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FLUID RESISTANCE ON A DISK OSCILLATING SINUSOIDALLY IN A LIQUID AT REST

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An experimental investigation was performed to study the fluid resistance acting on an oscillating disk over a wide range of Reynolds numbers. Data were reduced by the method of Fourier analysis to obtain average values of the added-mass and drag coefficients over one cycle of oscillation, i.e., k_{av} and C_{Dav} respectively.

The relationships between these coefficients and the modified Reynolds number, $d^2\omega/\nu$, were found to change between two regions of $d^2\omega/\nu$, where the flow pattern induced by the disk also changed. In the Reynolds number region where inner circulations are induced exclusively, k_{av} and C_{Dav} decreased with an increasing $d^2\omega/\nu$ and k_{av} was independent of the amplitude of oscillation. In the Reynolds number region where inner and outer circulations coexist, k_{av} was dependent only on the amplitude ratio, a/d; C_{Dav} was almost independent of $d^2\omega/\nu$ and correlated well with a/d, provided $d^2\omega/\nu>200$. The maximum force on the disk during a cycle of oscillation was also examined. Empirical equations for the added-mass, drag and maximum resistance coefficients were presented for each region.

The average power number, N_{Pav}, was defined and correlations for N_{Pav} were theoretically derived from those obtained for C_{Dav}.

Introduction

It is well known that the fluid forces on bodies in unsteady motion in fluids tend to exceed the average forces that may be expected from the laws of drag under steady conditions. This increase in force is induced by the inertia force due to the added mass and by an increase in the drag force due to the history of motion.

In a previous paper on the resistance of fluid to an oscillating circular cylinder,[7] correlations between the added-mass coefficient or the drag coefficient and oscillating parameters were proposed experimentally. Although some experimental approaches[3,8,9] have been made for the fluid resistances exerted on oscillating disks, these investigations have been restricted to cases where Reynolds numbers were large. Few are known which study in detail the relation between fluid force and flow field induced by a body over a wide range of Reynolds numbers.

The present investigation was undertaken with the aim of obtaining correlations for the added-mass and drag coefficients over wide ranges of oscillating conditions. The maximum force on the disk during a cycle was also examined.

1. Experimental Apparatus and Procedures

The apparatus and the experimental procedure were the same as those described in the previous paper.[7] A disk was forced to oscillate in a direction normal to the plane of the disk in a viscous fluid by means of a scotch-yoke mechanism which converted rotary motion into sinusoidal translation. The force on the disk was measured by a transducer with strain gauges. The signal from the transducer was passed through an amplifier with a low-pass filter of 10 Hz and was recorded on a pen-oscillograph. Mixtures of millet jelly and water were used as test fluid, their concentration being varied to provide a range of Reynolds numbers.

Ranges of experimental variables are shown in

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Table 1.

2. Calculation of Added-Mass and Drag Coefficients

The resistance of a body in accelerated motion in a fluid at rest is often expressed as follows:

$$F=-kM_0\frac{dv}{dt}-C_D\frac{1}{2}\rho v|v|S \tag{1}$$

where k and C_D are the added-mass coefficient and the drag coefficient, respectively. For a disk, the displaced mass of the fluid, M_0, is sometimes taken as that of a sphere of diameter equal to the disk diameter.[4)] Thus $M_0=\rho\pi d^3/6$, and $S=\pi d^2/4$.

For a disk oscillating sinusoidally perpendicular to the plane of the disk, the disk velocity is given by Eq. (2):

$$v=-a\omega\sin\omega t \tag{2}$$

Substitution of Eq. (2) into Eq. (1) gives

$$F=k\frac{\pi}{6}d^3\rho a\omega^2\cos\omega t + C_D\frac{1}{2}\rho(a\omega)^2\sin\omega t\,|\sin\omega t|\frac{\pi}{4}d^2 \tag{3}$$

Though k and C_D are functions of the time-dependent angular displacement, through Fourier analysis average values of these coefficients are given by:

$$k_{\mathrm{av}}=\frac{6}{\pi^2\rho a\omega^2 d^3}\int_0^{2\pi}F\cos\omega t d(\omega t) \tag{4}$$

$$C_{D\mathrm{av}}=\frac{3}{\pi\rho a^2\omega^2 d^2}\int_0^{2\pi}F\sin\omega t d(\omega t) \tag{5}$$

The maximum resistance force during a cycle of oscillation, $F_{\max}$, was nondimensionalized by:

$$C_{FV}=F_{\max}\Big/\frac{1}{2}\rho(a\omega)^2\frac{\pi}{4}d^2 \tag{6}$$

$$C_{FA}=F_{\max}\Big/\frac{\pi}{6}d^3\rho a\omega^2 \tag{7}$$

where C_{FV} and C_{FA} are the maximum resistance coefficients.

