HEAT TRANSFER BETWEEN WALL AND SOLID-WATER SUSPENSION FLOW IN HORIZONTAL PIPES

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Key Words: Multiphase Flow, Heat Transfer, Solid-Water, Two Phase Flow, Stationary Bed, Moving Bed, Asymmetric Suspension, Suspended Solid Particle, Slurry Reactor

Apparatus made of copper and acrylic pipes of 14 mm, 19 mm and 25 mm i.d. were constructed to study the fundamental characteristics of heat transfer from the wall to water suspensions of glass beads (d = 0.06 to 1.0 mm) or ion exchange resin (d = 0.8 mm) flowing through horizontal pipes. The operating range of Reynolds numbers and volume fraction of solid is from 3000 to 50,000 and from 0 to 0.1, respectively.

In the case of particle sizes larger than 0.35 mm in diameter, the heat transfer coefficient is always larger than that of a simple water flow and is not strongly affected by particle size, pipe diameter or volume fraction of the solid in the asymmetric suspension flow. In the case of particle sizes smaller than 0.15 mm in diameter, on the other hand, heat transfer reduction is found to occur in the range of Reynolds numbers from 6000 to 15,000.

The temperature profiles of fluids in the horizontal radial direction are symmetric with respect to the pipe axis even at low Reynolds numbers. In the vertical radial direction perpendicular to the pipe axis, the temperature profiles of fluids are asymmetric with respect to the pipe axis.

Introduction

Concentrated suspension flow has been applied to many chemical processes in the form of a slurry reactor combined with simultaneous transportation of solid material. Any attempt to design a slurry reactor necessarily requires a knowledge of its characteristics such as the behavior of the suspension, pressure drop and the rate of heat transfer related to the supply or removal of the heat of reaction. Several investigators[1,2,4–6] have reported on the heat transfer behavior of some suspension flows. Konno *et al.*[1,2] experimentally investigated the heat transfer characteristics of downward and upward flows of glass beads or ion exchange resin-water suspensions in vertical pipes and proposed an empirical correlation of the heat transfer coefficients. Orr and DallaValle[4] correlated their data on chalk slurries with a Dittus-Boelter type equation. Quader *et al.*[5] carried out an experimental investigation into the turbulent heat transfer to pseudoplastic titanium dioxide suspensions in pipes and discussed the limitations of the analogy between heat and momentum transfer. Salamone *et al.*[6] presented some data on turbulent heat transfer of powdered solids such as copper, carbon, chalk and silica which were suspended in water. They proposed a design equation for turbulent-flow heat transfer to solid-liquid suspensions inside pipes. The heat transfer characteristics of a horizontal solid-liquid suspension flow, however, have not been thoroughly clarified because of the complex behavior of the suspension.

The experimental investigation reported here is concerned with the effects of volume fraction, physical properties of solids and the mean velocity of a suspension on the rates of heat transfer between the pipe wall and the suspension flow in horizontal pipes. The mechanism of heat transfer in the suspension flow is also discussed on the basis of flow pattern and temperature profile in the fluid.

1. Experimental Apparatus and Procedure

A schematic diagram of the experimental apparatus is shown in **Fig. 1**. Three copper pipes, of 14, 19 and

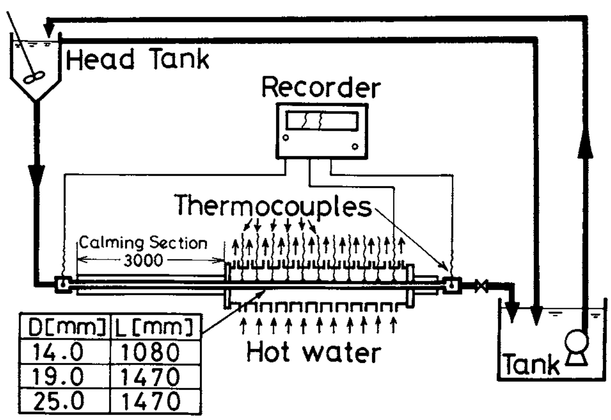

Fig. 1. Schematic diagram of experimental apparatus.

