INTERRELATIONS AMONG MIXING TIME, POWER NUMBER AND DISCHARGE FLOW RATE NUMBER IN BAFFLED MIXING VESSELS

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Key Words: Mixing, Mixing Vessel, Impeller, Mixing Time, Power Number, Discharge Flow Rate

Interrelations among the non-dimensional mixing time, the power number and the discharge flow rate number for baffled mixing vessels are experimentally investigated for paddles and turbines of various dimensions. The correlation equations with the geometrical dimensions of the impellers are presented for the mixing time, the power number and the discharge flow rate number, respectively. The power number can be directly correlated to the discharge flow rate number to the 1.34 power. The mixing time is proportional to the time required to circulate the discharged liquid once, as proposed by van de Vusse in 1955. The number of times of circulation of the liquid to reach the mixing time is correlated with the geometrical dimensions of the impellers. The relations of the non-dimensional mixing time with the power number or with the discharge flow rate number are presented. The impeller with larger diameter, larger breadth and greater number of blades is recommendable within the experimental range for turbulent mixing from the point of view of energy saving.

Introduction

Mixing characteristics of homogeneous liquid in an agitated vessel are measured as the time required to reach a certain low level of concentration deviation in a vessel after tracer liquid is poured into the vessel.[2] The mixing time is determined by means of conductivity measurement to detect the concentration deviation using ionized aqueous solution as a tracer.[5] The mixing time in non-dimensional form $n\theta_M$ usually depends on the geometrical shape of the vessel and the impeller in the turbulent region ($Re >$ several thousands), and the effects of Reynolds number and Froud number on $n\theta_M$ are negligible. Correlation equations for $n\theta_M$ based on dimensional analysis have been proposed by many investigators.[1,8]

The mixing time in the turbulent region in a mixing vessel is considered to be determined by the following factors.

1) Mixing due to direct shear action between liquid and an impeller.

2) Macro-mixing due to circulation flow in a vessel by discharge action of an impeller.

3) Mixing due to turbulent diffusion during the circulation of liquid.

4) Mixing due to molecular diffusion.

$n\theta_M$ mentioned above in the turbulent region in a baffled vessel is mainly determined by the mixing actions of 1), 2) and 3).

Received January 23, 1984. Correspondence concerning this article should be addressed to Y. Sano.

Correlation equations of $n\theta_M$ using power number N_p and/or discharge flow rate number N_{qd} (or circulation flow rate number), reflecting the factors noted above for mixing in the turbulent region, have been variously proposed. For example, Kamiwano *et al.*[5] reported a correlation equation in the form of addition of two terms, N_{qd} and $\sqrt{N_p/N_{qd}}$, which represent the effect of circulation flow and the effect of turbulent diffusion, respectively. Sasakura *et al.*[12] have given a relation between $n\theta_M$ and N_p, based on a model of series of perfect mixing vessels. Hiraoka and Ito[4] have given an equation of $n\theta_M$ vs. N_p from the point of view of the turbulent diffusion rate. However, van de Vusse[14] had already pointed out in 1955 that the mixing time in the turbulent region is determined by the average circulation time. For the mixing of viscous liquid, Nagata *et al.*[9] showed in 1957 that $n\theta_M$ is determined by the circulation flow rate in the laminar region. The reason that various equations have subsequently been proposed from the point of view of turbulent diffusion, instead of the concept of van de Vusse, is due to the lack of correct data for N_{qd} under baffled conditions. The data of N_{qd} measured by a Pitot tube[10] include considerable error, because of the difficulty of measurement for the three-dimensional flow in a baffled vessel.

In this report, $n\theta_M$, N_{qd} and N_p are measured with paddles and turbines of various dimensions in the turbulent region of the baffled vessels, and the interrelations among these parameters are investigated.