Procedure for the determination of each coefficient By use of a digitizer, both the forces and the phases of twenty-two arbitrary points lying on the force-time record chart involving a whole cycle were read and fed to a digital computer. Making corrections for the phase-lag due to the filter at each point, which was proportional to the frequency of oscillation, instantaneous forces were calculated by use of Lagrange's interpolation formula at each interval of 0.1π radian. By subtracting both the inertia forces due to the effective mass of the disk-transducer system and the sinusoidal buoyancy forces due to the supporting rod from the calculated forces, the fluid resistances were obtained at each phase of the oscillating cycle, from which the added-mass and the drag coefficients were determined by Eqs. (4) and (5), respectively. To evaluate C_{FV} defined by Eq. (6), the maximum resistance in the cycle of oscillation was determined by differentiating Eq. (3) with the added-mass and drag coefficients up to fifth harmonics.

Table 1. Experimental conditions

Diameter of disk	d [m]:	0.0451, 0.0600
Thickness of disk	t_D [m]:	0.0023, 0.0031
Amplitude of Oscillation	a [m]:	0.0050, 0.0100, 0.0200, 0.0400, 0.0700
Frequency	f [Hz]:	0.19–2.3
Kinematic viscosity of fluid	ν [m²/s]:	8.49×10^{-7}–8.07×10^{-4}

3. Results and Discussion

3.1 Flow pattern around a disk oscillating in a fluid

Kitano[6)] has photographically studied the flow field around a disk oscillating in a viscous fluid and has reported the critical conditions at which the transition of flow pattern occurs. His result is summarized in **Fig. 1**. For relatively low Reynolds numbers, inner circulations are formed in the vicinity of the disk, as illustrated in flow pattern (I) in Fig. 1. For larger Reynolds numbers, outer circulations are induced outside the inner circulations, as illustrated in flow pattern (II) in Fig. 1. Critical Reynolds numbers, $(d^2\omega/\nu)_c$, where the transition of flow pattern takes place, are dependent only on the amplitude ratio, a/d, and are well correlated by the following equation:

$$(d^2\omega/\nu)_c=27.0(a/d)^{-1.7} \tag{8}$$

3.2 Added-mass and drag coefficients

Figure 2 shows the relation between the added-mass coefficients, k_{av}, and the modified Reynolds number, $d^2\omega/\nu$, for $d^2\omega/\nu<(d^2\omega/\nu)_c$. The values of k_{av} are almost independent of the amplitude of oscillation. The theoretical value of the added-mass coefficient for a disk was evaluated by Lamb[4)] for an ideal fluid, and for the case of a disk moving perpendicular to the plane of the disk,[4)]

$$k=2/\pi \tag{9}$$

which means that the added-mass coefficient is independent of time. As $d^2\omega/\nu$ decreases, k_{av} takes remarkably larger value than the theoretical value given by Eq. (9).

By use of expansion technique, Kanwal[5)] proposed the theoretical value of the added-mass coefficient for an oscillating disk as follows:

$$k=\frac{32\sqrt{2}}{\pi^2}\left(\frac{d^2\omega}{\nu}\right)^{-1/2} \tag{10}$$

which means that the added-mass coefficient is inde-

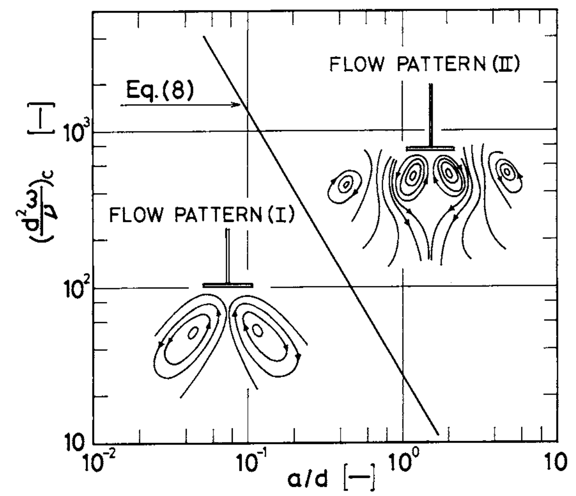

Fig. 1. Illustration and regime of flow pattern around disk.

