NATURAL FREQUENCY OF CHAIN BUBBLES FROM SINGLE NOZZLES

KATSUMI NAKAGAWA
Department of Chemical Engineering, Kogakuin University, Tokyo 160

Key Words: Bubble, Chain Bubble, Bubble Diameter Measurement, Bubble Natural Frequency, Fast Fourier Transform, Velocity Potential, Pulse Response Method, Floating Bubble Method

A theoretical equation for oscillation of chain bubbles was obtained by use of velocity potential. To examine theoretical considerations, experiments applying the pulse response method of sound and the floating bubble method were carried out. The numerical solutions were well in accord with the experimental results, and the validity of the theoretical considerations was demonstrated. The frequency of chain bubbles decreased with decreasing interbubble distance and with increasing number of bubbles in water.

Introduction

Many studies of the natural frequency of a single bubble have been made since the report by Minnaert.[3] But if there are more than one bubble in a liquid, the frequencies of these bubbles differ from that of a single bubble as the adjacent bubbles affect each other. Theoretical studies of the natural frequency of more than one spherical bubble in liquid have been made by Shima,[8,9] using a velocity potential around each of the bubbles (two and three bubbles), later by Morioka[4] (two bubbles), and by Foody *et al.*[1] (two to four bubbles), while the author[6] experimentally measured the natural frequencies of two bubbles, changing an interbubble distance in water. However, no paper has made reference to the natural frequency of chain bubbles continuously formed through a single nozzle in liquid.

The purpose of this paper is to investigate the effects of interbubble distances and the number of bubbles on the natural frequency of chain bubbles, and to discuss a method of measuring the volume mean diameter of chain bubbles.

1. Theory

Figure 1 shows a physical picture of the present model. There are i spherical bubbles of the same size in a vertical line in the liquid, and the interbubble distances are all the same. In addition, the following assumptions are made.

1) The static radius of the bubbles is small compared with the interbubble distance.

2) The effects of viscosity, compressibility and gravity are neglected.

The velocity potentials around more than one oscillating sphere in an inviscid and incompressible liquid were obtained by Lorentz.[2]

In this paper, the velocity potential Φ_m for the radial motion around the m-th bubble is approximately given by Eq. (1), based on Lorentz's method.[2]

$$\Phi_m = \Phi_{ma} + \Phi_{mb} + \Phi_{mc} \tag{1}$$

where

$$\Phi_{ma} = \sum_{n=1}^{m-1} -R_0^2 \dot{R}_n \left\{ \frac{1}{l(m-n)} - \frac{r_m \cos\alpha_m}{l^2(m-n)^2} \right\} + \sum_{n=m+1}^{i} -R_0^2 \dot{R}_n \left\{ \frac{1}{l(n-m)} + \frac{r_m \cos\alpha_m}{l^2(n-m)^2} \right\} \tag{1a}$$

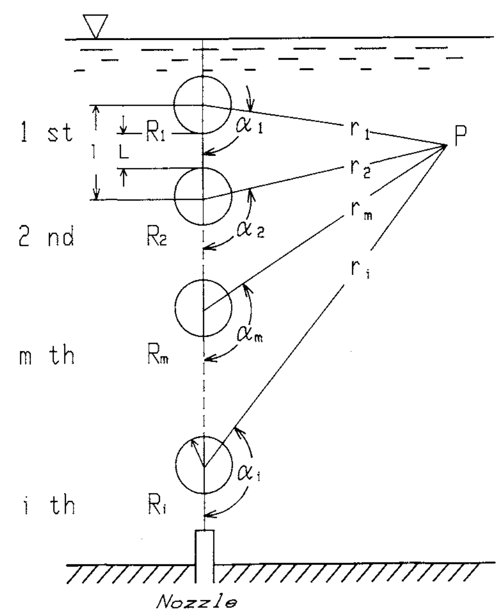

Fig. 1. Chain bubbles configuration.