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Table 1. Physical properties of solids

Material	Density [kg/m³]	Diameter [mm]	Calculated terminal velocity [m/s]	Specific heat [kJ/(kg·K)]
Glass beads	2500	0.06 0.15 0.35 1.0	0.003 0.018 0.055 0.164	0.75
Ion exchange resin	1250	0.8	0.034	2.64

25 mm i.d., are used for the heat transfer section. Measurement of the heat transfer coefficient is carried out at a constant wall temperature. The temperature measurement used in this work is described in detail in the previous paper.[1] The heat transfer coefficient is calculated by use of Eq. (1) in the same manner as in the previous paper.[1]

$$h=\frac{1}{\left[(2\pi r_o L)\left\{\frac{\Delta t_{lm}}{(W_f C_f+W_s C_s)(t_o-t_i)}-\frac{r_a-r_o}{k_{cu}(2\pi r_{av}L)}\right\}\right]} \tag{1}$$

The physical properties of the solid used in this work are listed in **Table 1**.

2. Experimental Results and Discussion

2.1 Heat transfer coefficients for solid-water suspensions

Figure 2 shows the measured heat transfer coefficient as a function of m_s for a 0.35 mm glass beads-water suspension flowing through a horizontal pipe of 19 mm i.d. The solid line shows the calculated value using the Sieder-Tate empirical equation.[7] The observed flow patterns are also shown schematically in Fig. 2 and the definition of the flow pattern is identical with that in the previous study.[8]

For the case of a simple water flow ($m_s=0$), close agreement between the observed data and the solid line is obtained.

The heat transfer coefficients of a solid-water suspension flow are always 10 to 30 percent higher than those of a simple water flow over the whole range of Reynolds numbers. For a moving bed and asymmetric suspension flow, it is assumed that the disturbance of the thermal boundary layer by the suspended solid particles increases the heat transfer rate between the suspension and the inside wall of a pipe. But in this region, the heat transfer coefficients are not strongly affected by the volume fraction of the solid. For a stationary bed flow, it is considered that the heat transfer rate increases extremely because of the presence of a much higher fluid velocity[8] than that of a simple water flow in the upper part of a pipe. In this

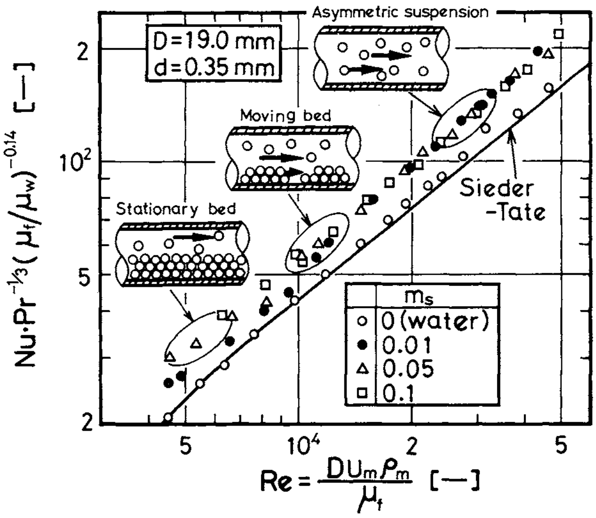

Fig. 2. Variation of modified Nusselt number of 0.35 mm glass beads-water suspension with Reynolds number as a function of volume concentration of solid.

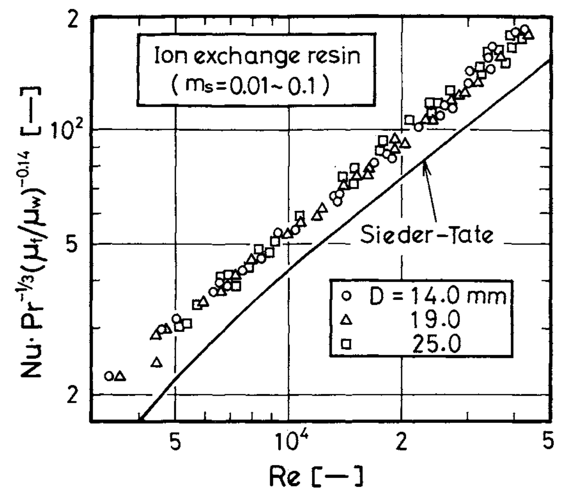

Fig. 3. Variation of modified Nusselt number of 0.8 mm ion exchange resin-water suspension with Reynolds number as a function of volume concentration of solid.

case, the effect of m_s on the heat transfer coefficient is observed, as shown in Fig. 2.

