EFFECT OF FEEDING AND/OR OUTLET POSITION OF FLOW IN CONTINUOUS STIRRED-TANK REACTOR

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Key Words: Mixing, Stirred Tank Reactor, Residence Time Distribution, Circulation Time Distribution, Feed Position, Outlet Position, Discharge Flow Rate, Flow Distribution, Flow Pattern, Circulation Flow Rate

The flow patterns in continuous stirred-tank reactors of three types, i.e., of three different feed and outlet port configurations, were observed under conditions where the feed rate was less than 10% of the discharge flow rate from impeller by the particle follower technique, and these patterns were compared with those in the batch stirred tank. The feed flow affects the flow pattern near the feeding port, up- and down-flow along the wall opposite the feeding port, and the flow rotating along the wall in the reverse direction to that of impeller rotation. Since the local flow patterns induced by the feed flow were not appreciable throughout the whole volume of the tank, they did not affect remarkably the residence and circulation time densities. The variation coefficient of the circulation time density function had a negligible effect on the residence time density function.

Introduction

Many studies have been made on the flow characteristics of the stirred-tank reactor (s.t.r.).[2] Most studies on continuous s.t.r. have been primarily concerned with the relationship between the residence time distribution of fluid and the geometrical and/or operational conditions, along with the proposal of its mathematical model.[1,3-5] In research on batch s.t.r., the circulation flow characteristics of fluid from and to the impeller region such as the circulation time and circulation flow rate have been the main concern.[3]

Despite the abundance of research reports, the flow features in continuous s.t.r. such as local distribution of fluid flow have scarcely been discussed in relation to the comprehensive flow characteristics such as the residence time. However, it is pointed out that the in- and out-streams of the continuous s.t.r. give rise to considerably different performances.

The aim of the present study is, therefore, firstly to characterize experimentally the flow in continuous s.t.r. with varying feeding and outlet positions of fluid by use of the particle follower technique within the practical operational range, and secondly to discuss the flow characteristics in relation to the circulation and residence time distributions. The operational conditions studied are confined within the range of feed rate less than 10% of the discharge flow rate from impeller, since the most practical continuous-flow s.t.r.s are operated within this range.

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1. Experimental

1.1 Experimental apparatus and experimental conditions

An acrylic cylindrical baffled tank of 28 cm inside diameter and 30 cm height, fitted with a flat-blade impeller of 10 cm diameter and 3.8 cm blade width, was used. The liquid volume in the tank was $1.85 \times 10^{-2}\ \mathrm{m^3}$. Three different geometrical configurations of feeding and outlet port were used, expressed as T-, C- and Z-type respectively as shown in **Fig. 1**.

Water at room temperature was used for the fluid. The feed rate of water and the rotational speed of impeller were varied within the range of 0–$8.0 \times 10^{-5}\ \mathrm{m^3/s}$ and 1.0–2.5 $\mathrm{s^{-1}}$, respectively. The flow in the tank was in the turbulent region ($Re = 1.0 \times 10^4$–2.5×10^4). Experimental conditions are summarized in **Table 1**.

1.2 Measurement method

The experiments were conducted in the stationary state, that is, measurement was carried out sufficiently after the start of s.t.r. operation.

The flow distribution and residence time of fluid were measured by a particle-follower method using a 1.2 mm-diameter polystyrene particle, the density of which was adjusted to that of water. The particle was assumed to behave in the same manner as the fluid.

Flow distribution To characterize the flow the tank was divided into nine conceptual compartments, with the impeller region called I, and each compartment was numbered as shown in **Fig. 2**. One tracer polys-

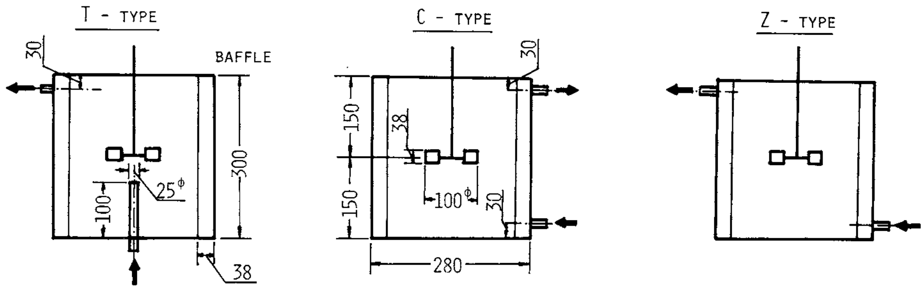

Fig. 1. Experimental apparatus (size in mm).