Received December 5, 1984. Correspondence concerning this article should be addressed to K. Nakagawa.

$$\Phi_{mb}=\sum_{n=1}^{m-1}\frac{1}{2}R_0{}^5\dot{R}_n\frac{\cos\alpha_m}{r_m{}^2l^2(m-n)^2}+\sum_{n=m+1}^{i}-\frac{1}{2}R_0{}^5\dot{R}_n\frac{\cos\alpha_m}{r_m{}^2l^2(n-m)^2} \tag{1b}$$

$$\Phi_{mc}=-\frac{R_0{}^2\dot{R}_m}{r_m} \tag{1c}$$

The velocity potential Φ_m given by Eq. (1) should be satisfied by the following boundary condition at the respective bubble surface, i.e., at $r_m=R_m$.

$$\left|\frac{\partial\Phi_m}{\partial t}+\frac{1}{2}\left(\frac{\partial\Phi_m}{\partial r_m}\right)^2+\frac{1}{2r_m{}^2}\left(\frac{\partial\Phi_m}{\partial\alpha_m}\right)^2\right|_{r_m=R_m}=\frac{p_\infty-p_{w,m}}{\rho} \tag{2}$$

where t is time, p_∞ is pressure in liquid far from bubbles, $p_{w,m}$ is pressure at the surface of the m-th bubble, and ρ is density of the liquid. Considering surface tension, and assuming an adiabatic change of the gas inside a bubble, $p_{w,m}$ is given by the following equation.

$$p_{w,m}=p_m\left(\frac{R_0}{R_m}\right)^{3\gamma}-\frac{2\sigma}{R_m} \tag{3}$$

The pressure p_m inside the bubble can be related to the static radius R_0 by Plesset,[7] as follows.

$$p_m=p_\infty+\frac{2\sigma}{R_0} \tag{4}$$

By substituting Eqs. (3) and (4) into Eq. (2), and neglecting the terms that include angle α_m and cross terms in the obtained equation, an equation for the zero-mode oscillation of the m-th bubble is given as follows.

$$-\ddot{R}_mR_m-\frac{3}{2}(\dot{R}_m)^2-\sum_{\substack{n=1\\n\neq m}}^{i}\frac{2R_n\dot{R}_n{}^2+R_n{}^2\ddot{R}_n}{|l(m-n)|}=\frac{p_\infty}{\rho}\left\{1-\left(\frac{R_0}{R_m}\right)^{3\gamma}\right\}+\frac{2\sigma}{\rho R_0}\left\{\frac{R_0}{R_m}-\left(\frac{R_0}{R_m}\right)^{3\gamma}\right\} \tag{5}$$

For the purpose of dimensionless expression of Eq. (5), the following quantities are introduced:

$$\beta_m=\frac{R_m}{R_0} \tag{6}$$

$$\tau=\frac{t}{\sqrt{\dfrac{\rho}{p_\infty}}R_0} \tag{7}$$

Then Eq. (5) can be reduced to

$$-\beta_m\left(\frac{d^2\beta_m}{d\tau^2}\right)-\frac{3}{2}\left(\frac{d\beta_m}{d\tau}\right)^2-\sum_{\substack{n=1\\n\neq m}}^{i}\frac{W}{|m-n|}\left\{2\beta_n\left(\frac{d\beta_n}{d\tau}\right)^2+\beta_n{}^2\left(\frac{d^2\beta_n}{d\tau^2}\right)\right\}$$

$$=\left(1-\frac{1}{\beta_m{}^{3\gamma}}\right)+S\left(\frac{1}{\beta_m}-\frac{1}{\beta_m{}^{3\gamma}}\right) \tag{8}$$

where

$$W=\frac{R_0}{l} \tag{9}$$

$$S=\frac{2\sigma}{p_\infty R_0} \tag{10}$$

Considering that the amplitude of oscillation is very small, ε_m is defined as follows.

$$\varepsilon_m=\beta_m-1 \tag{11}$$

Neglecting the cross terms in Eq. (8) and supposing that ε_m is very small, the next equation is obtained.

$$\frac{d^2\varepsilon_m}{d\tau^2}+\sum_{\substack{n=1\\n\neq m}}^{i}\frac{W}{|m-n|}\cdot\frac{d^2\varepsilon_n}{d\tau^2}+\{3\gamma+S(3\gamma-1)\}\varepsilon_m=0 \tag{12}$$

Equation (12) is for the oscillation of the m-th bubble among i bubbles, and includes Shima's solutions[8,9] for two and three bubbles of equal radius as a particular solution.