For the different sizes of glass beads and pipes, the heat transfer coefficients obtained follow the same trend as those exhibited by the 0.35 mm glass beads-water suspension flow.

The flow of an ion exchange resin suspension was also examined. In this case no stationary bed is observed under operating conditions. **Figure 3** illustrates the variation of the heat transfer coefficients for the ion exchange resin suspension versus Reynolds numbers. It is noted that the heat transfer coefficients follow the same trend as those observed with the glass beads-water suspension and are not affected by either pipe diameter or volume fraction of the solid.

Figure 4 shows the heat transfer coefficient for a 0.06 mm glass beads-water suspension flowing through a 25 mm i.d. pipe. It is clearly seen that the

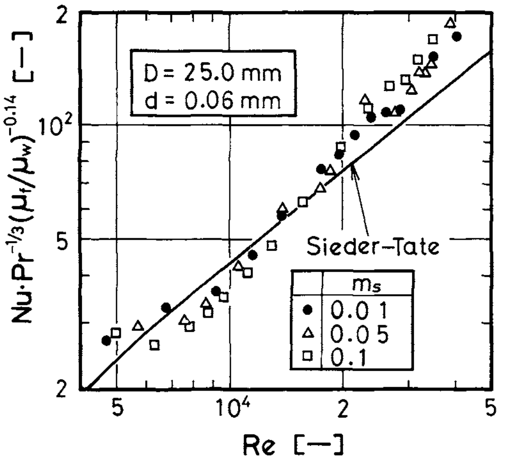

Fig. 4. Variation of modified Nusselt number of 0.06 mm glass beads-water suspension with Reynolds number as a function of volume concentration of solid.

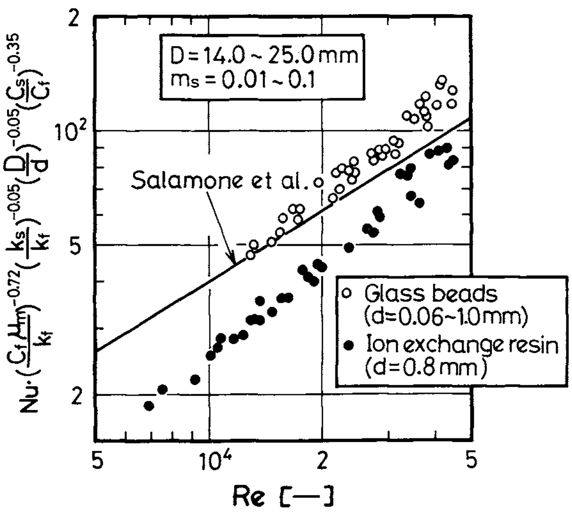

Fig. 5. Comparison of Nusselt numbers obtained experimentally and calculated by correlation of Salamone *et al.*

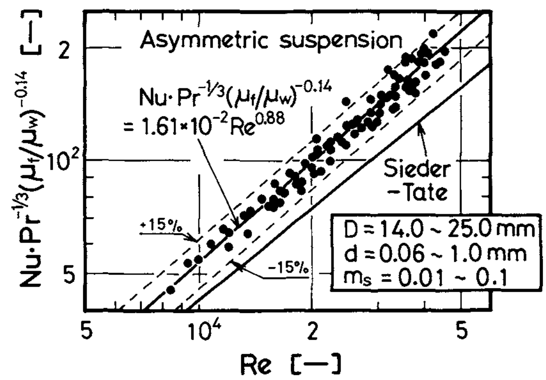

Fig. 6. Relationship between $NuPr^{-1/3}(\mu_f/\mu_w)^{-0.14}$ and Re for an asymmetric suspension flow.

heat transfer reduction phenomenon occurs in the range of Reynolds numbers from 6000 to 14,000. In this case, an increase of the volume fraction of the solid decreases the heat transfer coefficient of the suspension. Beyond a certain Reynolds number, however, an increase of the volume fraction of the solid tends to increase the heat transfer coefficient of the suspension. This reduction phenomenon is also found in the case of the 0.15 mm glass beads suspension flow.

