

Fig. 3. Change of energy spectrum function  $E(k, t)$  with time.

dimensional analysis, are comparatively similar in performance to the empirical equation.

In conclusion, it is necessary to decide suitably uncertain constants  $\gamma$  for application of the previous hypotheses to an explanation of the turbulent energy transfer mechanism. The hypotheses still involve a practical problem, since they have not clarified the procedure for deciding the constants  $\gamma$ . In that respect, the empirical equation poses no problem.

#### Nomenclature

$E(k, t)$  = three-dimensional energy spectrum function [ $\text{cm}^3 \cdot \text{s}^{-2}$ ]

|                        |  |                                       |
|------------------------|--|---------------------------------------|
| $I_\lambda$            | = $(1/4\nu)(d\lambda_f^2/dt)$ , parameter representing the state of turbulence decay | [—]                                   |
| $k(r, t)$              | = triple velocity-correlation function   | [—]                                   |
| $M$                    | = mesh length of grid  | [cm]                                  |
| $R_M$                  | = $\bar{U}M/\nu$ , Reynolds number based on mesh length of grid                      | [—]                                   |
| $R_\lambda$            | = $u'\lambda_f/\nu$ , turbulent Reynolds number                                      | [—]                                   |
| $r$                    | = distance between two points  | [cm]                                  |
| $T(k, t)$              | = energy transfer function   | [ $\text{cm}^3 \cdot \text{s}^{-3}$ ] |
| $t$                    | = time   |                                       |
| $\bar{U}$              | = time-mean velocity of main flow  | [ $\text{cm} \cdot \text{s}^{-1}$ ]   |
| $u'$                   | = root-mean-square fluctuation velocity  | [ $\text{cm} \cdot \text{s}^{-1}$ ]   |
| $W(k, t)$              | = energy transfer function in integral form  | [ $\text{cm}^2 \cdot \text{s}^{-3}$ ] |
| $x$                    | = distance from turbulence-generating grid   | [cm]                                  |
| $\gamma$               | = constant used in energy transfer hypothesis  | [—]                                   |
| $\epsilon$             | = energy dissipation by turbulence per unit of mass                                  | [ $\text{cm}^2 \cdot \text{s}^{-3}$ ] |
| $\kappa$               | = wavenumber   | [ $\text{cm}^{-1}$ ]                  |
| $\lambda_f$            | = longitudinal integral scale of turbulence  | [cm]                                  |
| $\lambda_f, \lambda_g$ | = longitudinal and lateral microscale of turbulence                                  | [cm]                                  |
| $\nu$                  | = kinematic viscosity of fluid   | [ $\text{cm}^2 \cdot \text{s}^{-1}$ ] |
| $\psi$                 | = $r/\lambda_f$ , normalized distance between two points                             | [—]                                   |

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(Presented at the 45th Annual Meeting of The Society of Chemical Engineers, Japan, at Osaka, April, 1980.)

## TRANSITION FROM FROTH TO SPRAY ON PERFORATED PLATES

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**Key Words:** Distillation, Fluid Mechanics, Perforated Plate, Phase Inversion, Froth, Spray

The regimes of froth and spray are mostly found in industrial perforated plates having relatively large free areas. As spray dynamics and mass transfer are

significantly different in these regimes,<sup>1,3-5)</sup> attention in recent investigations of perforated plate performance has been drawn to the froth-to-spray transition.<sup>2,8,12,13)</sup> Recently, it has also been reported that the spray regime represents a stable regime of oper-

Received June 2, 1983. Correspondence concerning this article should be addressed to T. Miyahara.

ation at high gas velocity.<sup>6)</sup> Though Lockett<sup>7)</sup> has tried to correlate the previous experimental results of froth-to-spray transition, he has failed to predict the dependence of the transition on physical properties of the liquid.

In the present study, correlations of the froth-to-spray transitions are obtained by using the previous transition results for single orifice,<sup>9)</sup> and the difference between bubbling-to-jetting transition at a single orifice and froth-to-spray transition on a plate was studied experimentally by increasing the number of holes.

## 1. Experimental Apparatus and Procedure

The experimental apparatus and the procedure used are the same as those described in a previous paper,<sup>9)</sup> with orifice diameters extending over the range 0.204–0.603 cm, the number of holes of 2–7 and pitch of 0.332–1.475 cm. All of the orifices used were made of brass plate. Geometries of these orifices are listed in Table 1. Water was used as liquid.