2. Numerical Calculation

To solve the linear ordinary differential equation with constant coefficients of Eq. (12), we put

$$\varepsilon_m=a_m e^{j\sqrt{\mu}\tau} \tag{13}$$

The following equation for μ was obtained.

$$\begin{vmatrix} \mu+K & W\mu & \frac{W}{2}\mu & \cdots & \frac{W}{i-1}\mu \\ W\mu & \mu+K & W\mu & \cdots & \cdot \\ \frac{W}{2}\mu & W\mu & \mu+K & \cdots & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & W\mu \\ \frac{W}{i-1}\mu & \frac{W}{i-2}\mu & \cdot & \cdots & \mu+K \end{vmatrix}=0 \tag{14}$$

When μ_j is defined as the j-th root of characteristic equation (14), the j-th frequency of each bubble among the chain bubbles is shown by the following equation.

$$F_{j,i}=\frac{1}{2\pi R_0}\sqrt{\frac{p_\infty}{\rho}\mu_j}\,,\qquad 1\leq j\leq i \tag{15}$$

When the interbubble distance of the chain bubbles becomes infinite, the natural frequency of a bubble F_0 is given by the following equation.[10]

$$F_0=\frac{555}{2R_0\sqrt{\rho/\gamma}} \tag{16}$$

Table 1. Calculated values of $F_{j,i}/F_0$ and L/D_0
$D_0=0.46\,\text{cm}$, $S=2.86\times10^{-6}$, $L=(40-i\cdot D_0)/i$

		i	2	3	4	5	6	7	8
$F_{j,i}/F_0$	j	1	0.988	0.971	0.951	0.931	0.911	0.892	0.871
		2	1.012	1.009	1.002	0.991	0.979	0.966	0.952
		3		1.022	1.021	1.017	1.010	1.001	0.991
		4			1.032	1.033	1.030	1.025	1.019
		5				1.048	1.043	1.041	1.037
		6					1.050	1.052	1.054
		7						1.058	1.061
		8							1.068
L/D_0			19.5	12.9	9.5	7.5	6.2	5.3	4.5

Hence, taking $F_{j,i}/F_0$ as dimensionless, the next equation is obtained.

$$Z_{j,i}=\frac{F_{j,i}}{F_0} \tag{17}$$

For example, for i bubbles of $D_0=0.46\,\text{cm}$ arranged in a chain at a depth of 40 cm, the values $Z_{j,i}$ at $m=1$ calculated from Eq. (17) are summarized in **Table 1**. It is shown in this Table that each of these bubbles has theoretically i frequencies, the same as the number of bubbles which compose a chain.

3. Experimental Apparatus and Procedure

A schematic diagram of the experimental apparatus is shown in **Fig. 2**. It had the following components.

1) Water tank, $70\times70\times110$ (height) cm^3, made of glass.

2) Component which generates a sinusoidal pulse of sound.

3) Digital signal analyzer which transforms a response wave received by a hydrophone into a frequency.

4) Component which measures interbubble distance by photography, using a strobe synchronized with the pulse.

5) Component which measures volume mean diameter of bubbles by photography, using the floating bubble method.[5]

6) Bubble formation component which disperses air from a compressor into water through a single nozzle.

The experimental procedure was the same as described in the previous paper[6] except for the method of bubble formation. The frequencies of the chain bubbles were measured over a range of 0.44 to 0.81 cm D_0 and of 1.7 to 18.9 L/D_0.