Salamone *et al.*[6] proposed the following empirical equation, based on the dimensional analysis of a fine particle-water suspension.

$$Nu = 0.131 Re^{0.62}\left(\frac{C_f \mu_m}{k_f}\right)^{0.72}\left(\frac{k_s}{k_f}\right)^{0.05}\left(\frac{D}{d}\right)^{0.05}\left(\frac{C_s}{C_f}\right)^{0.35} \tag{2}$$

where $14{,}000 \leqq Re \leqq 140{,}000$, $3.4 \leqq C_f \mu_m / k_f \leqq 12.7$, $0.53 \leqq k_s/k_f \leqq 583$, $282 \leqq D/d \leqq 10{,}500$ and $0.09 \leqq C_s/C_f \leqq 0.22$.

Figure 5 shows a comparison of the present data with the above correlation. The heat transfer coefficients for the suspension of glass beads can be predicted by Eq. (2), but none of the heat transfer coefficients for the suspension of ion exchange resin particles which were not investigated by Salamone *et al.* are found to be estimated by Eq. (2), as shown in Fig. 5.

Since Eq. (2) shows poor agreement, the heat transfer coefficients for an asymmetric suspension flow are correlated on the basis of the Sieder–Tate equation.[8] **Figure 6** shows the relationship between $NuPr^{-1/3}(\mu_f/\mu_w)^{-0.14}$ and Re for an asymmetric suspension flow. It is found from the figure that $NuPr^{-1/3}(\mu_f/\mu_w)^{-0.14}$ is proportional to Re on a logarithmic graph. Evaluating the exponent of the Reynolds number and the coefficient from experimental data, the following empirical equation is proposed for asymmetric suspension flow.

$$NuPr^{-1/3}(\mu_f/\mu_w)^{-0.14} = 1.61 \times 10^{-2} Re^{0.88} \tag{3}$$

where $8000 \leqq Re \leqq 50{,}000$, $0.01 \leqq m_s \leqq 0.1$ and $0.0024 \leqq d/D \leqq 0.071$.

As shown in Fig. 6, the experimental data are correlated by Eq. (3) within $\pm 15\%$ error.

2.2 Fluid temperature profiles

Figures 7 and **8** illustrate the fluid temperature profiles of a 0.35 mm glass beads-water suspension flow in both vertical and horizontal radial directions for 25 mm i.d. and 14 mm i.d. test pipe, respectively. In the horizontal radial direction the temperature profiles of the fluid are symmetric with respect to the pipe axis even at low Reynolds numbers. In the vertical radial direction, on the other hand, the temperature profiles of fluid are asymmetric with respect to the pipe axis compared with those of a simple water flow. For stationary bed flow at $Re = 7500$ for $D = 25$ mm and $Re = 4500$ for $D = 14$ mm, the temperature profiles of fluid in the bottom of a pipe are almost linear. However, for asymmetric suspension flow at

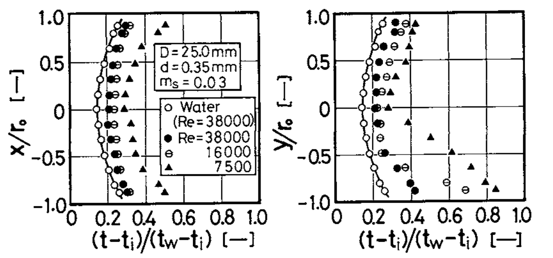

Fig. 7. Temperature profiles of 0.35 mm glass beads-water suspension flow in a horizontal pipe.