Table 1. Experimental conditions

Q_F [m³/s]	N [s⁻¹] 1.0	1.7	2.5
	Re [—] 1.0×10^4	1.7×10^4	2.5×10^4
0	Batch	Batch	Batch
1.7×10^{-5}	T, C, Z	Z	Z
5.3×10^{-5}	T, C, Z	Z	Z
8.0×10^{-5}	T, C, Z	Z	Z

T = experiment with T-type tank
C = experiment with C-type tank
Z = experiment with Z-type tank

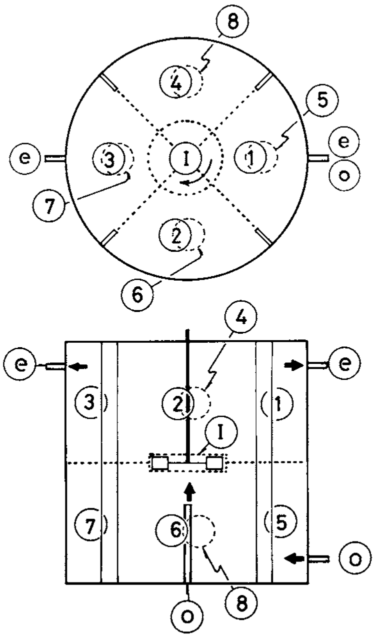

Fig. 2. Compartmentalization of tank.

tyrene particle was injected just inside the feeding port and its trajectory was followed by eye and timed by watch, and read onto a voice recorder. The same procedure was repeated sufficiently many times, e.g., at least 1000 times. Some of the particles thus observed passed through all the compartments and others not through all before flowing out through the outlet. In this range of experiment no difficulty was experienced in visualization by means of ordinary lighting procedure.

The particles which passed through compartment j from compartments i to k were considered. Let the component of flow rate between a pair of compartments i and k via compartment j be called q_{ijk}. Since the flow in each compartment shall not be discussed, each compartment is considered as a black box. Therefore, the following relationship is assumed.

$$q_{ijk} = a \cdot n_{ijk} \tag{1}$$

where n_{ijk} is the number of observation that the particle flowed from compartment i to k through j. The constant a is inversely proportional to the observation time and the number of observation. Let the compartment $j=3$ be the one which has the exit port; then the suffix $k=e$ means the outlet. For this case the following relationship is written and the constant a is determined by this equation.

$$Q_F \equiv \sum_{i=1}^{8} q_{i3e} + q_{I3e} = a\left(\sum_{i=1}^{8} n_{i3e} + n_{I3e}\right) \tag{2}$$

where Q_F means the water feed rate since water is incompressible and the operation is at the stationary state.

Discharge flow rate The flow rates from the impeller region I to compartment j, and from compartment j to the impeller region are expressed as

$$\sum_{k=1}^{8} q_{Ijk} + q_{Ije} \quad \text{and} \quad \sum_{i=1}^{8} q_{ijI} + q_{ojI},$$

respectively. The discharge flow rate from impeller, Q_D, is obtained by summing up all component flow rates from the compartments adjacent to the impeller region as

$$Q_D = \sum_{\substack{k,j=1\\k\neq j}}^{8} q_{Ijk} + \sum_{j=1}^{8} q_{Ije} = \sum_{\substack{i,j=1\\i\neq j}}^{8} q_{ijI} + \sum_{j=1}^{8} q_{ojI}. \tag{3}$$