1. Experimental Apparatus and Procedure

Measurements of the mixing time, the power and the discharge flow rate were performed with the apparatus shown in **Fig. 1**. Two agitated vessels of diameters 0.2 m and 0.4 m were used. The liquid used was tap water, filled into the vessel to a height equal to the vessel's diameter. Four baffles of a breadth of 1/10 the vessel diameter were installed on the side wall of the vessel. The impellers used in the experiments were paddles and turbines, shown schematically in **Fig. 2**. Their dimensions are presented in **Table 1**, in which starred values are used for the measurements of $n\theta_M$, N_p and N_{qd} and others are used only for $n\theta_M$ and N_p. The impeller was installed at a level of half the liquid depth on the center axis.

The method for measurement of mixing time is almost the same as that of Kamiwano *et al.*[5] Two electrode cells made of platinum wire (shown in Fig. 1), used to detect the electrical conductivity of the liquid, were placed at the corners of the upstream side of two baffles installed opposite one another on the vessel wall. This is considered as the place showing the largest elapsed time to reach a given degree of deviation of concentration by preliminary experiments. The two electrode cells were put in a bridge using A.C. of 2.13 kHz. The difference in resistance between the cells, which corresponds to the concentration difference between the two cell positions, was recorded by a rapid-response recorder. The mixing time was determined by the point on the recorder chart at which the signal reaches a final value after the tracer liquid is poured into the vessel (the degree of relative deviation of concentration $<1\%$). The tracer liquid was 1/2 N NaCl aqueous solution. For the vessel of 0.4 m diameter, 50 ml of the tracer was poured into the vessel from the free surface at a position near the shaft of the impeller. For the vessel of 0.2 m diameter, one-eighth that amount of the tracer liquid was used. In the experiments, successive measurements were performed without changing liquid in the vessel until the concentration of NaCl in the vessel liquid reached 6×10^{-3} N. The resultant mixing times were reproducible within an error of 15%. Measurements under the same mixing conditions were repeated twice and the average value is adopted as the mixing time.

The power of the impeller was measured by a torque transducer using strain gauge method, placed at the shaft of the impeller.

The discharge flow rate of the impeller is measured by the flow follower method, first developed by Porcelli and Marr[11] for a propeller in a vessel. This method has been followed successfully by Sato and Taniyama[13] for turbines and paddles of radial discharge flow type in baffled vessels. A polyethylene particle of 2 mm diameter, colored and adjusted in density to equal that of the liquid, is used as a flow follower. The number of times m_q in which the particle passes through the cylindrical part defined by rotation of the impeller blade is counted in a finite time T by observing the particle. The discharge flow rate q of the impeller is calculated by the following equation.

$$q=m_qV/T \tag{1}$$

where V denotes the volume of liquid in the vessel.

All measurements are performed in the turbulent region of $Re>5\times10^3$.

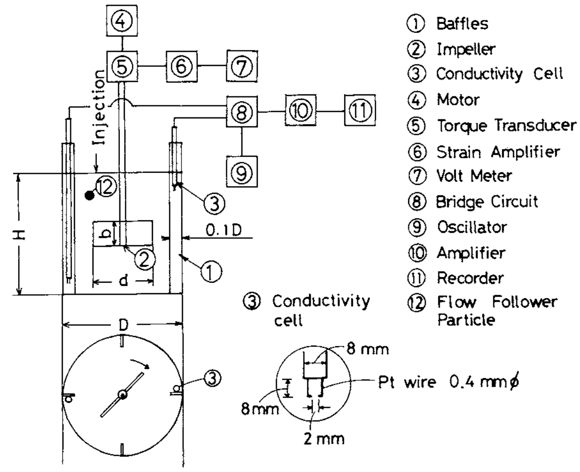

Fig. 1. Experimental apparatus.

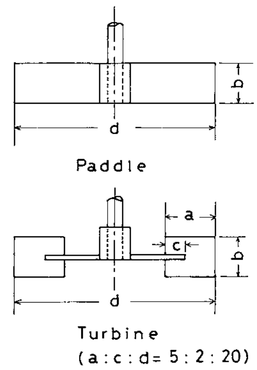

Fig. 2. Impeller.