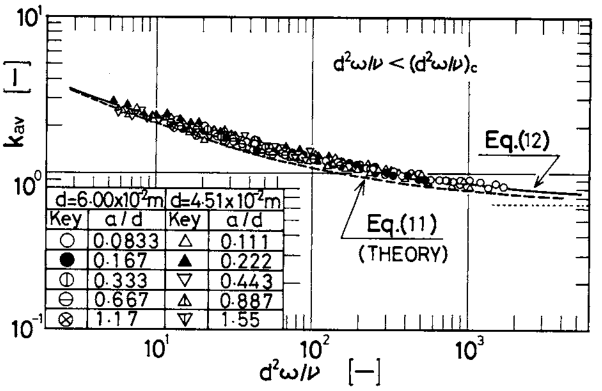

Fig. 2. Comparison of experimental and theoretical added-mass coefficients before transition of flow field.

pendent of the phase of oscillation. On account of the assumption of slow motion, Eq. (10) is only applicable for very small Reynolds numbers. When $d^2\omega/\nu$ is sufficiently large, the added-mass coefficient is expected to take values close to that obtained by potential flow theory,[4] as seen in Fig. 2; so that in order to extend the above theoretical result given by Kanwal to higher Reynolds numbers, we modify Eq. (9) as follows:

$$k_{av}=\frac{2}{\pi}+\frac{32\sqrt{2}}{\pi^2}\left(\frac{d^2\omega}{\nu}\right)^{-1/2} \tag{11}$$

It is seen from Fig. 2 that Eq. (11) gives fairly good agreement with the experimental results. However, Eq. (11) predicts slightly smaller values than the experimental ones. This may be due to neglect of higher terms in the inner and outer expansions. Assuming that the value of k_{av} approaches $2/\pi$ with increasing $d^2\omega/\nu$, an empirical equation for k_{av} in the range of $d^2\omega/\nu<(d^2\omega/\nu)_c$ was obtained, by the least square method, as follows:

$$k_{av}=\frac{2}{\pi}+4.42\left(\frac{d^2\omega}{\nu}\right)^{-0.43}, \quad 0.083<a/d<1.55 \tag{12}$$

The data were well correlated by Eq. (12), which is shown by a solid line in Fig. 2.

After transition of flow field, i.e., for $d^2\omega/\nu>(d^2\omega/\nu)_c$, k_{av} was found to be almost independent of the Reynolds number at constant values of a/d. Under this condition, k_{av} is a function of a/d alone. The relation between k_{av} and a/d for flow pattern (II) is shown in **Fig. 3**, in which the values plotted are the averages of all data. Making an approximation with straight lines, the data for $d^2\omega/\nu>(d^2\omega/\nu)_c$ can be correlated by the following equations.

$$k_{av}=1.10(a/d)^{0.12}, \quad 0.083<a/d<0.2 \tag{13}$$

$$k_{av}=1.64(a/d)^{0.37}, \quad 0.2<a/d<1.55 \tag{14}$$

Recently, Bernardinis *et al.*[1] have made a theoretical approach to oscillatory flow around a disk on the basis of the discrete vortex model. Their results for the added-mass coefficient are indicated by squares in Fig. 3 and show that the effect of a/d on the added-mass coefficient is similar to that in this experiment. However, agreement between the data and the theory is unsatisfactory, especially at large a/d. The disagreement may be due to the sharpness of the disk edge in the theory.