Exchanging flow rate between the upper and lower regions of tank The flow rate which passes through a conceptual horizontal plane across the impeller, Q_B, is obtained by

$$
\begin{aligned}
Q_B &= \sum_{i=1}^{4}\sum_{j=5}^{8}\left(\sum_{k=1}^{8} q_{ijk}+q_{ijI}+q_{ije}\right)+\sum_{i=1}^{4}\sum_{k=5}^{8} q_{iIk} \\
&= \sum_{i=5}^{8}\sum_{j=1}^{4}\left(\sum_{k=1}^{8} q_{ijk}+q_{ijI}+q_{ije}\right)+\sum_{i=5}^{8}\sum_{k=1}^{4} q_{iIk}-Q_F .
\end{aligned} \tag{4}
$$

Circulation flow rate One circulation flow is defined as the flow with which a tracer particle is discharged from the impeller region and returned to the impeller region for the first time. The circulation time is defined by the time that a particle experiences in one circulation flow. Let the observation number of particles which have circulation time between t_c and $t_c+\Delta t$ be m_c; then the circulation time density is expressed as

$$
g(t_c)=\frac{m_c}{M_c}\frac{1}{\Delta t} \tag{5}
$$

where M_c is the total observation number of particles which passed through the impeller region.

Residence time density The residence time density of fluid is defined as the time that a particle injected into the tank from feeding port resides within the tank before flowing out through the outlet. Let the observation number of the particle which had residence time between t_r and $t_r+\Delta t$ be m_r; then the residence time density function is expressed by

$$
g(t_r)=\frac{m_r}{M_r}\frac{1}{\Delta t} \tag{6}
$$

where M_r means the total observation number.

2. Results and Discussion

2.1 Distribution of flow rate

Figures 3(a)–(d) show examples of flow rate matrices, the component of which,

$$
Q_{jk}\left(=\sum_{i=1}^{8} q_{ijk}+q_{ojk}\right),
$$

is the flow rates from compartment j to k for batch and continuous flow C-, Z- and T-type s.t.r., respectively. The size of the circles in the figure represents the magnitude of flow rate. The circles on the matrix elements "from o to 5" (Fig. 3(b) and (c)) and "from o to I" (Fig. 3(d)) represent the inflow, and "from 1 to e" (Fig. 3(b)) and "from 3 to e" (Fig. 3(c) and (d)) represent the out-flow. The flow in batch tank (Fig. 3(a)) is characterized by the following four flows:

i) flow to impeller region from each compartment (S)

ii) discharge flow to each compartment from impeller region (D)

iii) flow along tank wall, i.e., primary flow (P_u, P_l)

iv) exchanging flow between a pair of compartments, adjacent to each other across the horizontal plane through the impeller region (B_{ul}, B_{lu})