Table 1. Dimensions of impeller

Paddle	
d/D	0.30*, 0.40*, 0.50*, 0.60*, 0.70*
b/D	0.05*, 0.10*, 0.15*, 0.20*, 0.30
n_p	2*, 4*, 6*
Turbine	
d/D	0.4*, 0.5*, 0.6*, 0.7*
b/D	0.1*, 0.15*, 0.2*, 0.3, 0.4
n_p	2*, 4*, 6*, 8*
D [m] = 0.4*, 0.2*	

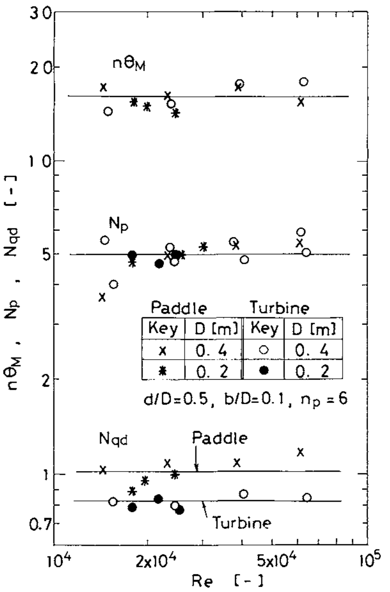

Fig. 3. Dependency of $n\theta_M$, N_p and N_{qd} on Re.

2. Experimental Results and Discussion

2.1 Non-dimensional mixing time, power number and discharge flow rate number

The plots of the reciprocal of the observed mixing time with the rotational speed are all linear, and the non-dimensional mixing time $n\theta_M$ is constant throughout the experimental range.

Typical data of the non-dimensional mixing time $n\theta_M$, the power number N_p and the discharge flow rate number N_{qd} are shown in **Fig. 3**. These values show certain constant values independent of Re and are only the functions of geometrical shape of the impeller. N_p has almost the same values for the paddle and the turbine of the same d/D, b/D and n_p. But N_{qd} for the paddle is a little larger than that for the turbine. As for $n\theta_M$, the data in Fig. 3 are almost the same for both impellers, but generally $n\theta_M$ for paddles tends to show smaller values than those for turbines, as shown later.

The correlations of $n\theta_M$, N_p and N_{qd} with the geometrical parameters of d/D, b/D and n_p are shown in **Figs. 4** and **5**, for the paddles and the turbines, respectively. As for the values of $n\theta_M$ for paddles, the data by Sasakura *et al.*[12] are also plotted in Fig. 4 with the same form of $(d/D)^{-1.67}(b/D)^{-0.74}n_p^{-0.47}$. Their data are well correlated by the present relation of the geometrical parameters, but the values themselves are different from the present data. This may be due to the difference in degree of relative deviation of concentration for the definition of mixing time. The relation of the mixing time θ_M and the degree of relative deviation of concentration in the turbulent region can be presented by the following equation given by Hiby.[2]

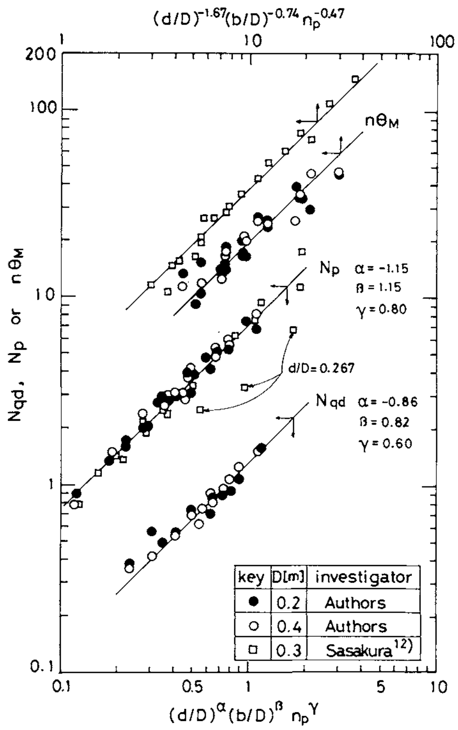

Fig. 4. Correlation of $n\theta_M$, N_p and N_{qd} with geometrical dimensions of paddle.

