass transfer in the liquid film.

The values of k_{f1} and k_{f2} at a certain flow rate were calculated from Eq. (21) with physical properties of the solution and ions and with the values of j_d obtained by Eqs. (22) or (23). The diffusivities for infinite dilution were adopted as the diffusivities of

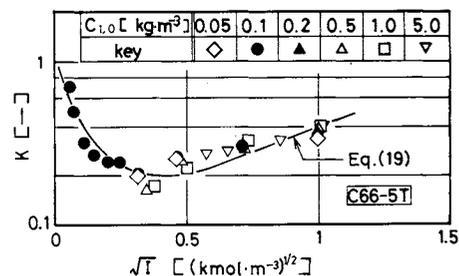


Fig. 6. Relation between cation exchange equilibrium constant and root of ionic strength of bulk solution.

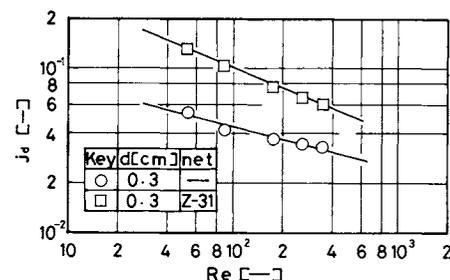


Fig. 7. Correlation between j_d -factor and Re number.

ions in the solution, $D_1 = 7.46 \times 10^{-10} \text{m}^2 \cdot \text{s}^{-1}$ and $D_2 = 9.31 \times 10^{-9} \text{m}^2 \cdot \text{s}^{-1}$.

3.3 Self-diffusion coefficients in membrane

Under the conditions described in 2.2, the transport of respective ion through the membrane was simplified to the diffusion of the single ion. For further simplification, the distribution coefficient must be constant in the concentration range of the present work. The experimental conditions of 2.2 were selected to adopt the above criteria. Then λ_{01} indicated 10.5 and λ_{02} indicated 2.22. The self-diffusion coefficient of species i was calculated from the following equations with the apparent value \bar{D}_i (Appendix 3).

$$\bar{D}_i = -\frac{V_A \delta}{2\lambda_{0i} St} \ln \frac{2C_{iA} - C_{i0A}}{C_{i0A}} \quad (24)$$

$$\bar{D}_i = \frac{\delta}{\lambda_{0i}} \left(\frac{\delta}{\bar{D}_i \lambda_{0i}} - \frac{2}{k_{fi}} \right)^{-1} \quad (25)$$

Figure 8 shows the results of the arrangement of Eq. (24). The values of \bar{D}_1 and \bar{D}_2 were obtained as $2.07 \times 10^{-11} \text{m}^2 \cdot \text{s}^{-1}$ and $1.86 \times 10^{-10} \text{m}^2 \cdot \text{s}^{-1}$, respectively.

3.4 Osmotic water

Figure 9 shows the relation between the flux of osmotic water, J_w , and the difference of ionic strength ($I_B - I_A$). Since the flow rate and physical conditions of each compartment were equal in this work, there was no hydrostatic pressure between the two compartments. The water transport might be caused by the difference of osmotic pressure between the two compartments. The value of ($I_B - I_A$) was used as the value related to the difference of osmotic pressure. J_w

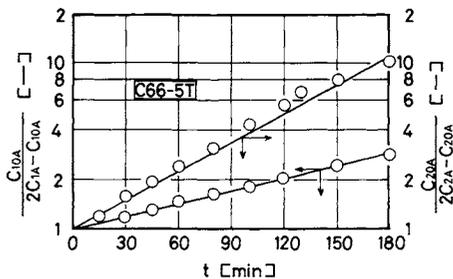


Fig. 8. Determination of self-diffusion coefficient in membrane.