Making the ratio of the drag term to the inertia term due to added mass in Eq. (3), we obtain the following equation:

$$\frac{(\text{drag force})_{max}}{(\text{inertia force})_{max}}\propto\frac{C_D}{k}\frac{a}{d} \tag{15}$$

If a/d is sufficiently small, the relative order of magnitude of fluid resistance due to the effect of added mass becomes so large[7] that the value of k_{av} is almost equal to the value of C_{FA} defined by Eq. (7). Hara and Yokoyama[3] have empirically investigated the fluid force acting on a disk oscillating with relatively small amplitude of oscillation and have examined the relation between the maximum force during a cycle of oscillation and oscillatory parameters. In Fig. 3, C_{FA} values obtained by them are also plotted. Their data lie very close to values of k_{av} predicted by Eq. (13). It is expected from Fig. 3 that the value of k_{av} approaches the theoretical value of $2/\pi$ for low values of a/d.

In **Fig. 4** the drag coefficient C_{Dav} is plotted against $d^2\omega/\nu$ for various values of the parameter a/d. The figure indicates that C_{Dav} increases as a/d decreases and also that in the higher $d^2\omega/\nu$ range, C_{Dav} is exclusively dependent upon a/d and is not affected by $d^2\omega/\nu$. In Fig. 4 the critical Reynolds numbers, $(d^2\omega/\nu)_c$, calculated from Eq. (8) for each a/d are also shown by dotted lines. For $a/d=0.222$ and 0.111, C_{Dav} appears to be almost independent of $d^2\omega/\nu$ above each value of $(d^2\omega/\nu)_c$. For $a/d=0.443$, 0.887 and

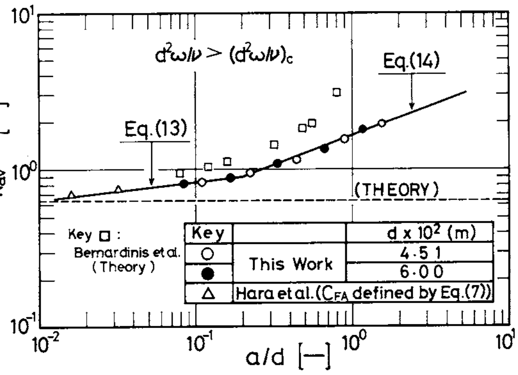

Fig. 3. Correlations of added-mass coefficient with amplitude ratio after transition of flow field.

1.55, however, C_{Dav} takes a constant value at $d^2\omega/v \approx 200$ in every case. This means that the lowest value of $d^2\omega/v$, where C_{Dav} is independent of $d^2\omega/v$, agrees with $(d^2\omega/v)_c$ for $(d^2\omega/v) > 200$ and 200 for $(d^2\omega/v)_c < 200$. $(d^2\omega/v)_c \gtreqless 200$ is equivalent to $a/d \lesseqgtr 0.308$ from Eq. (8).

For $d^2\omega/v < (d^2\omega/v)_c$, C_{Dav} was found to be nearly inversely proportional to the amplitude ratio, a/d, and this relationship is shown in **Fig. 5**. The data are well correlated by Eq. (16), which is indicated by a solid line in Fig. 5.

$$C_{Dav} = 8.52(\log(d^2\omega/v))^{-2.1}(a/d)^{-0.93} \qquad (16)$$

$$0.083 < a/d < 1.55$$

In the range of $d^2\omega/v > (d^2\omega/v)_c$ $(a/d < 0.308)$ and $d^2\omega/v > 200$ $(0.308 < a/d)$, C_{Dav} is independent of $d^2\omega/v$ and a function of a/d alone as mentioned above. This relationship is shown in **Fig. 6**, in which the value of C_{Dav} are the averages of all experimental data for each value of a/d. The data for small a/d were not plotted because of large scattering of the data. From Fig. 6, one obtains the following empirical equation:

$$C_{Dav} = 2.37(a/d)^{-0.57} \qquad (17)$$

$$d^2\omega/v > (d^2\omega/v)_c \quad \text{for } 0.222 < a/d < 0.308$$

$$d^2\omega/v \times 200 \quad \text{for } 0.308 < a/d < 1.55$$

In Fig. 6 the theoretical results of C_D obtained by Bernardinis *et al.*[1] are also plotted for comparison. While the variation of the drag coefficient with a/d in the theory is similar to that in the experiments, as was also seen for the added-mass coefficient in Fig. 3, the theory gives much larger values than the experiments.