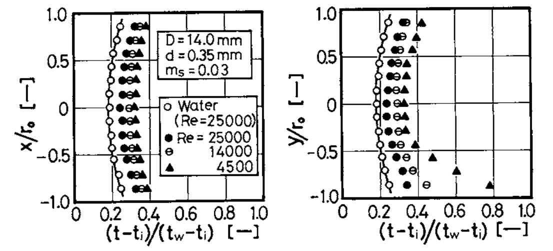

Fig. 8. Temperature profiles of 0.35 mm glass beads-water suspension flow in a horizontal pipe.

large Reynolds numbers ($Re=38{,}000$ for $D=25$ mm and $Re=28{,}000$ for $D=14$ mm), the fluid temperature profiles in the vertical radial direction are almost symmetric with respect to the pipe axis.

2.3 Heat transfer coefficient for a stationary bed flow

To examine the mechanism of heat transfer in a stationary bed flow, the flow is divided into two parts: the asymmetric suspension and the particle bed, as shown in **Fig. 9**. The following assumptions were made: 1) The fluid velocity in the particle bed is zero. 2) Heat is transmitted only by conduction in the particle bed. 3) Flow of fluid is plug flow in the asymmetric suspension region.

In Fig. 9, since the heat flow (Q_m+Q_b) into a suspension from the inside wall should be equal to the net heat $\{(W_f+W_s)C_m(t_o-t_i)\}$ carried away by a suspension at the steady state condition, the heat balance is

$$Q_m+Q_b=(W_f+W_s)C_m(t_o-t_i) \tag{4}$$

By neglecting the thermal resistance of the wall of a pipe, the average heat transfer coefficient h'_m can be expressed as

$$h'_m=\frac{(W_f+W_s)C_m(t_o-t_i)}{\pi DL\cdot\Delta t_{lm}}=\frac{(Q_m+Q_b)}{\pi DL\Delta t_{lm}} \tag{5}$$

As Q_m and Q_b are given, h'_m can be calculated from Eq. (5).

If the effective thermal conductivity of the particle bed is taken as k_b, the rate of heat transfer through an element of the pipe wall, $r_o d\theta dz$, can be given by Eq. (6):

$$q_b=\left(k_b\frac{\partial t}{\partial r}\right)_{r=r_o} r_o d\theta dz \tag{6}$$

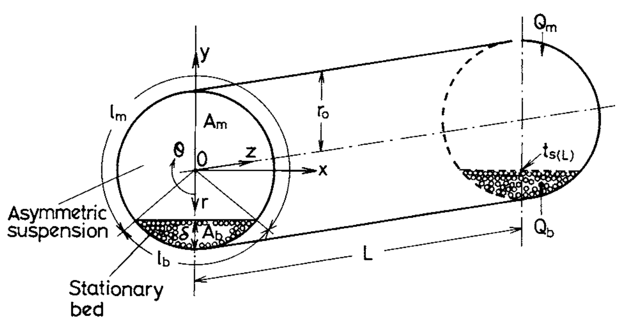

Fig. 9. Schematic representation of the model.

Hence, if k_b and $(\partial t/\partial r)$ are known, Q_b can be obtained by integration of Eq. (6) over $z=l_b$ to L. The effective thermal conductivity k_b is assumed to be given by Eq. (7) of Kunii *et al.*[3)]

$$k_b=k_f\left\{\varepsilon+\frac{1-\varepsilon}{\psi+\frac{2}{3}\left(\frac{k_f}{k_s}\right)}\right\} \tag{7}$$

To calculate $(\partial t/\partial r)_{r=r_o}$ in Eq. (6), no temperature distribution in the θ direction is supposed. The temperature t_s of the boundary surface between the asymmetric suspension and the stationary bed is also assumed to be given by the linear relation $t_s(z)=t_i+(t_s(L)-t_i)z/L$ along the pipe axis. Therefore, if the thickness of the stationary bed is taken as δ, $(\partial t/\partial r)_{r=r_o}$ in Eq. (6) is approximated by:

$$-\left(\frac{\partial t}{\partial r}\right)_{r=r_o}=\frac{t_w-\{t_i+(t_s(L)-t_i)z/L\}}{\delta} \tag{8}$$

The integration of Eq. (6) over $z=l_b$ to L, considering Eqs. (7) and (8), gives the following equation for Q_b.

$$Q_b=k_b l_b L\left(\frac{t_w-t_i}{\delta}+\frac{t_w-t_s(L)}{\delta}\right)\Big/2 \tag{9}$$

where l_b is a function of m_s.