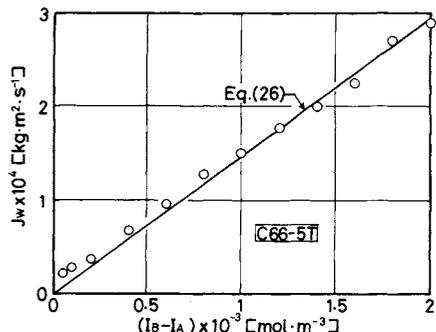


Fig. 9. Relation between flux of osmotic water and difference of ionic strength between feed and strip solutions.

was proportional to the value of $(I_B - I_A)$.

$$J_w = 1.46 \times 10^{-7} (I_B - I_A) \quad (26)$$

3.5 Dimensionless time-dependent change of enrichment ratio

Figure 10 shows the effect of liquid flow rate on the enrichment rate. The solid line indicates the values calculated with the conditions that intra-membrane transport was the rate-determining step and that no osmotic water existed. The observed enrichment ratio approached the solid line with increasing liquid flow rate and insertion of nets, and agreed well with the calculated ratio by use of liquid film resistance and the flux of osmotic water, described by various lines other than the solid line.

Figures 11–13 show comparisons between the observed enrichment rates and the calculated ones. The value of α_0 is the ratio of the equivalent of the hydrogen ion in the strip solution to that of the cupric ion in the feed solution.¹⁴⁾

$$\alpha_0 = V_{B0} C_{20B} / (2V_{A0} C_{10A}) \quad (27)$$

From the charge balance, the condition of $\alpha_0 \geq 1$ is required to exchange cupric ion with hydrogen ion. The enrichment rate increased with increment of n and α_0 . The solid lines were calculated with the same conditions as those in Fig. 10. The broken and one-dotted lines are respectively calculated values without and with regard to osmotic water. The observed values in Figs. 11–13 agreed well with the calculated ones indicated by one-dotted lines.

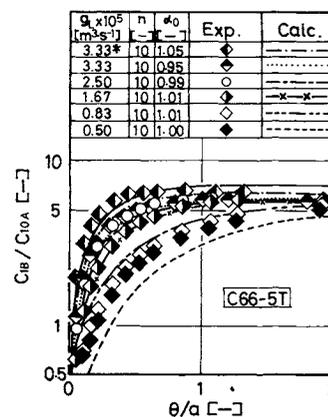


Fig. 10. Effect of liquid flow rate on enrichment rate. For the case marked with a star, nets are inserted in channel.

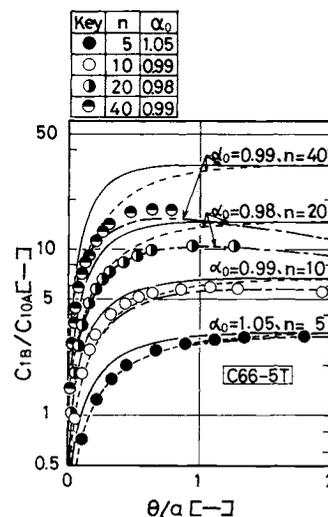


Fig. 11. Comparison between observed enrichment ratio and calculated one for α_0 of about 0.5. Solid lines are calculated values when intra-membrane flux of ions is rate-determining and without regarding osmotic water. Broken and one-dotted lines are calculated values without and with regard to osmotic water, respectively.

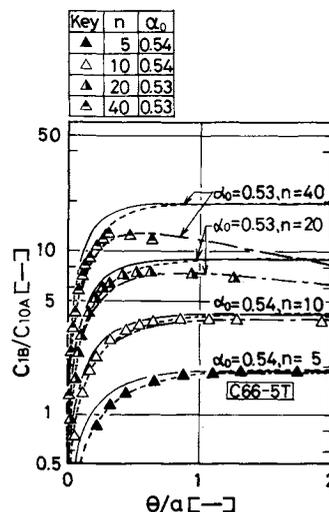


Fig. 12. Comparison between observed enrichment ratio and calculated one for α_0 of about 1.0. Lines are calculated with the same conditions as those in Fig. 11.

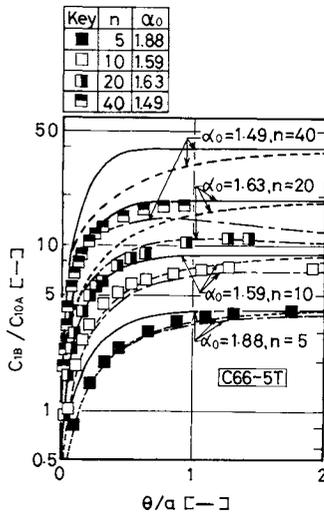


Fig. 13. Comparison between observed enrichment ratio and calculated one for α_0 of about 1.5. Lines are calculated with the same conditions as those in Fig. 11.