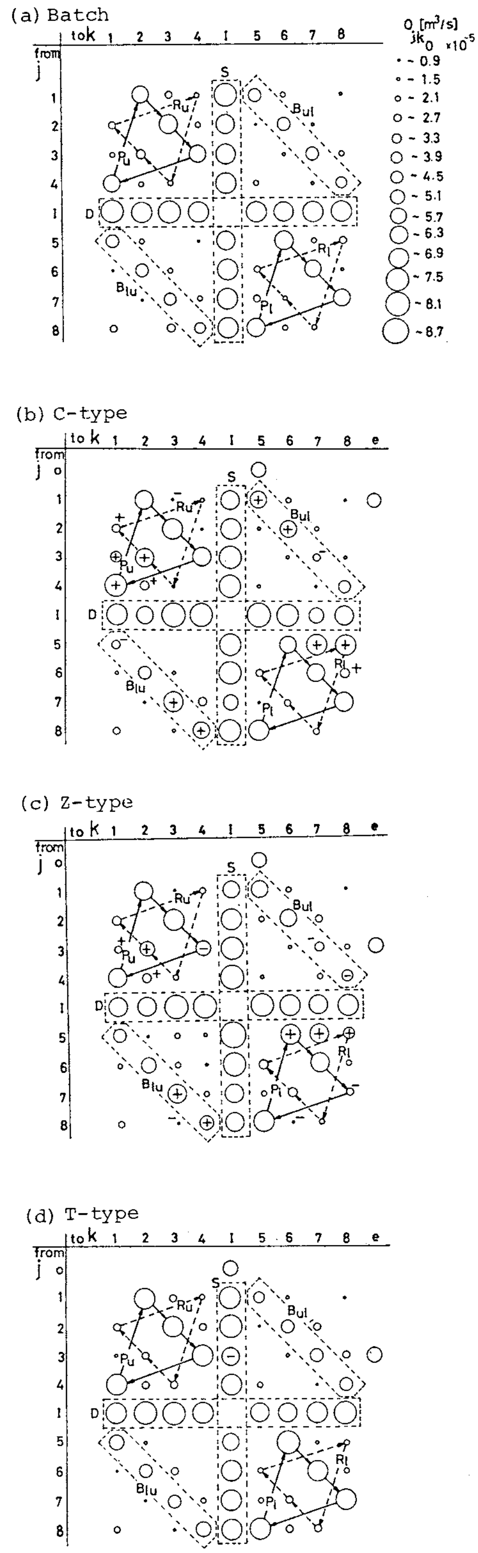

Fig. 3. Flow Distribution in tank. ($N=1.0\,\mathrm{s}^{-1}$; $Q_F=8.0\times10^{-5}$ m/s: The symbols "+" and "−" show distinctive increase and decrease respectively in flow rate compared with batch tank.)

Symbols in parentheses stand for the names of regions shown in the figures. All these flows in the batch tank are geometrically symmetrical with respect to the impeller region.

The flow pattern in C-type stirred tank (Fig. 3(b)) was affected extremely by the injection flow from the feeding port and showed an asymmetrical flow distribution. The symbols + and − in Figs. 3(b) and (c) show, respectively, the distinctive increase and decrease in flow rate compared with that of batch operation. Although not so extreme a difference was observed in the primary flow (P_u, P_1), a remarkable difference was observed in the exchanging flow rate (B_{u1}, B_{1u}) and the flow rate directed reversely to the primary flow (R_u, R_1). In other words, in the C-type tank the following two flows were induced.

$$\text{c1)}\quad \begin{tikzcd} & & 7 \arrow[r] & 3 \arrow[r] & 2 \arrow[dr] & \\ o \arrow[r] & 5 \arrow[ur] \arrow[r] & 8 \arrow[rr] & & 4 \arrow[r] \arrow[lll, bend left] & 1 \end{tikzcd}$$

$$\text{c2)}\quad \begin{tikzcd} 4 \arrow[r] & 2 \arrow[r] & 6 \arrow[r] & 8 \arrow[lll, bend left] \end{tikzcd}$$

Flow c1) was induced directly by the radial momentum increase of the injection flow from the feeding port. The flow was asymmetrical with respect to the conceptual plane which included the feeding port, outlet and the impeller axis. This was due to flow c2), which was directed perpendicularly to the conceptual plane.

In the Z-type tank (Fig. 3(c)) the flow rates from compartment 7 to 8 and from compartment 3 to 4 were less than those obtained in the C-type tank, and the third flow Z1) became dominant.

$$\text{Z1)}\quad o \longrightarrow 5 \longrightarrow 7 \longrightarrow 3$$

The difference in the flow distribution between C- and Z-type tanks was due to the difference in the positions of the outlet. In the Z-type tank the flow c2) was not explicitly discerned.

The flow pattern in the T-type tank (Fig. 3(d)) was very similar to that of the batch tank except that a slight decrease was observed in the rate of flow to the impeller region from compartment 3, to which the exit port was connected. Almost all the injected fluid from the feeding port in the T-type tank was taken into the impeller region just after injection, and its effect on the discharge flow rate was negligibly small since the feed rate was not so large compared with the discharge flow rate from the impeller under the present experimental condition (less than 10%).