3.3 Maximum resistance coefficient

Figure 7 shows the correlation of maximum resistance coefficient for $d^2\omega/v < (d^2\omega/v)_c$. The data are well approximated by the following equation:

$$C_{FV} = 7.72(\log(d^2\omega/v))^{-1.54}(a/d)^{-0.95} \qquad (18)$$

$$0.0833 < a/d < 1.55$$

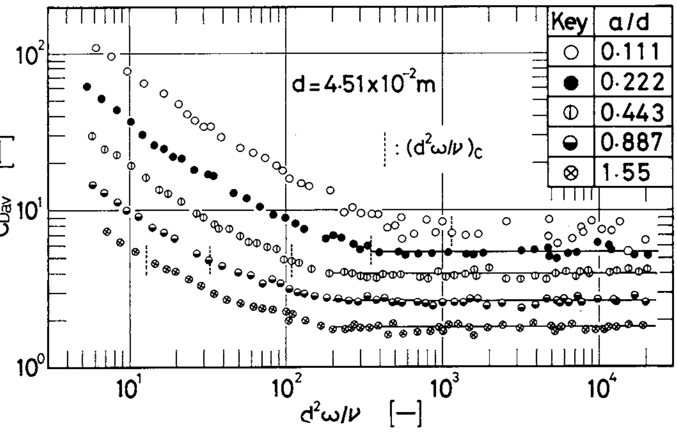

Fig. 4. Variation of drag coefficient with Reynolds number as parameter of amplitude ratio.

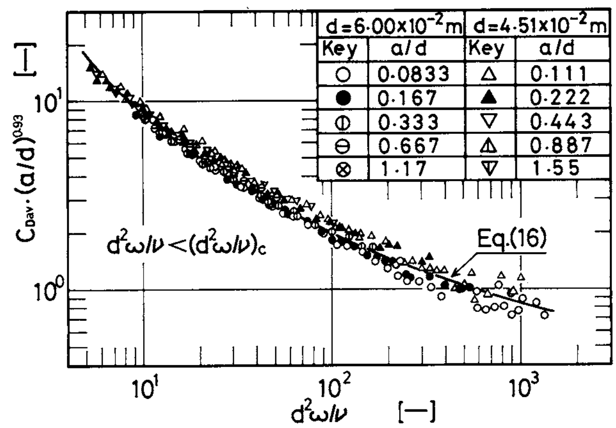

Fig. 5. Correlation of drag coefficient before transition of flow field.

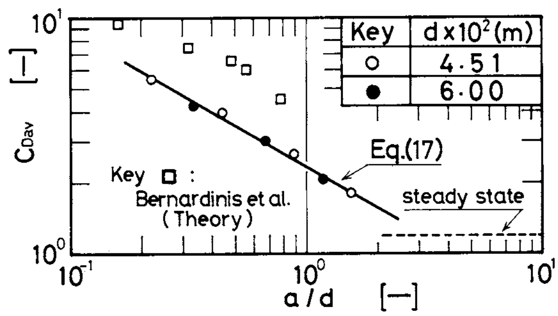

Fig. 6. Relation between drag coefficient and amplitude ratio after transition of flow field.

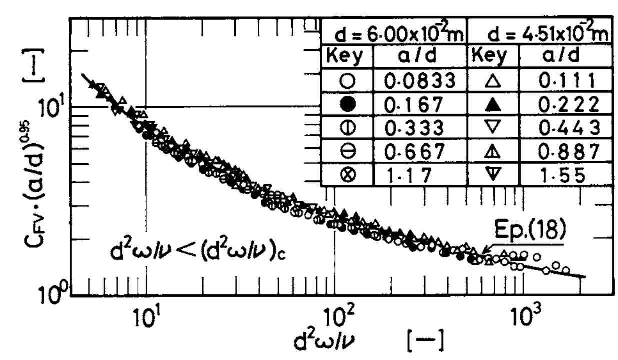

Fig. 7. Correlation of maximum resistance coefficient before transition of flow field.

The relation between C_{FV} and a/d is very similar to that for C_{Dav} as is shown in Fig. 5.