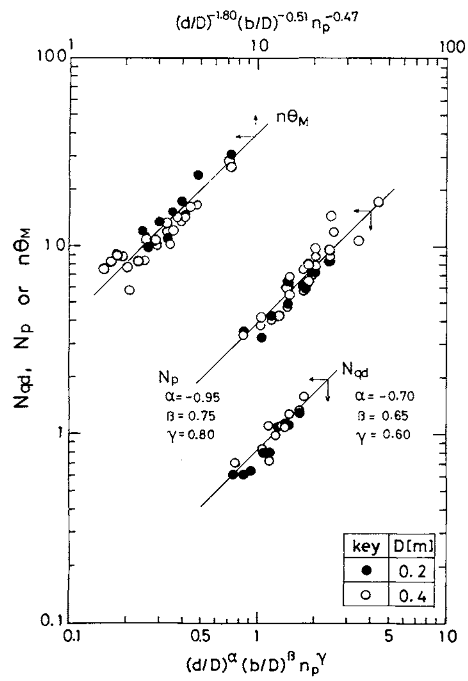

Fig. 5. Correlation of $n\theta_M$, N_p and N_{qd} with geometrical dimensions of turbine.

$$\ln(\Delta c/c_m) = K_1 - K_2\theta_M \tag{2}$$

Kamiwano *et al.*[5] reported experimental data of this relation for a paddle of $d/D=0.5$, $b/D=0.1$ and $n_p=8$ at a rotational speed of $n=1.0$ rps which fit Eq. (2) satisfactorily. If we estimate the degree of relative deviation of concentration based on the data by

Kamiwano *et al.*,[5] it is 0.1% for Sasakura's[12] and 1.5% for ours.

The values of N_p for paddles measured by Sasakura *et al.*[12] are also compared with our data in Fig. 4. They are in good agreement with ours in the same correlation form, except the data by Sasakura[12] for $d/D=0.267$.

The correlations for $n\theta_M$, N_p and N_{qd} in Figs. 4 and 5 are expressed by the following equations.

For paddles

$$n\theta_M=2.1(d/D)^{-1.67}(b/D)^{-0.74}n_p^{-0.47} \tag{3}$$

$$N_p=7.3(d/D)^{-1.15}(b/D)^{1.15}n_p^{0.80} \tag{4}$$

$$N_{qd}=1.3(d/D)^{-0.86}(b/D)^{0.82}n_p^{0.60} \tag{5}$$

For turbines

$$n\theta_M=3.8(d/D)^{-1.80}(b/D)^{-0.51}n_p^{-0.47} \tag{6}$$

$$N_p=3.6(d/D)^{-0.95}(b/D)^{0.75}n_p^{0.80} \tag{7}$$

$$N_{qd}=0.80(d/D)^{-0.70}(b/D)^{0.65}n_p^{0.60} \tag{8}$$

The constants of 2.1 and 3.8 for the equations of $n\theta_M$ will change according to the degree of relative deviation of concentration for the definition of the mixing time.

2.2 The relation between N_p and N_{qd}

The relations betweens N_p and N_{qd} for the paddles and the turbines are shown in **Figs. 6** and **7**, respectively. The present experimental data are well correlated uniquely for both paddles and turbines. The correlation equations are shown as follows.