4. Results and Discussion

4.1 Power spectra of chain bubbles

The power spectrum of chain bubbles was obtained from a response wave by using fast Fourier transform (FFT), and an example is shown in **Fig. 3**. As the

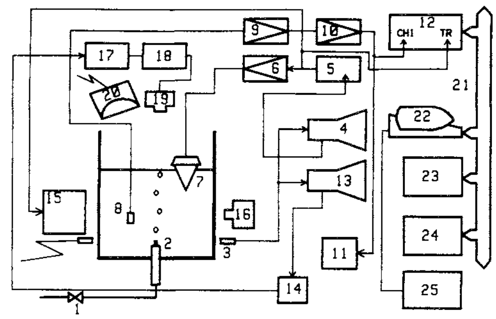

Fig. 2. Block diagram of experimental apparatus: 1, needle valve; 2, nozzle; 3, phototransistor; 4, synchroscope; 5, pulse generator; 6, power amplifier; 7, speaker; 8, hydrophone; 9, pre-amplifier; 10, DC amplifier; 11, digital counter; 12, D.S. analyzer; 13, synchroscope; 14, relay; 15, strobe; 16, 35 mm camera; 17, timer; 18, camera control unit; 19, film running camera; 20, stroboscope; 21, HP-IB; 22, desktop computer; 23, printer; 24, graphic plotter; 25, digitizer.

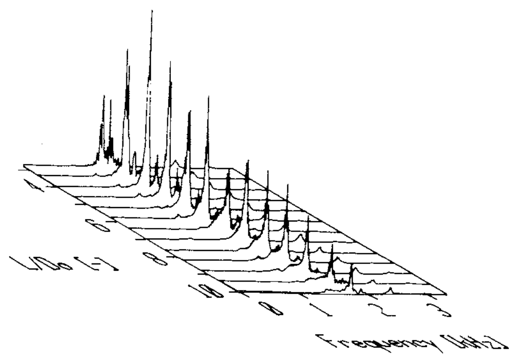

Fig. 3. A sample of power spectra from chain bubbles. As interbubble distance L/D_0 is decreased, the frequency of the highest peak in the power spectrum decreased.

interbubble distance L/D_0 was smaller than 10, several peaks appeared. Hence, we determined the frequency F_{exp} from the centroid in a distribution curve.

It is also observed in Fig. 3 that with decreasing value of L/D_0, F_{exp} decreases.

4.2 Demonstration of fundamental frequencies of chain bubbles

The experimental data of the frequency ratios

Table 2. Fundamental frequency ratios $F_{1,i}/F_0$
$D_0 = 0.55\,\mathrm{cm}$, $S = 2.86 \times 10^{-6}$

i	2	3	4	5	6	7	8	9	10	11
$F_{1,i}/F_0$	0.988	0.971	0.951	0.931	0.910	0.890	0.871	0.851	0.834	0.815
L/D_0	19.5	12.8	9.5	7.5	6.1	5.2	4.5	3.9	3.5	3.1

Table 3. Fundamental frequency ratios $F_{1,i}/F_0$
$D_0 = 0.75\,\mathrm{cm}$, $S = 2.86 \times 10^{-6}$

i	2	3	4	5	6	7	8	9	10	11
$F_{1,i}/F_0$	0.988	0.970	0.951	0.929	0.907	0.887	0.863	0.846	0.828	0.808
L/D_0	19.3	12.6	9.3	7.3	5.9	5.0	4.1	3.7	3.3	2.9

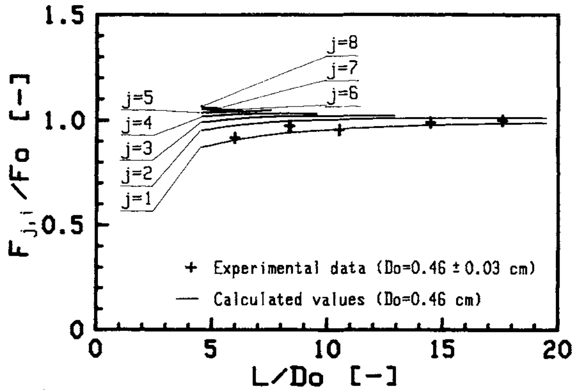

Fig. 4. Comparison of calculated values with experimental data.

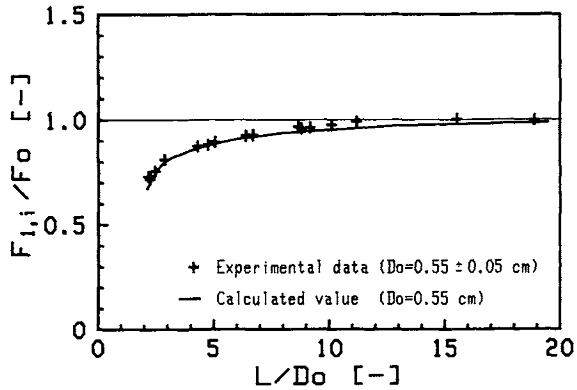

Fig. 5. Comparison of calculated fundamental frequency ratios with experimental data.