The special flow patterns c1)–Z1) were the effect of excess radial momentum introduced by the feeding flow rate. The other flow patterns were scarcely affected by the feeding rate. It is easily conjectured that the effect of the excess radial momentum introduced by the feeding flow on the local flow pattern will be decreased if the discharge flow rate from the impeller becomes greater.

Figure 4 shows examples of the influence of impeller speed on flow distribution in the Z-type tank. As seen from the figure the effect of the inflow was almost diminished when the feed rate is less than 5% of the discharge flow rate.

The above results indicate that under the conditions examined in this work the flow pattern observed in batch experiments can be realized in a continuous flow s.t.r. irrespective of impeller speed by using a T-type tank, and independent of tank geometry at feed rate less than 5% (at most) of the discharge flow rate from the impeller.

Figure 5 shows an example of impeller discharge flow rate Q_D in relation to the net exchanging flow rate Q_B defined by Eq. (4). The discharge rate number ($=Q_D/N\cdot d^3$) was about 0.75, independently of flow rate. Although the effect of feed rate on local flow patterns was observed in the C- and Z-type tanks as seen in Fig. 3, the effect of feed rate on the exchanging flow rate was scarcely observed. The discharge flow rate was practically unaffected by the feeding flow rate in the three types of tanks examined.

2.2 Residence and circulation time densities

Figure 6 shows an example of the residence time density function (r.t.d.f.) and circulation time density function (c.t.d.f.) under the same experimental condition, under which the mean circulation time is very small, i.e., less than one percent at most of the mean residence time.

The normalized c.t.d.f. and r.t.d.f. with respect to mean circulation and mean residence time, respectively, were considered independent of both feed rate and impeller speed. This was confirmed by the fact that the coefficients of variation of both density functions $\sigma/\bar{t}$ were independent of the experimental conditions as shown in **Fig. 7**.

For further understanding of the relation between the r.t.d.f. and the c.t.d.f., let the following four flow elements be considered.

a) direct flow from inlet to impeller region (oI)

b) one circulation flow, i.e., flow from impeller region to the impeller region for the first time (II)

c) direct flow from impeller to outlet (Ie)

d) direct flow from inlet to outlet without passing through the impeller region (oe)

Let the passage time density functions for each flow from a) to d) by $f(t)$, $c(t)$, $h(t)$ and $k(t)$, respectively. The r.t.d.f., $r(t)$, is represented by these passage time density functions as

$$r(t)=\alpha_{oe}k(t)+\alpha_{oI}f(t)*\left\{\beta_{Ie}+\beta_{II}\gamma_{Ie}c(t) * \sum_{l=0}^{\infty}(\gamma_{II}c(t))^{*l}\right\}*h(t) \tag{7}$$

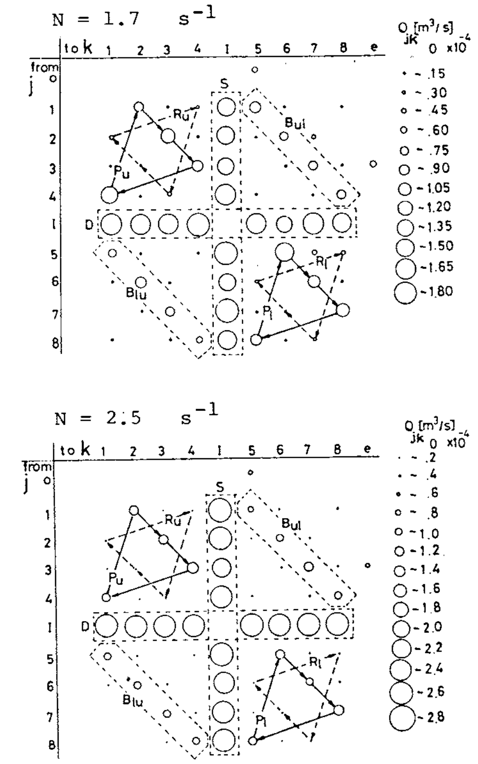

Fig. 4. Effect of impeller speed on flow distribution of Z-type tank.