In the region of $d^2\omega/\nu > (d^2\omega/\nu)_c$ ($a/d < 0.308$) and $d^2\omega/\nu > 200$ ($0.308 < a/d$), C_{FV} was almost independent of $d^2\omega/\nu$ and a function of a/d alone in the same manner as C_{Dav}. This relationship is shown in **Fig. 8**, in which the values of C_{FV} are the averages of all experimental data for each a/d, and the experimental results by Hara and Yokoyama[3] are plotted. In Fig. 8, the values of C_{FV} calculated from the data of maximum force during a cycle of oscillation by Ueno and Kishioka[8] are also plotted. Our experimental result agrees well with theirs. The relationship between C_{FV} and a/d is given as follows:

$$C_{FV} = 3.13(a/d)^{-0.61} \tag{19}$$

$d^2\omega/\nu > (d^2\omega/\nu)_c$ for $0.083 < a/d < 0.308$

$d^2\omega/\nu > 200$ for $0.308 < a/d < 1.55$

If we assume that the maximum resistance F_{max} is nearly equal to the inertia force due to the theoretical added mass, i.e., $(2/\pi)\cdot(\rho\pi d^3/6)\cdot(a\omega^2)$, C_{FV} is estimated by Eq. (6) as follows:

$$C_{FV} = \frac{8}{3\pi}\frac{d}{a} \tag{20}$$

The dotted line in Fig. 8 represents Eq. (20). As is clear from the data of Hara and Yokoyama,[3] C_{FV} tends to approach the theoretical value by Eq. (20) with decreasing a/d. The reason is that the fluid force almost entirely consists of the inertial force due to the added mass when a/d is small and the Reynolds number is relatively large. On the other hand, as a/d is increased over the experimental condition, C_{FV} is expected to approach the steady-state value of the drag coefficient indicated by the dotted line.

3.4 Average power number

The instantaneous power consumption is calculated by multiplying the fluid resistance by the disk velocity:

$$P = F\cdot v \tag{21}$$

We define the average power number as follows:

$$N_{Pav} = P_{av}/\rho(a\omega)^3 d^2 \tag{22}$$

where P_{av} is the average power consumption over a cycle of oscillation.

Average power consumption, which is due to the drag force, was calculated by the following equation.

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} P d(\omega t) \tag{23}$$

From Eqs. (2), (3), (21), (22) and (23), the power number is given by:

$$N_{Pav} = C_{Dav}/6 \tag{24}$$

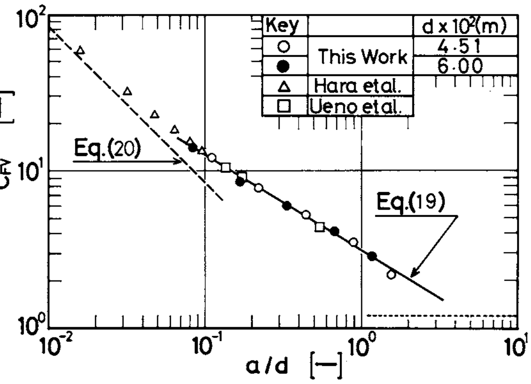

Fig. 8. Relationship between maximum resistance coefficient and amplitude ratio after transition of flow field.

Accordingly, the correlating equations for N_{Pav} are given as follows, corresponding to Eqs. (16) and (17), respectively:

$$N_{Pav} = 1.42(\log(d^2\omega/\nu))^{-2.1}(a/d)^{-0.93} \tag{25}$$

$0.083 < a/d < 1.55$

$$N_{Pav} = 0.40(a/d)^{-0.57} \tag{26}$$

$d^2\omega/\nu > (d^2\omega/\nu)_c$ for $0.222 < a/d < 0.308$

$d^2\omega/\nu > 200$ for $0.308 < a/d < 1.55$

Figure 9 shows correlations of N_{Pav} for $d^2\omega/\nu < (d^2\omega/\nu)_c$. As was expected, data showed good agreement with Eq. (25), which is indicated by a solid line in Fig. 9. In **Fig. 10**, the observed power consumptions were compared with the theoretical values by Eqs. (22) and (26) for the range of $d^2\omega/\nu > (d^2\omega/\nu)_c$ ($a/d < 0.308$) and $d^2\omega/\nu > 200$ ($0.308 < a/d$) to show agreement.