The relationship of Donnan equilibrium was expressed as follows.

$$K_A(C_{2A}^2/C_{1A}) = K_B(C_{2B}^2/C_{1B}) \quad (28)$$

In simple calculation, the equilibrating concentrations were obtained by the charge balance of ions and Eq. (28) with $K_A = K_B$, $C_{10B}/C_{10A} = 0$ and $C_{20A}/C_{20B} = 0$. The enrichment ratio obtained by the above calculation was almost the same as the maximum value of solid lines in Figs. 10–13. This might be caused by the small difference between K_A and K_B in this work. For the experiment of $n=40$ and $\alpha_0=1$, the results of $K_A=0.253$ and $K_B=0.472$ were obtained with $I_A=0.0471 \text{ kmol} \cdot \text{m}^{-3}$ and $I_B=1.256 \text{ kmol} \cdot \text{m}^{-3}$, respectively. The experimental values of maximum enrichment ratio were smaller than the value calculated from the relationship of Donnan equilibrium. The difference between these values increased with increasing n and α_0 . This was due to the fact that the HCl concentration in the strip solution increased with increasing n and α_0 and the flux of osmotic water increased.

Conclusion

The cation exchange equilibrium constants of cupric and hydrogen ions were obtained as a function of the ionic strength of the bulk solution. The maximum values of the enrichment ratio obtained experimentally became smaller than the values calculated from the relationship of Donnan equilibrium of Eq. (28). The enrichment rate was found to be affected by liquid-film resistance, the flux of ions through the membrane and the flux of osmotic water. The flux of osmotic water was found to be proportional to the difference of ionic strength between the two solutions. The experimental changes of enrichment ratio with

time agreed well with the values calculated by using the measured values of physical properties of the membrane, self-diffusion coefficients, distribution coefficients of ions, liquid film resistances and the flux of osmotic water.

Appendix 1. Fundamental equation

The solutions were recycled from storage tanks to a dialyzer. The mass balance of species 1 which moves through the membrane and decreases in concentration is given as follows.

$$d(V_d C_{1d})/dt + d(V_t C_{1t})/dt = -J_1 S \quad (A-1)$$

where the subscripts d and t indicate a dialyzer and a tank, respectively. The concentration change in a dialyzer is obtained by mass balance in steady-state operation.

$$C_{1t} - C_{1d} = S J_1 / g_L \quad (A-2)$$

Under the conditions of the present experiments, the relation of $C_{1t} = C_{1d}$ is obtained. By using the relation of $C_{1t} = C_{1d}$ and $V = V_t + V_d$, Eq. (A-1) is rearranged to the following equation.

$$d(V C_{1t})/dt = -J_1 S \quad (A-3)$$

Appendix 2. Ionic transport flux through membrane

When we assume that the diffusion flux of co-ion and the osmotic water transport based on the difference of hydrostatic pressure are both negligible, the transport flux of the ions i , J_i , can be derived from the following Nernst-Planck equation.

$$J_i = -\bar{D}_i d\bar{C}_i/dx - z_i \bar{C}_i \bar{D}_i F / (RT) d\phi/dx \quad (A-4)$$

By also assuming no net electric current and the electroneutrality constraint in the membrane, the following equation is obtained for species 1.

$$J_1 = -\bar{D}'_{12} d\bar{C}_1/dx \quad (A-5)$$

where the differential interdiffusion coefficient \bar{D}'_{12} is

$$\bar{D}'_{12} = \frac{\bar{D}_1 \bar{D}_2 (4\bar{C}_1 + \bar{C}_2)}{4\bar{C}_1 \bar{D}_1 + \bar{C}_2 \bar{D}_2} \quad (A-6)$$

By eliminating \bar{C}_2 with the relation of $\bar{C}_2 = Q - 2\bar{C}_1$ and by integrating Eq. (A-5) with respect to x from 0 to δ and \bar{C}_1 from \bar{C}_{1A} to \bar{C}_{1B} , Eq. (5) is finally obtained.