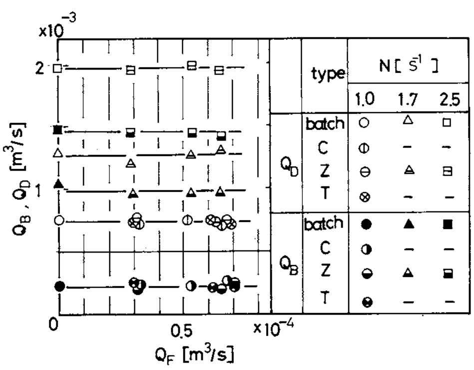

Fig. 5. Discharge flow rate from impeller Q_D and exchanging flow rate Q_B.

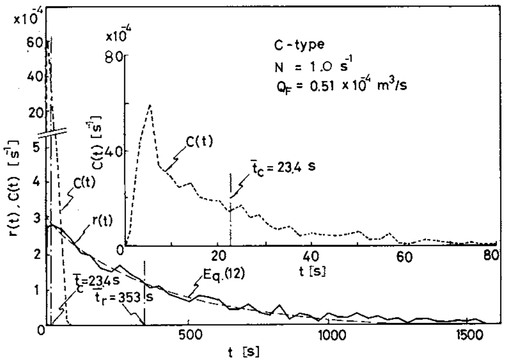

Fig. 6. An example of residence time density and circulation time density.

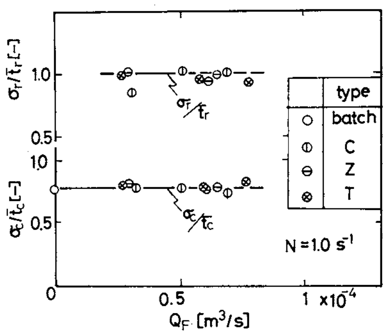

Fig. 7. Relative standard deviation of residence time density and circulation time density.

Where α_{oI} $(=1-\alpha_{oe})$ and α_{oe} are the fractions of feed flow which passed through the impeller region and that which did not pass through the impeller region before flowing out of the tank, respectively; β_{Ie} and β_{II} $(=1-\beta_{Ie})$ are the fractions of flow which were distributed from flow a) onto the flow b) and c), respectively; γ_{Ie} and γ_{II} $(=1-\gamma_{Ie})$ are the fractions of flow which were distributed from circulation flow b) onto flow c) and circulation flow b) again, respectively; $*$ and $*l$ are the convolution and l-times convolution integrals, respectively.

The fractions α and β were obtained by the number ratio of particles which flowed with each fluid flow. **Figure 8** shows the measured ratios α and β. The result shows that the ratios are constant, independently of the conditions, as

$$\left.\begin{array}{ll}\alpha_{oe}\doteqdot 0, & \alpha_{oI}\doteqdot 1.0\\ \beta_{Ie}\doteqdot 0, & \beta_{II}\doteqdot 1.0\end{array}\right\} \tag{8}$$

Under the conditions of Eq. (8), the mean residence time $\bar{t}_r$ and the variance of the r.t.d.f. σ_r^2 are obtained respectively as

$$\bar{t}_r=\alpha_{oe}\bar{t}_{oe}+\alpha_{oI}(\bar{t}_{oI}+\bar{t}_{Ie})+\frac{\beta_{II}}{\gamma_{Ie}}\alpha_{oI}\bar{t}_c$$

$$\doteqdot\frac{1}{\gamma_{Ie}}\bar{t}_c \tag{9}$$

$$\sigma_r{}^2=\overline{t_r{}^2}-\bar{t}_r{}^2\doteqdot\bar{t}_r{}^2 \tag{10}$$

From Eq. (9) the ratio of the mean circulation time to the mean residence time is obtained as

$$\frac{\bar{t}_c}{\bar{t}_r}\doteqdot\gamma_{Ie}=\frac{\text{feed flow rate}}{\text{circulation flow rate}}$$

$$=\frac{Q_F}{Q_D-Q_F} \tag{11}$$

Figure 9 shows the relationship between $\bar{t}_c/\bar{t}_r$ and γ_{Ie}, and shows that Eq. (11) represents the relation satisfactorily. From Fig. 7 it is seen that Eq. (10), i.e., $\sigma_r^2/\bar{t}_r^2\doteqdot 1$, holds under the conditions examined. These results suggest that Eq. (7) is reduced to