On the other hand, to estimate the heat flow rate Q_m given by Eq. (10) the average heat transfer coefficient h_m is calculated from Eq. (3) using a Reynolds number based on an equivalent diameter D_e $(=4\cdot A_m/l_m)$ of the flow path.

$$Q_m=h_m\cdot l_m\cdot L\cdot\Delta t_{lm} \tag{10}$$

From Eqs. (5), (9) and (10), the average heat transfer coefficient h'_m of stationary bed flow can be expressed by Eq. (11).

$$h'_m=h_m\left(\frac{l_m}{\pi D}\right)+k_b\left(\frac{l_b}{\pi D}\right)\left(\frac{t_w-t_i}{\delta}+\frac{t_w-t_s(L)}{\delta}\right)\Big/2\Delta t_{lm} \tag{11}$$

From Eq. (11), it can be seen that the heat transfer coefficient of stationary bed flow is influenced by the volume fraction of the solid. This agrees well with the results obtained at low Reynolds numbers, as shown in Fig. 2. However, as the mean velocity of a suspen-

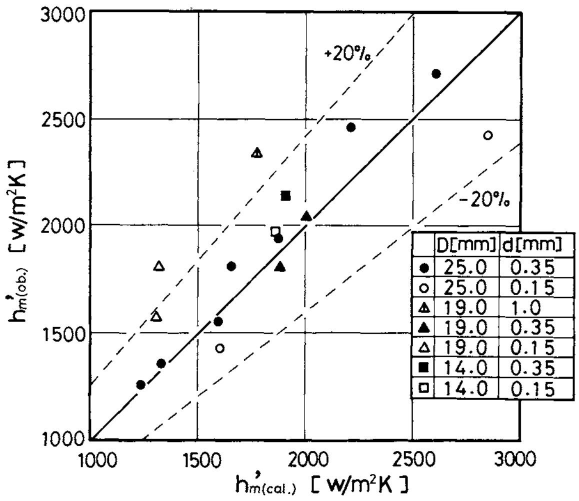

Fig. 10. Comparison of heat transfer coefficients obtained experimentally and calculated by Eq. (11) for a stationary bed flow.

sion increases, l_b becomes extinct. This indicates that the heat transfer coefficient for asymmetric suspension flow is not affected by m_s. This also agrees well with the results obtained at high Reynolds numbers, as shown in Fig. 2.

To test the validity of Eq. (11), the heat transfer coefficients of stationary bed flow are calculated by Eq. (11) using the observed values of l_m, l_b, δ, t_w, t_i and Δt_{lm}. **Figure 10** compares the calculated heat transfer coefficient h'_m with the experimental values as functions of pipe diameter and particle size. Relatively good agreement of the values is observed. Furthermore, it is found that the contribution of the second term of Eq. (11) to the overall heat transfer coefficient is not greater than 3%. Therefore, it is revealed that the suspension flow above the stationary bed strongly influences the heat transfer coefficient of stationary bed flow.

Conclusion

Experiments were conducted to clarify the heat transfer behavior of a horizontal solid-water suspension flow. The following results were obtained:

1) The heat transfer coefficient of an asymmetric suspension flow is always higher than that of a simple water flow. The empirical equation (3) is proposed to predict the heat transfer coefficient of an asymmetric suspension flow. It is also found that the heat transfer coefficient for stationary bed flow is expressed by Eq. (11).

2) The temperature profiles in the vertical radial direction in the outlet of the test section are very different from those of a simple water flow and are asymmetric with respect to the pipe axis except at high fluid velocity and small volume fraction of the solid. The temperature profiles in the horizontal radial direction are found to be similar to those of a simple water flow.