## 2. Results and Discussion

Figure 1 shows photographs taken in the case of the orifice having two holes. Obviously, we can find meniscus, steady jet, pulsating jet and transient jet expected for a single orifice.<sup>9)</sup> The transient jet is similar to the pulsating jet whose diameter varies from the orifice diameter to a few times that length, whereas it occasionally breaks, the discontinuity of the jet being noticed. As shown in Fig. 1(A), two jets issuing from two holes do not contact each other at large  $P/D$ . However, the interaction between them is noticed by visual observation. On the other hand, Fig. 1(B) is for  $P/D$  of 1.74, the value of which is smaller than that in Fig. 1(A). Two jets contact, then coalesce and behave like a jet issuing from a single hole. Therefore, the interaction is very violent.

Transition points were determined here by using the change of free entrainment rate and finding the phenomena shown in Fig. 1. The free entrainment was determined by the rate of liquid supply equal to the entrainment at steady state. These have been used previously by the present authors<sup>9)</sup> to characterize the single-orifice bubbling-to-jetting transition.

In a previous paper,<sup>9)</sup> the transient to pulsating and the pulsating-to-steady jetting transition were correlated well with the liquid depth to hole diameter ratio  $H/D$ , the modified Froude number  $Fr^*$  and the Bond number  $Bo$  proposed by Payne *et al.*<sup>10)</sup>

The transitions from pulsating to steady jetting for multiple orifices listed in Table 1 were in good agreement with the experimental correlation for single orifice.<sup>9)</sup>

In operation of industrial perforated plates, transient jetting is presumed to take place at random in

Table 1. Geometries of orifices

| Orifice number | Diameter $D$ [m]      | Pitch $P$ [m]          | $P/D$ [—] | Thickness [m]         | Hole number |
|----------------|-----------------------|------------------------|-----------|-----------------------|-------------|
| 1              | $3.16 \times 10^{-3}$ | $3.40 \times 10^{-3}$  | 1.08      | $0.95 \times 10^{-3}$ | 2           |
| 2              | $3.03 \times 10^{-3}$ | $4.63 \times 10^{-3}$  | 1.53      | $0.80 \times 10^{-3}$ | 2           |
| 3              | $3.00 \times 10^{-3}$ | $5.23 \times 10^{-3}$  | 1.74      | $1.00 \times 10^{-3}$ | 2           |
| 4              | $3.12 \times 10^{-3}$ | $6.87 \times 10^{-3}$  | 2.20      | $0.95 \times 10^{-3}$ | 2           |
| 5              | $3.11 \times 10^{-3}$ | $9.05 \times 10^{-3}$  | 2.91      | $0.95 \times 10^{-3}$ | 2           |
| 6              | $3.14 \times 10^{-3}$ | $11.03 \times 10^{-3}$ | 3.51      | $0.95 \times 10^{-3}$ | 2           |
| 7              | $3.11 \times 10^{-3}$ | $12.94 \times 10^{-3}$ | 4.16      | $0.90 \times 10^{-3}$ | 2           |
| 8              | $3.07 \times 10^{-3}$ | $14.75 \times 10^{-3}$ | 4.80      | $0.90 \times 10^{-3}$ | 2           |
| 9              | $6.03 \times 10^{-3}$ | $6.96 \times 10^{-3}$  | 1.15      | $0.70 \times 10^{-3}$ | 2           |
| 10             | $6.03 \times 10^{-3}$ | $13.01 \times 10^{-3}$ | 2.16      | $1.00 \times 10^{-3}$ | 2           |
| 11             | $3.20 \times 10^{-3}$ | $5.36 \times 10^{-3}$  | 1.68      | $1.00 \times 10^{-3}$ | 3           |
| 12             | $3.09 \times 10^{-3}$ | $8.86 \times 10^{-3}$  | 2.87      | $1.00 \times 10^{-3}$ | 3           |
| 13             | $3.10 \times 10^{-3}$ | $12.95 \times 10^{-3}$ | 4.18      | $1.00 \times 10^{-3}$ | 3           |
| 14             | $2.00 \times 10^{-3}$ | $3.32 \times 10^{-3}$  | 1.66      | $0.95 \times 10^{-3}$ | 7           |
| 15             | $2.01 \times 10^{-3}$ | $6.00 \times 10^{-3}$  | 2.99      | $0.60 \times 10^{-3}$ | 7           |
| 16             | $2.04 \times 10^{-3}$ | $8.49 \times 10^{-3}$  | 4.16      | $0.90 \times 10^{-3}$ | 7           |
| 17             | $3.04 \times 10^{-3}$ | $4.95 \times 10^{-3}$  | 1.63      | $0.95 \times 10^{-3}$ | 7           |
| 18             | $3.04 \times 10^{-3}$ | $8.90 \times 10^{-3}$  | 2.93      | $1.00 \times 10^{-3}$ | 7           |

the froth regime because the gas velocity through a hole in the froth regime fluctuates for the pulsation of the liquid on a plate.<sup>9)</sup> Therefore, the transition from froth to spray on a perforated plate is presumed to be similar to that from transient to pulsating jetting.