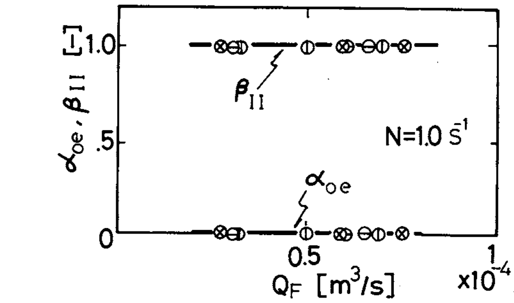

Fig. 8. Fraction of flow rate. (Keys are the same as in Fig. 7.)

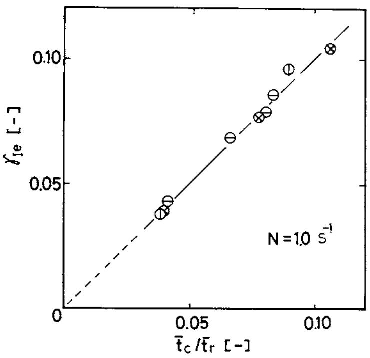

Fig. 9. Relationship between γ_{Ie} and $\bar{t}_c/\bar{t}_r$. (Keys are the same as in Fig. 7.)

$$r(t) \doteqdot \frac{1}{\bar{t}_r} e^{-(t/\bar{t}_r)} \tag{12}$$

under the conditions of Eq. (8), and the time constant $\bar{t}_r$ is represented in terms of the mean circulation time $\bar{t}$ as

$$\bar{t}_r = \frac{\bar{t}}{\gamma_{Ie}}.$$

The broken line in Fig. 6 represents the calculated relation of Eq. (12).

The most interesting results of the feed effect on the residence time is that the variance of the r.t.d.f. was unaffected by the c.t.d.f. under the condition of $Q_F < 0.1 Q_D$.

This result is closely related to the fact that, although the effect of the feed flow on the flow features was evident when the observation scale was 1/8 tank volume, it became undiscernible when the scale became larger i.e., half tank volume.

Figure 10 shows, as a function of the feed rate, the apparent volume V_{nc} defined by Eq. (13).

$$V_{nc} = V - (Q_D - Q_F)\bar{t}_c \tag{13}$$

where V is the total liquid volume in the tank. The apparent volume does not contribute to the circulation flow.

Since the vessel examined was considered to have no dead volume, the apparent volume V_{nc} is expressed

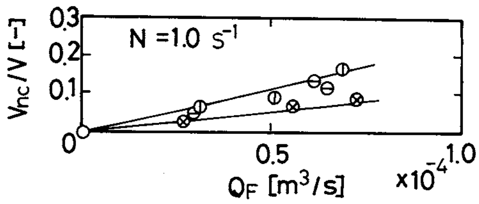

Fig. 10. Relationship between V_{nc} and Q_F. (Keys are the same as in Fig. 7.)

as

$$V_{nc} = q_{oI}\bar{t}_{oI} + q_{oe}\bar{t}_{oe} + q_{Ie}\bar{t}_{Ie}\,. \tag{14}$$

Figure 10 shows that the apparent volumes of C- and Z-type tanks were about twice that of the T-type tank. This was due to the effect of inflow and the symmetrical flow pattern of flow pathway with respect to the impeller region since the flow rate q_{oe} was considered to be negligibly small.

The results obtained in this work suggest that when any rate process having a constant comparable for the time scale of fluid motion in volume (e.g., 1/8 volume of the tank in this experiment) is considered, a model representing the local flow pattern is needed.

