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(Presented at the 15th Autumn Meeting of The Society of Chemical Engineers, Japan at Kanazawa, October 1981).

MODIFIED DERIVATIVE DECOUPLING CONTROL OF NONLINEAR MULTIVARIABLE PROCESS

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Key Words: Process Control, Noninteracting Control, Nonlinear Multivariable Process, Derivative Decoupling Control, Mixing Tank, Liquid Level, Temperature, Optimal Control, Inverse Nyquist Array

A method is developed for on-line noninteracting control of a nonlinear multivariable process to handle constraints on the control variables based on the derivative decoupling control approach. An extension of the proposed modified derivative decoupling control method to load changes (unknown disturbance) is also treated. This modified method is investigated for noninteracting control of laboratory-scale mixing tanks in series both by digital simulation and by experiment using an on-line microcomputer. Liquid levels and temperature in tanks mixing hot and cold water inflow streams are controlled. The results are compared with those obtained by instantaneously optimal control and by the inverse Nyquist array technique. Both simulated and experimental performances of the modified derivative decoupling controller are found to be better in comparison with those obtained by the controller based on the inverse Nyquist array. Further, experiments showed that the modified derivative decoupling controller can reduce interaction to a negligible level.

Introduction

Complex industrial processes are of an essentially multivariable nature and would demand treatment as such if a simpler approach were not adequate in most cases. However, due to the presence of interactions in the multivariable system, controllers tuned for a single loop often must be retuned by trial and error to avoid destabilizing the closed-loop responses. It has been noted in the literature that multivariable control design techniques[4] do provide controller synthesis strategies which directly treat the interactions among multiple inputs and outputs, while the concept is complex in structure and application. Besides the complexity, almost no technique except optimal control theory can effectively handle constraint conditinos on the control variables from the theoretical point of view.

Liu[3] presented an approach for noninteracting control based on the decoupling of the state derivatives satisfying the given constraints on control variables. Hutchinson and McAvoy[2] applied this derivative decoupling method to the servo control of an experimental heat exchanger system in which the objective was to control the two state variables by manipulation of two inputs. However, a detailed examination of this control method has revealed the general difficulties encountered in obtaining the controller equations.[1,2,5] In addition, this derivative decoupling control method is limited to a setpoint change.

This paper is concerned with a method to overcome

Received September 16, 1982. Correspondence concerning this article should be addressed to W. K. Lee.

some of the general difficulties involved in designing a controller based on the derivative decoupling control approach and its extension to an unknown load change. To illustrate this modified derivative decoupling control method, a laboratory-scale mixing process is used where two liquid levels and temperature are controlled. The performance of a controller based on this modified derivative decoupling control is evaluated via digital simulation and experiments using an on-line microcomputer and is compared with those obtained by instantaneously optimal control[6] and by the inverse Nyquist array technique.[7]

1. Theoretical Approaches

The derivative decoupling approach for noninteracting process control is reviewed briefly,[2,3] and a modified method and its extension to a load change are presented here.

1.1 Derivative decoupling control

The uncontrolled process can be characterized by a nonlinear state-vector differential equation model of the form

$$\dot{\boldsymbol{X}}=\boldsymbol{F}(\boldsymbol{X}, \boldsymbol{U}, t) \tag{1}$$

where $\boldsymbol{X}=n\times 1$ column vector of state variables; $\boldsymbol{U}=n\times 1$ column vector of manipulated variables; $\boldsymbol{F}=$column vector of nonlinear functions $f_i\ (\boldsymbol{X}, \boldsymbol{U}, t)$; $t=$time. The control objective is the control of $\boldsymbol{X}$ by the manipulation of $\boldsymbol{U}$ satisfying the inequality constraints.

$$u_i^{\min}\leqq u_i\leqq u_i^{\max}\quad i=1,\cdots n$$

With control, the manipulative input $\boldsymbol{U}$ becomes a function of the measurable state $\boldsymbol{X}$ and the setpoint vector $\boldsymbol{R}$. As a result, the controlled process dynamics can be written as a function of the error $\boldsymbol{E}$, where $\boldsymbol{E}=\boldsymbol{R}-\boldsymbol{X}$.

$$\dot{\boldsymbol{X}}=\boldsymbol{G}(\boldsymbol{E}, t) \tag{2}$$

If $\boldsymbol{G}(\boldsymbol{E}, t)$ were specified such that

$$\boldsymbol{F}(\boldsymbol{X}, \boldsymbol{U}, t)=\boldsymbol{G}(\boldsymbol{E}, t) \tag{3}$$

by design as a column vector whose i-th component is a function of only the corresponding element e_i and t, then the state derivatives would be decoupled. This also means that the state elements are decoupled.

However, it is noted[5] that there are three general difficulties which can be encountered in obtaining u_i controller equations from this system of equations. The first difficulty is the specification of $\boldsymbol{G}(\boldsymbol{E}, t)$. Obviously, there are an infinite number of choices for each of the g_i elements. Liu[3] arbitrarily assumed that a suitable form for all of the state derivatives was a type of proportional response:

$$\dot{x}_i=a_ie_i\,,\quad i=1,\cdots n \tag{4}$$

where a_i are unspecified proportional coefficients. He then proposed an algorithm for finding the value of a_i that would iteratively reduce the magnitudes starting from initial arbitrarily large values. However, constraints on the control variables require an off-line calculation of the complete solution of the nonlinear process equations or an on-line approach involving integration through each time step.

The second difficulty is that it may not be possible to solve the equations for each of the u_i as explicit functions of state variables and setpoints.