Figure 2 shows the correlation of the experimental results of the transition from transient to pulsating jetting by using multiple orifices shown in Table 1 and the transition from froth to spray on a perforated plate observed by previous investigators.<sup>5,11,12,14,15)</sup> From Fig. 2, which shows the case of  $Bo > 4$ , the transition from froth to spray is correlated well with the empirical equation for single orifice expressed by the solid line.<sup>9)</sup> In the range of  $Bo < 4$  shown in Fig. 3(A), however, the dimensionless liquid depth  $H/D$  for a perforated plate is somewhat larger than that for single orifice, and the following empirical equation is obtained.

$$H/D = 0.2(Fr^*)^{1.23}(Bo)^{0.8};$$

$$1.5 < Bo < 4, \quad 1.5 < Fr^* < 10 \quad (1)$$

The dashed line is the empirical equation for single orifice presented in the previous paper.<sup>9)</sup> The difference mentioned above is presumably due to the interaction between adjacent jets as shown in Fig. 1.

The data used in Figs. 2 and 3(A) include a wide range of gas-liquid systems, only half of which are air-water, and the available data from distillation systems. The correlations correctly reflect the trends of the experimental results of individual workers.

Figure 3(B) shows the difference between single and multiple orifices by increasing the number of holes. Obviously, by the increase of pitch between holes and by the decrease of gas velocity, the experimental values of  $(H/D)/Bo^{0.8}$  are expressed by the relationship

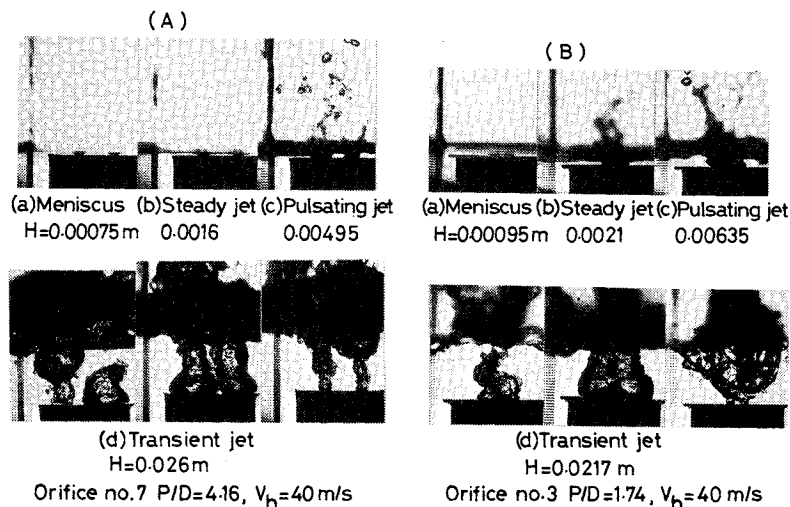


Fig. 1. Photographs of flow patterns at two holes.

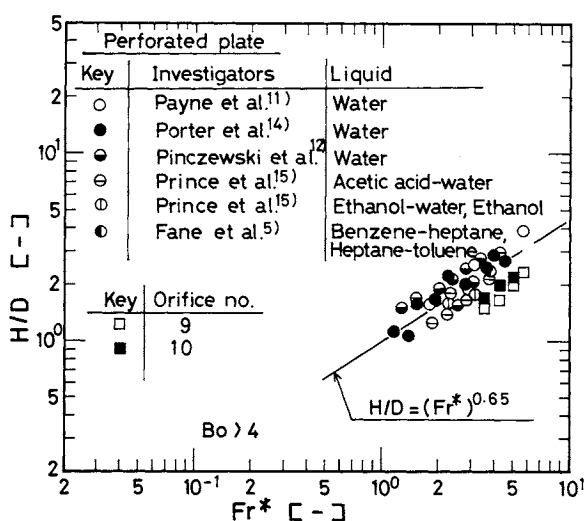


Fig. 2. Correlation of froth-to-spray transition.