Conclusion

The flow patterns in continuous stirred-tank reactors having three types of feed and outlet port configurations were observed under conditions where the feed rate was less than 10% of the discharge flow rate from impeller by the particle follower technique and these patterns were compared with those in the batch stirred tank.

The following points were found:

1) In the C- and Z-type tanks the effect of the feeding flow on the flow pattern was observed near the feeding port and along the wall opposite the feeding port.

2) In the C-type tank, in addition to the effect stated in (1), induced flow rotating along the wall inversely to the direction of impeller rotation was observed.

3) In the T-type tank no remarkable effect was observed, and therefore the flow pattern was very similar to that of a batch tank.

4) The local flow patterns stated in (1) and (2) were not experienced appreciable throughout the whole volume of every type tank, and therefore the variance of the residence time density function was unaffected by the circulation time density function under the experimental conditions.

Acknowledgment

We are thankful to Mr. Yuzo Shimizu, now with Shiseido Co., who conducted the experimental work.

Nomenclature

a	= coefficient of flow distribution	[m^3/s]
$c(t)$	= circulation time density function	[s^{-1}]
d	= impeller diameter	[m]
$f(t)$	= passage time density function	[s^{-1}]
$h(t)$	= passage time density function	[s^{-1}]
$k(t)$	= passage time density function	[s^{-1}]
M	= total observation number	[—]
m	= observation number	[—]
N	= impeller speed	[s^{-1}]
n	= observation number	[—]
Q_B	= exchanging flow rate	[m^3/s]
Q_D	= discharge flow rate	[m^3/s]
Q_F	= feed rate	[m^3/s]
Q_{ij}	= flow rate from compartment i to j	[m^3/s]
q	= flow component	[m^3/s]
Re	= impeller Reynolds number	[—]
$r(t)$	= residence time density function	[s^{-1}]
t	= time	[s]
$\bar{t}$	= mean time	[s]
Δt	= time interval	[s]
V	= liquid volume	[m^3]
V_{nc}	= apparent volume independent of circulation	[m^3]
α	= fraction of flow rate	[—]
β	= fraction of flow rate	[—]
γ	= fraction of flow rate	[—]
σ	= standard deviation of time distribution	[s]

⟨Subscripts⟩

c	= circulation
e	= exit
i, j, k	= compartments i, j and k
I	= impeller region
o	= inlet
r	= residence

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AXIAL DISPERSION OF LIQUID IN LIQUID FLUIDIZED BEDS IN THE LOW REYNOLDS NUMBER REGION

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Key Words: Fluidization, Liquid Fluidized Bed, Fluid Mixing, Axial Dispersion Coefficient, Energy Dissipation Rate, Particle, Void Fraction

Axial dispersion coefficients of liquid were measured for liquid fluidized beds in the low Reynolds number region by the residence–time–curve method. Particles used were polystyrene and glass beads of diameter from 239 to 1887 μm. The experimental results were successfully correlated by the following empirical equation:

$$E_z/\nu = 500\varepsilon^{0.43}\exp\{-20.5(0.75-e)^2\}$$

$$0.41 < e < 0.93\,, \qquad 0.15 < Re_0 < 100\,, \qquad 8\times10^{-5} < \varepsilon < 0.44$$

where E_z is the axial dispersion coefficient, ν the kinematic viscosity of liquid, ε the energy dissipation rate [m^2/s^3], e the void fraction and Re_0 the particle Reynolds number based on superficial velocity. Most of the published correlations estimate too large values of the axial dispersion coefficient over the low Reynolds number region except in the case of glass beads of small diameter.

Introduction

Liquid fluidized beds have been applied not only to conventional operations such as ion exchange and crystallization, but also to special fields such as bioreaction processing.[3] Much work[1,2,8,11,16] has been reported on the axial dispersion coefficient of liquid in liquid fluidized beds, information about which is fundamental to the design of the equipment. Typical investigations recently reported are those of Wen *et al.*[2,17–19] and Shemilt *et al.*[7–9] Chung and

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