For paddles: $$N_p=4.3N_{qd}^{1.34} \tag{9}$$

For turbines: $$N_p=6.6N_{qd}^{1.34} \tag{10}$$

The data for paddles by Sato and Taniyama[13] are well correlated by Eq. (9) but the data by Kamiwano *et al.*[5,10] in which N_{qd} was measured by a Pitot tube scatters considerably, as shown in Fig. 6. The data for the impeller of a special type of burmargin by Sato and Taniyama[13] lie a little lower with the same slope, which reflects slightly superior performance of the impeller compared to the paddles. Moreover, the correlation line for the turbines in Fig. 7 lies on the upper side compared with that for the paddles in Fig. 6. This means that the discharge performance of the turbine is slightly inferior to that of the paddle.

Investigations of the relation between N_p and N_{qd} are rather few. Sato and Taniyama[13] showed that the ratio of N_p and N_{qd} is almost the same for paddles and turbines of the same d/D, b/D and n_p, and suggested the proportionality of N_p and N_{qd}. But their data for the paddles shown in their paper should be expressed by Eq. (9). Nagase *et al.*[6,7] have suggested that N_p should be proportional to the square of N_{qd}, based on the theoretical consideration of conservation of

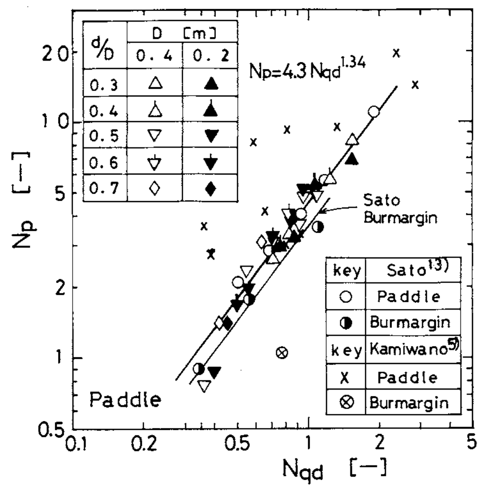

Fig. 6. N_p vs. N_{qd} for paddle.

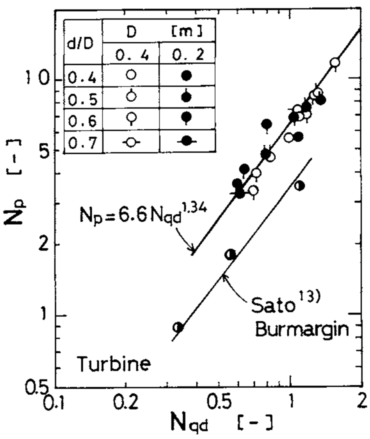

Fig. 7. N_p vs. N_{qd} for turbine.

angular momentum. This was supported by Hiraoka and Ito.[3] The present results show that N_p is proportional to $N_{qd}^{1.34}$ for both paddles and turbines.

2.3 Number of times of circulation of discharged liquid to reach the mixing time

If it is assumed that the mixing time is influenced mainly by the convection of discharge flow by an impeller in a vessel, the mixing time is proportional to V/q, as proposed by Nagata *et al.*[9] for the laminar region and by van de Vusse[14] for the turbulent region.

$$\theta_M=mV/q \tag{11}$$

where m is the number of times of circulation of the discharged liquid in a vessel to reach a certain low level of the degree of concentration deviation. Typical data for the relation are shown in **Fig. 8**. The proportionality between θ_M and V/q is quite satisfactory. The values of m are from 2 to 4 and depend on the geometrical factors of the impeller, as shown in **Fig. 9**. The correlation equation for m is as follows.

$$m=3.8(d/D)^{0.5}(n_pb/D)^{0.1} \tag{12}$$

From Eqs. (11) and (12), the reciprocal of $n\theta_M$ is

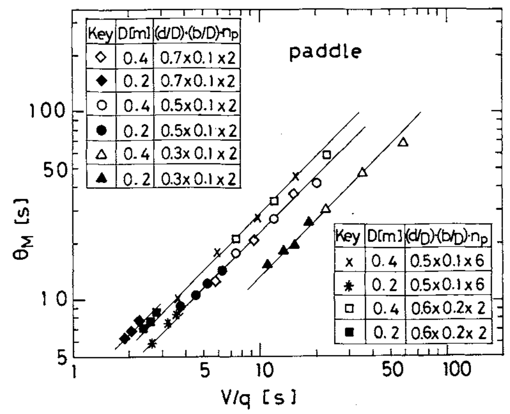

Fig. 8. Relation between mixing time and circulation time of discharged liquid.