Appendix 3. Calculation of self-diffusion coefficient

The flux J_1 of Eq. (5) was simplified under the conditions of \bar{C}_{1A} , $\bar{C}_{1B} \ll Q$.

$$J_1 = (\bar{D}_1/\delta)(\bar{C}_{1A} - \bar{C}_{1B}) \quad (A-7)$$

For a constant value of distribution ratio,

$$\lambda_{01} = \bar{C}_{1A}/C_{1AS} = \bar{C}_{1B}/C_{1BS} \quad (A-8)$$

Since no osmotic water is transported, the flux J_1 can be derived by combining Eqs. (A-7), (A-8), (11), (13) and (15) with the relation $k_{f1} = k_{f1A} = k_{f1B}$.

$$J_1 = \frac{C_{1A} - C_{1B}}{\frac{\delta}{\bar{D}_1 \lambda_{01}} + \frac{2}{k_{f1}}} \quad (A-9)$$

By substituting Eq. (A-9) into Eq. (1) with the relation of $C_{1A} + C_{1B} = C_{10A}$ for $V_A = V_B$, Eqs. (24) and (25) are finally derived. A similar procedure can be applied to species 2.

Nomenclature

a	$= C_{10A}/(Q/2)$	[—]
C	$=$ concentration	[mol · m ⁻³]

D	= diffusion coefficient	$[\text{m}^2 \cdot \text{s}^{-1}]$
\bar{D}'	= differential interdiffusion coefficient	$[\text{m}^2 \cdot \text{s}^{-1}]$
d	= thickness of channel	$[\text{m}]$
F	= Faraday's constant	$[\text{C} \cdot \text{mol}^{-1}]$
g_L	= liquid flow rate	$[\text{m}^3 \cdot \text{s}^{-1}]$
I	= ionic strength	$[\text{mol} \cdot \text{m}^{-3}]$
i_L	= limiting current density	$[\text{A} \cdot \text{m}^{-2}]$
J	= ionic transport flux	$[\text{mol} \cdot \text{m}^{-2} \cdot \text{s}^{-1}]$
J_f	= mass transfer flux in liquid film	$[\text{mol} \cdot \text{m}^{-2} \cdot \text{s}^{-1}]$
J_w	= flux of osmotic water	$[\text{kg} \cdot \text{m}^{-2} \cdot \text{s}^{-1}]$
j_d	= j -factor	$[-]$
K	= cation exchange equilibrium constant	$[-]$
k_f	= mass transfer coefficient in liquid film	$[\text{m} \cdot \text{s}^{-1}]$
m	= \bar{D}_1/\bar{D}_2 , ratio of self-diffusion coefficient	$[-]$
n	= V_{A0}/V_{B0} , volumetric ratio	$[-]$
Q	= ion exchange capacity	$[\text{mol} \cdot \text{m}^{-3} \cdot \text{wet Mem.}]$
R	= gas constant	$[\text{J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}]$
Re	= $2dU_L\rho_L/\mu_L$, Reynolds number	$[-]$
S	= dialysis area of membrane	$[\text{m}^2]$
Sc	= $\mu_L/(\rho_L D)$, Schmidt number	$[-]$
Sh	= $2dk_f/D$, Sherwood number	$[-]$
T	= temperature	$[\text{K}]$
t	= time	$[\text{s}]$
U_L	= superficial liquid velocity in dialyzer	$[\text{m} \cdot \text{s}^{-1}]$
V	= total volume of solution	$[\text{m}^3]$
x	= coordinate of membrane thickness	$[\text{m}]$
z	= charge number	$[-]$
α_0	= $V_{B0}C_{20B}/(2V_{A0}C_{10A})$, exchange equivalent ratio	$[-]$
δ	= membrane thickness	$[\text{m}]$
ϕ	= potential	$[\text{V}]$
θ	= $S\bar{D}_1 t/(\delta V_{A0})$, dimensionless time	$[-]$
λ	= \bar{C}/C , distribution coefficient	$[-]$
λ_0	= constant value of distribution coefficient	$[-]$
μ_L	= viscosity of solution	$[\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}]$
ρ_L	= density of solution	$[\text{kg} \cdot \text{m}^{-3}]$
ρ_w	= density of water	$[\text{kg} \cdot \text{m}^{-3}]$

<Subscripts>

0	= initial value
1	= cupric ion

2	= hydrogen ion
A	= feed
B	= strip
d	= dialyzer
i	= species i
S	= interface of membrane
t	= tank
w	= osmotic water

<Superscripts>

-	= value in membrane
~	= apparent value

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(Presented in part at 19th Autumn Meeting of The Society of Chemical Engineers, Japan at Nagoya, Oct. 17, 1985.)