F_{exp}/F_0 for the volume mean diameters $D_0 = 0.46 \pm 0.03\,\mathrm{cm}$ are plotted against L/D_0 in **Fig. 4** together with the numerical solution. These curves were calculated from the values $F_{j,i}/F_0$ and L/D_0 in Table 1 using a cubic spline interpolator. The experimental data, i.e., natural frequency ratios, agreed with the curve obtained from the smallest value, $F_{1,i}/F_0$, i.e., the fundamental frequency ratios in Table 1. Consequently, it was evident that a fundamental frequency $F_{1,i}$ of the chain bubbles could be measured by applying the pulse response method of sound.

4.3 Effects of L/D_0 on fundamental frequency

For the bubble diameters $D_0 = 0.55$ and $0.75\,\mathrm{cm}$, the frequency ratios $F_{1,i}/F_0$ calculated from Eq. (17) are summarized in **Tables 2** and **3**, respectively, and the curves obtained by the cubic spline interpolator from this numerical solution are shown together with the experimental data in **Figs. 5** and **6**, respectively. These numerical solutions agreed well with the data obtained by the pulse response method of sound, the same conclusion reached for the numerical solutions of $D_0 = 0.46\,\mathrm{cm}$ in the previous section. Consequently, the effectiveness of the theoretical equation, Eq. (12), was demonstrated experimentally.

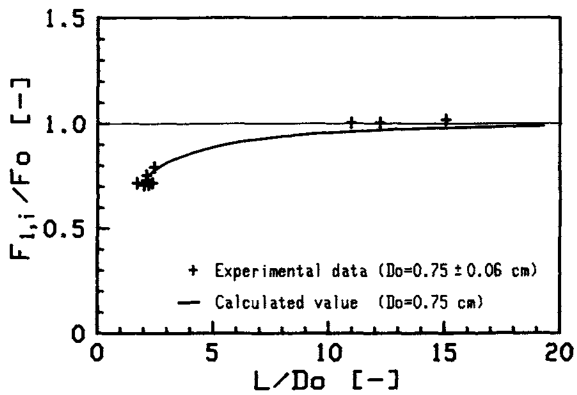

Fig. 6. Comparison of calculated fundamental frequency ratios with experimental data.

Furthermore, examining the numerical values in Tables 1, 2, and 3, it was evident that the values $F_{1,i}/F_0$ were not affected by the volume mean diameter of bubble D_0.

4.4 Effect of number of bubbles on fundamental frequency

The effect of the number of bubbles in a chain on the fundamental frequency, as calculated from Eq. (12), are shown in **Fig. 7**. For any chain, the values

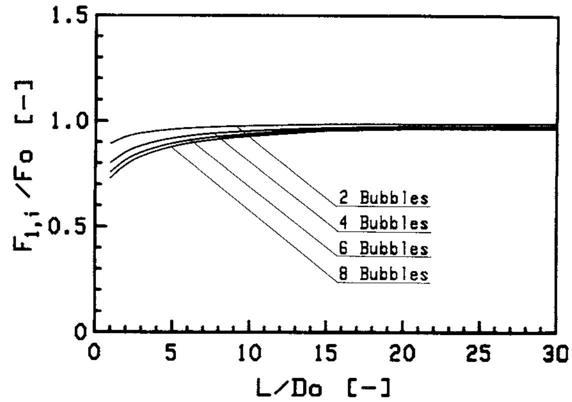

Fig. 7. Variation of $F_{1,i}/F_0$ with L/D_0. Each of these curves was calculated from Eq. (12) using the number of bubbles as parameter.

$F_{1,i}/F_0$ decreases with decreasing interbubble distance L/D_0. Even if the interbubble distance L/D_0 is the same, it can be seen that the value $F_{1,i}/F_0$ decreases with increasing number of bubbles in the chain.