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Nomenclature

A	= cross-sectional area of pipe	[m²]
C	= specific heat	[kJ/(kg·K)]
D	= pipe diameter	[m]
d	= particle diameter	[m]
h	= heat transfer coefficient	[W/(m²·K)]
$h'_{m(ob)}$	= experimental value by Eq. (1)	[W/(m²·K)]
$h'_{m(cal)}$	= calculated value by Eq. (11)	[W/(m²·k)]
k	= thermal conductivity	[W/(m·k)]
L	= length of test section	[m]
l	= wetted perimeter	[m]
m_s	= delivered volume fraction of solid	[—]
Nu	= Nusselt number (hD/k_f)	[—]
Pr	= Prandtl number ($C_m\mu_f/k_f$)	[—]
Q	= rate of heat flow	[W]
Re	= Reynolds number ($DU_m\rho_m/\mu_f$)	[—]
r	= radial distance	[m]
r_o	= inner radius of pipe	[m]
r_a	= outer radius of pipe	[m]
r_{av}	= logarithmic mean radius of pipe $\{(r_a-r_o)/\ln(r_a/r_o)\}$	[m]
t	= temperature	[K]
t_i	= inlet bulk temperature	[K]
t_o	= outlet bulk temperature	[K]
t_s	= surface temperature of stationary bed	[K]
t_w	= wall temperature of test section	[K]
Δt_{lm}	= logarithmic mean temperature difference $\{(t_w-t_i)-(t_w-t_o)\}/[\ln\{(t_w-t_i)/(t_w-t_o)\}]$	[K]
U	= mean velocity	[m/s]
W	= mass flow rate	[kg/s]
x	= horizontal distance from pipe center	[m]
y	= vertical distance from pipe center	[m]
z	= distance from inlet of test section	[m]
θ	= angle	[rad]
μ	= viscosity	[Pa·s]
μ_w	= viscosity evaluated at t_w	[Pa·s]
ρ	= density	[kg/m³]
δ	= thickness of stationary bed	[m]
ε	= porosity of stationary bed	[—]
ψ	= constant $f(\varepsilon, k_f/k_s)$	[—]

⟨Subscripts⟩

b	= stationary bed
f	= fluid
s	= solid particel
m	= solid-water suspension

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DIFFUSION STUDIES IN INORGANIC SOLID–SOLID SYSTEM

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Diffusivity measurements between two inorganic solids, zinc oxide and zinc aluminate, in the absence of reaction was undertaken with particle size, compaction pressure, diffusion temperature and diffusion time as variables. The present work was mainly concerned with the experimental techniques of measurement and the establishment of the concentration profile for diffusion of zinc oxide into the zinc aluminate through which fundamental as well as concentration and temperature diffusivities were determined. Activation energies required for fundamental as well as concentration and temperature diffusion were calculated and reported. Generalized correlations were also established.

Introduction

Zinc aluminate is formed between the reactants zinc oxide and aluminum oxide by solid-state reaction. Diffusion of each of the reactants is essential for the solid-state reaction to proceed. It is in this context that information on diffusivities and their measurement are considered necessary.

The mechanism of diffusion in solid–solid systems can be classified into three categories:

(1) Volume or bulk diffusion within the individual particles;

(2) boundary diffusion at particle–particle interface, or along the grain boundaries in polycrystalline substances, or at the interface between two dissimilar pellets; and

(3) pore surface diffusion along the surface of the particles.

An attempt is made in the present study to indicate the factors governing the relative importance of various mechanisms of diffusion.

The experimental study in this part refers to the measurement of diffusivities in single cylindrical pellets when the pellets of either of the reactants and product are kept with their end faces in contact with each other. This procedure is chosen so as to avoid the consideration of change in the interfacial area. In this way, it is felt easier to correlate the experimental data by a suitable theory and to make successful predictions.

Very few studies are available in the literature regarding diffusion mechanisms in solid–solid systems.[1–5] The work on inorganic systems mainly consists of studies on oxides and ionic crystals.[2,6–22]

Among the diffusion studies in organic systems, one recently reported was on the diffusion of phthalic anhydride in a phthalyl derivative of sulphathiazole by Arrowsmith and Smith.[23] They report pore surface diffusion.

The present studies on diffusion are confined to the regular geometry of the interfacial area based on single cylindrical pellets. Each of the reactant pellets is kept in contact with pellets of product. Diffusivities are obtained as functions of particle size, compaction pressures, diffusion temperature and concentration of

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