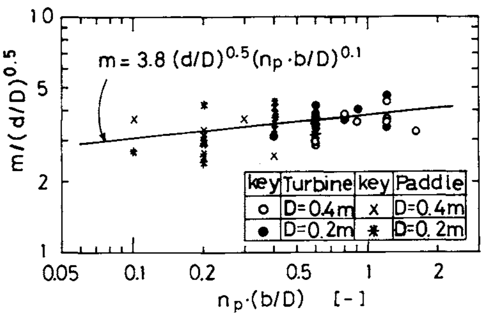

Fig. 9. Correlation of number of times of circulation with geometrical dimensions of impeller.

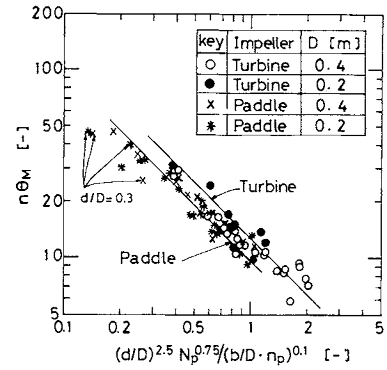

Fig. 10. Relation of $n\theta_M$ and N_p for present data.

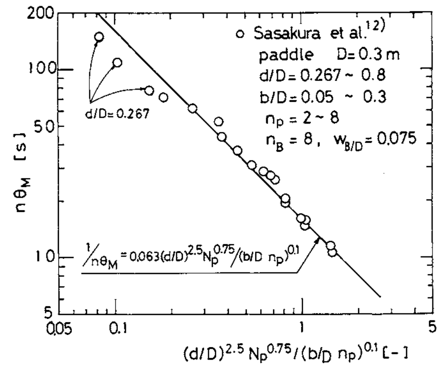

Fig. 11. Relation of $n\theta_M$ and N_p for the data by Sasakura *et al.*[12]

directly proportional to N_{qd}. The values of m measured in the turbulent region under baffled conditions are comparable to those in the laminar region reported by Nagata *et al.*,[9] in which m was from 2.6 to 3 for ribbon mixers and screw mixers.

2.4 The relation between $n\theta_M$ and N_p

The relation between $n\theta_M$ and N_p is obtained by Eqs. (11) and (12) with Eq. (9) or Eq. (10).

$$1/n\theta_M = C(d/D)^{2.5} N_p^{0.75}(n_p b/D)^{-0.1} \qquad (13)$$

The constant C was 0.11 and 0.082 for paddles and turbines, respectively, which will depend on the degree of relative deviation of concentration for the definition of the mixing time.

Equation (13) is shown with the experimental data in **Fig. 10**. The data of $n\theta_M$ and N_p by Sasakura *et al.*[12] are plotted in the same form in **Fig. 11**. The success of the plots in Figs. 10 and 11 supports the concept that the mixing in the turbulent region depends on the macro-mixing due to circulation flow in a vessel by the discharge action of the impeller.

2.5 Scale-up criterion for mixing

As seen in Eqs. (3) and (6), $n\theta_M s$ are only functions of the geometrical shape of the impeller; then $n\theta_M$ itself is a criterion of mixing for geometrically similar mixing vessels. To obtain equal values of θ_M, n must be the same for both a larger vessel and a smaller one. This situation is the same as that for mixing in the laminar region.[9] In the laminar region, an equal value of power consumption per unit volume of liquid P_v also becomes a criterion for mixing, because N_p is inversely proportional to Re. However, in the fully developed turbulent region under baffled conditions, N_p is independent of Re and then P_v increases with D^2 for constant $n\theta_M$.