Conclusion

A theoretical equation for the oscillation of chain bubbles in an inviscid and incompressible liquid was obtained by assuming that the gas in a bubble followed the adiabatic change law and by considering the effect of surface tension. But the effects of compressibility, viscosity and gravity were neglected. This equation includes the solution of Shima[8,9] on two and three bubbles with equal radii. The frequency of the chain bubbles decreased with increasing number of bubbles and with decreasing interbubble distance, and these results agreed well with the experimental data. Consequently, the effectiveness of this theoretical equation was demonstrated experimentally. It was also shown that the volume mean diameter of the chain bubbles could be measured on line, applying the pulse response method of sound.

Acknowledgment

The author is most grateful to Prof. Y. Yamamoto of the Department of Chemical Engineering in Kogakuin University for many timely discussions concerning this work. This research was supported by the Kogakuin University's Special Research Grant.

Nomenclature

D_0	= volume mean diameter of a bubble	[cm]
F_{exp}	= frequency of chain bubbles obtained by measurement	[Hz]
$F_{j,i}$	= j-th frequency of i bubbles	[Hz]
$F_{1,j}$	= fundamental frequency of i bubbles	[Hz]
F_0	= natural frequency of a bubble	[Hz]
H	= depth of water	[cm]
K	= $3\gamma+S(3\gamma-1)$	[—]
L	= distance between surfaces of bubbles, $(H-i\cdot D_0)/i$	[cm]
l	= distance between origins of bubbles	[cm]
p_∞	= pressure in water far from bubbles	[Pa]
$p_{w,i}$	= pressure in water at bubble surfaces	[Pa]
p_i	= $p_\infty+2\sigma/R_0$	[Pa]
r	= distance from origin	[cm]
R_i	= radii of bubbles	[cm]
R_0	= static radii of bubbles	[cm]
$\dot{R}$	= dR/dt	[$\mathrm{cm\cdot s^{-1}}$]
S	= $(2\sigma/R_0)/p_\infty$	[—]
t	= time	[s]
W	= R_0/l	[—]
$Z_{j,i}$	= $F_{j,i}/F_0$	[—]
α	= angle (see Fig. 1)	[—]
β_i	= R_i/R_0	[—]
γ	= ratio of specific heats of gas in a bubble	[—]
ε_i	= $(R_i-R_0)/R_0$, $\lvert\varepsilon_i\rvert<1$	[—]
μ	= roots of the characteristic Eq. (14)	[—]
ρ	= density of water	[$\mathrm{kg\cdot m^{-3}}$]
σ	= surface tension of water	[$\mathrm{Nm^{-1}}$]
τ	= $t/\sqrt{\rho\cdot p_\infty^{-1}}\cdot R_0$	[—]
Φ	= velocity potentials	[$\mathrm{m^2\cdot s^{-1}}$]

⟨Subscripts⟩

i	= number of bubbles	[—]
j	= $1\leq j\leq i$	[—]
m	= m-th bubble from top	[—]

Literature Cited

1) Foody, B. E. and P. W. Huber: *J. App. Mech., Trans. ASME*, **48**, 727 (1981).
2) Lorentz, H. A.: "Lectures on Theoretical Physics," Vol. 1, p. 58, MacMillan and Co., Ltd. (1927).
3) Minnaert, M.: *Philos. Mag.*, **16**, 235 (1933).
4) Morioka, M.: *J. Nucl. Sci. Technol.*, **11**, 32 (1974).
5) Nakagawa, K.: *Kagaku Kogaku Ronbunshu*, **10**, 557 (1984).
6) Nakagawa, K.: *Kagaku Kogaku Ronbunshu*, **11**, 563 (1985).
7) Plesset, M. S.: *J. App. Mech.*, **16**, 277 (1949).
8) Shima, A.: *J. of Basic Eng., Trans. ASME, Ser. D*, **93**, 426 (1971).
9) Shima, A.: *Mem. Inst. High Speed Mech., Tohoku Univ.*, **31**, 81 (1972).
10) Yamamoto, Y. and K. Nakagawa: *Kagaku Kogaku Ronbunshu*, **5**, 374 (1979).