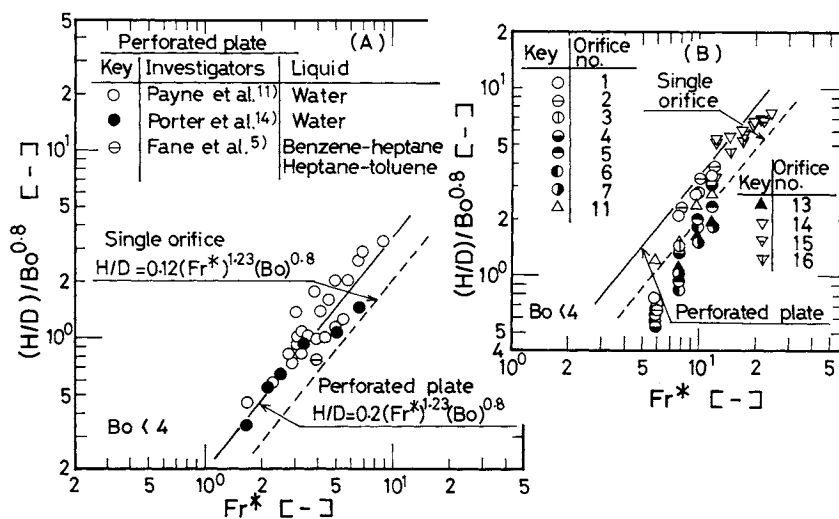


Fig. 3. Correlation of froth-to-spray transition.

for single orifice, while increasing gas velocities and decreasing the pitch give agreement of the experimental results with Eq. (1). Therefore, the results for

perforated plates in the range of  $Bo < 4$  are presumed to be correlated with Eq. (1) because of the interaction between adjacent jets.

## Acknowledgment

The authors thank Mr. Shinji Ohkura for his assistance in the experimental measurements.

## Nomenclature

|                  |   |                      |
|------------------|---|----------------------|
| $Bo$             | = Bond number $(=(\rho_l - \rho_g)gD^2/\sigma)$                                 | [—]                  |
| $D$              | = hole diameter   | [m]                  |
| $Fr^*$           | = modified Froude number<br>$(=[\rho_g V_h^2 / \{(\rho_l - \rho_g)gD\}]^{1/2})$ | [—]                  |
| $g$              | = gravitational acceleration  | [m/s <sup>2</sup> ]  |
| $H$              | = liquid depth  | [m]                  |
| $V_h$            | = gas velocity through hole   | [m/s]                |
| $\rho_g, \rho_l$ | = density of gas and liquid   | [kg/m <sup>3</sup> ] |
| $\sigma$         | = surface tension   | [N/m]                |

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(Presented at the 48th Annual Meeting of The Society of Chemical Engineers, Japan, at Kyoto, April 6, 1983.)

# EXACT SOLUTION TO NO<sub>2</sub> ABSORPTION

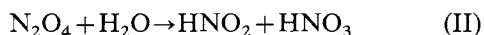
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**Key Words:** Absorption, Mass Transfer, Nitrogen Dioxide, Film Theory, Chemical Reaction

One of the most comprehensively investigated gas-liquid reaction systems is the absorption of NO<sub>2</sub> into an aqueous solution. Only in some limiting cases do analytical expressions exist to describe the mass transfer rate for absorption. The present paper describes an exact solution to the problem of absorbing NO<sub>2</sub> into an aqueous solution, in which case the NO<sub>2</sub> concentration is allowed to vary over a wide range.

Two reactions occur simultaneously in the liquid film:



From a practical point of view, kinetic studies are performed in the presence of alkali or a buffer at a pH value above 7; alkali stabilizes HNO<sub>2</sub> due to nitrite ion formation.

For further details on the problem, it is referred to a series of papers by Aoki *et al.*,<sup>1)</sup> and Komiyama and Inoue,<sup>2,3)</sup> which probably represent the best up-to-date-knowledge on the subject.

The general conservation equations for the two reactions can be represented by a single expression:

$$2D_B \frac{d^2 B}{dx^2} + D_A \frac{d^2 A}{dx^2} - 2k_{II} B = 0 \quad (1)$$

where  $A$  and  $B$  denote NO<sub>2</sub> and N<sub>2</sub>O<sub>4</sub>, respectively. Reaction (I) is instantaneous. Hence:

$$B = K_I A^2 \quad (2)$$

The following variable transformations are introduced:

$$A^* = \delta A + 2B \quad (3)$$

$$\psi = \delta A^* / dx \quad (4)$$

Received May 19, 1983. Correspondence concerning this article should be addressed to H. T. Karlsson.