Conclusions

1) In the Reynolds number region where inner circulation only exists ($d^2\omega/\nu < (d^2\omega/\nu)_c$), k_{av} is a function of $d^2\omega/\nu$ only, which given by Eq. (12), whereas in the region where outer circulation is induced in addition to the inner circulation ($d^2\omega/\nu > (d^2\omega/\nu)_c$), k_{av} is represented as functions of a/d only, as given by Eqs. (13) and (14) below and above $a/d = 0.2$ respectively.

2) The dependencies of C_{Dav} and C_{FV} on a/d greatly change between the two regions corresponding to the flow patterns. In the region of $d^2\omega/\nu < (d^2\omega/\nu)_c$, C_{Dav} and C_{FV} are functions of both $d^2\omega/\nu$ and a/d and are correlated by Eqs. (16) and (18) respectively. When $d^2\omega/\nu > (d^2\omega/\nu)_c$, C_{Dav} and C_{FV} are functions of a/d alone as given by Eqs. (17) and (19) respectively, provided that $d^2\omega/\nu > 200$.

3) The average power consumptions were well predicted with N_{Pav} by Eqs. (25) and (26), which were theoretically derived from the correlations for C_{Dav}.

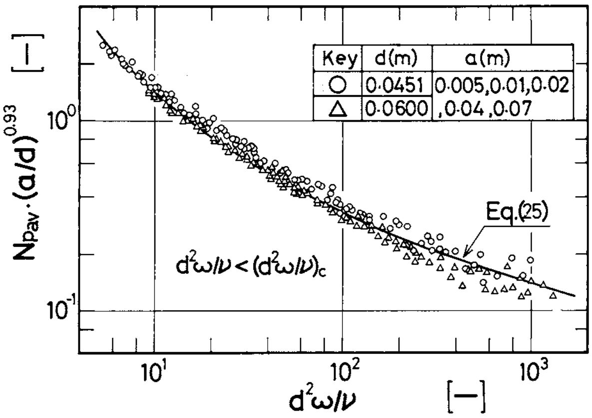

Fig. 9. Correlation of average power number for $d^2\omega/\nu < (d^2\omega/\nu)_c$.

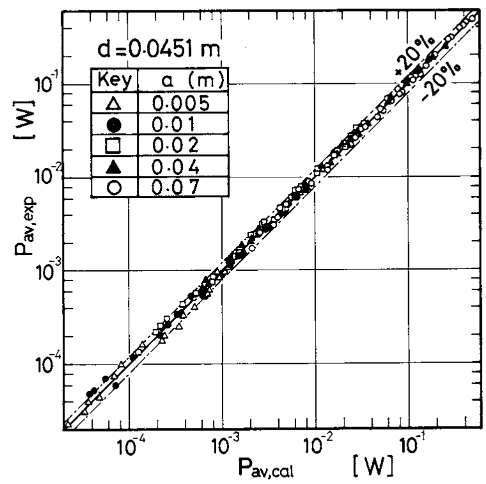

Fig. 10. Comparison of average power consumption predicted by Eqs. (22) and (26) with experimental values after transition of flow pattern.

Nomenclature

a	= amplitude of oscillation	[m]
C_D	= drag coefficient	[—]
C_{FV}	= maximum resistance coefficient defined by Eq. (6)	[—]
C_{FA}	= maximum resistance coefficient defined by Eq. (7)	[—]
d	= diameter of disk	[m]
f	= frequency of oscillation	[Hz]
F	= fluid resistance	[N]
k	= added-mass coefficient	[—]
M_O	= mass of fluid displaced by a body	[kg]
N_P	= power number	[—]
P	= power	[W]
S	= effective area of body	[m^2]
t	= time	[s]
t_D	= thickness of disk	[m]
v	= velocity of disk	[m/s]
ρ	= density of fluid	[kg/m^3]
ν	= kinematic viscosity of fluid	[m^2/s]
ω	= angular frequency of oscillation	[rad/s]

⟨Subscripts⟩

av	= average over a cycle of oscillation
c	= critical value for transition of flow pattern
cal	= calculation
exp	= experiment
max	= maximum during a cycle of oscillation

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