2.6 Mixing energy and recommendable shape of impeller

By means of mixing energy, defined as the energy consumed during θ_M per unit mass of liquid $E_m = P_v\theta_M/\rho$, the following equation is derived from Eqs. (3) and (13) for paddles.

$$E_m^{1/2}\theta_M/D = 9.1(d/D)^{-0.56}(b/D)^{-0.55} n_p^{-0.33} \qquad (14)$$

For turbines, a similar equation is also derived.

It is pointed out that θ_M is inversely proportional to the square root of E_m for mixing of the turbulent region. From Eq. (14), an impeller with larger diameter, larger breadth and greater number of blades, operated at lower rotational speed, is recommendable within the experimental range of $0.3 < (d/D) < 0.7$, $0.05 < (b/D) < 0.3$ and $2 < n_p < 8$ under baffled conditions of $Re > 5 \times 10^3$, from the point of view of energy saving. This means that mixing by an impeller of larger discharge action for a given mixing energy is preferable because it avoids wasteful consumption of energy due to excessive shear action between the impeller and the liquid. From Fig. 10 the paddles appear to be slightly superior to the turbines from the point of view of energy saving.

Conclusions

The mixing time, the power of the impeller and the discharge flow rate are measured for paddles and turbines of various dimensions $0.3<(d/D)<0.7$, $0.05<(b/D)<0.3$ and $2<n_p<8$ under baffled conditions of $Re>5\times10^3$.

1) The non-dimensional mixing time, the power number and the discharge flow rate number in the turbulent region are independent of Reynolds number and are correlated with the dimensions of the paddles and the turbines, respectively.

2) The power number is proportional to the discharge flow rate number to the 1.34 power and the relations are presented in Eqs. (9) and (10) for paddles and turbines, respectively.

3) The mixing time is proportional to the time required to circulate the discharged liquid once, as proposed by van de Vusse.[14] The number of times of circulation of the discharged liquid to reach the mixing time is correlated by Eq. (12).

4) The non-dimensional mixing time is related directly the power number, as shown in Fig. 10.

5) An impeller with larger diameter, larger breadth and greater number of blades is recommendable within the experimental range from the point of view of energy saving. The paddles seem to be slightly superior to the turbines.

Acknowledgment

The authors appreciate the assistance of student colleagues (T. Mukai, H. Matsuda, K. Kurakazu and S. Kirita).

Nomenclature

b	= width of impeller	[m]
c_m	= average concentration	[mol/m^3]
Δc	= concentration deviation	[mol/m^3]
D	= vessel diameter	[m]
d	= diameter of impeller	[m]
E_m	= mixing energy per unit mass of liquid	[J/kg]
m	= number of times of circulation of discharged liquid to reach the mixing time	[—]
m_q	= pass number of flow follower particle through impeller for time T	[—]
N_p	= power number, $P/\rho n^3 d^5$	[—]
N_{qd}	= discharge flow rate number, q/nd^3	[—]
n	= rotational speed of impeller	[r.p.s.]
n_p	= number of impeller blades	[—]
n_B	= number of baffle plates	[—]
P	= mixing power consumption	[W]
P_v	= power consumption per unit volume of liquid	[W/m^3]
q	= discharge flow rate	[m^3/s]
Re	= Reynolds number of impeller, $d^2 n/\nu$	[—]
T	= measuring time for discharge flow rate by flow follower method	[s]
V	= volume of liquid in a vessel	[m^3]
W_B	= width of baffle plate	[m]
θ_M	= mixing time	[s]
ν	= kinetic viscosity of liquid	[m^2/s]
ρ	= density of liquid	[kg/m^3]

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