The third difficulty occurs in the common process case where only a small subset of the inputs are manipulative and only a small subset of the state elements are actually controlled.

1.2 Modified derivative decoupling

Here it is assumed that the right-hand side of Eq. (1) has no explicit term of t. A linearized form of Eq. (1) can be written as

$$\dot{\boldsymbol{X}}=A\boldsymbol{X}+B\boldsymbol{U} \tag{5}$$

with initial conditions $\boldsymbol{X}(0)=\boldsymbol{0}=\boldsymbol{U}(0)$ where $A, B=n\times n$ constant matrix.

Taking the Laplace transform of Eq. (5) and rearranging yields

$$\boldsymbol{U}(s)=B^{-1}(sI-A)\boldsymbol{X}(s) \tag{6}$$

From Eq. (6) one obtains

$$u_i(s)=\sum_j(\hat{b}_{ij}s+h_{ij})x_j(s) \tag{7}$$

where

$$B^{-1}=[\hat{b}_{ij}]_{n\times n}\,,\qquad -B^{-1}A=[h_{ij}]_{n\times n}$$

Consider the control objective of setpoints change from $\boldsymbol{X}(0)$ to $\boldsymbol{X}_d$. Assume that $x_j(s)$ has the following form based on the derivative decoupling control method:

$$x_j(s)=d_j\left(\frac{1}{s}-\frac{1}{s+\lambda_j}\right),\quad j=1,\cdots n \tag{8}$$

where $d_j=j$-th component of X_d; $\lambda_j=$positive real constants. Combining Eqs. (7) and (8) gives

$$u_i(s)=\sum_j\left(\frac{h_{ij}}{s}+\frac{\hat{b}_{ij}\lambda_j-h_{ij}}{s+\lambda_j}\right)d_j \tag{9}$$

The inverse transformation of $u_i(s)$ can be written in the form

$$u_i(t)=\sum_j d_jh_{ij}(1-e^{-\lambda_jt})+\sum_j\hat{b}_{ij}\lambda_jd_je^{-\lambda_jt} \tag{10}$$

That is,

$$u_i(t)=\sum_j h_{ij}x_j(t)+\sum_j\hat{b}_{ij}\lambda_j[d_j-x_j(t)] \tag{11}$$

Let us consider a case of setting all λ_i equally to obtain monotonic behavior. Then, Eq. (10) becomes

$$u_i(t)=\sum_j d_j h_{ij}(1-e^{-\lambda t})+\lambda\sum_j \hat{b}_{ij} d_j e^{-\lambda t} \tag{12}$$

In this case $u_i(t)$ approaches steady state monotonically, since $u_i(t)$ is expressed as a linear combination of $e^{-\lambda t}$. Thus it can be observed that setting all λ_i equally makes the behavior of $u_i(t)$ monotonic.

Since $u_i(0)=\lambda\sum_j \hat{b}_{ij} d_j$ at $t=0$ from Eq. (12), λ must satisfy the following inequality:

$$u_i^{\min}\leq\lambda\sum_j \hat{b}_{ij} d_j\leq u_i^{\max}, \quad i=1,\cdots n \tag{13}$$

Since large values of λ_i would provide faster monotonic behavior for $u_i(t)$, it would be desirable to choose the value of λ as the maximum satisfying Eq. (13). Because of the monotonic behavior of $u_i(t)$, it never violates the constraint if λ is chosen as mentioned above. Therefore as time elapses, λ can be chosen larger and larger.

One can obtain the value of λ as time elapses using the following on-line procedure:

Step 1:

$$0\leq t<t_1: \quad x_i(0)=0 \qquad i=1,\cdots n$$

$$u_i(0)=\lambda\sum_j \hat{b}_{ij} d_j$$

$$u_i^{\min}\leq\lambda\sum_j \hat{b}_{ij} d_j\leq u_i^{\max}$$

Choose λ as the maximum value satisfying the above equation.

Step 2:

$$t_1\leq t<t_2:$$

$$u_i^{\min}\leq\sum_j h_{ij} x_j(t_1)+\lambda\sum_j \hat{b}_{ij}[d_j-x_j(t_1)]\leq {}_i^{\max} \tag{14}$$

Choose λ as the maximum value satisfying Eq. (14).

Step 3:

$$\text{If } \left|\frac{x_j(t_f)-d_j}{d_j}\right|\leq n_j \text{ for all } j \text{ or some } j,$$

Stop the procedure and maintain λ as the value for the interval $t_{f-1}\leq t<t_f$ to reach steady state. Here n_j is a constant chosen by user.

In each time interval the manipulated variable $u_i(t)$ can be expressed as

$$u_i(t)=\sum_j h_{ij} x_j(t)+\lambda^{(p)}\sum_j \hat{b}_{ij}[d_j-x_j(t)], \quad t_p\leq t<t_{p+1}$$

where

$$\lambda^{(p)}=\lambda \quad \text{chosen at step} \quad p\,.$$

It can be easily seen from this procedure that λ becomes larger as time elapses. However, it would not be desirable to make all λ_i equal in the case where the magnitudes of $\hat{b}_{ij} d_j$, $i, j=1,\cdots n$, differ greatly from each other.

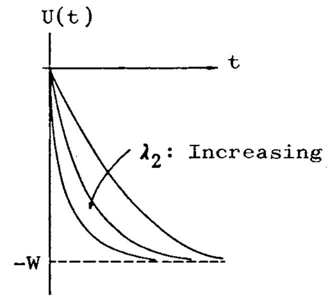

Fig. 1. Monotonic behavior of control action with $a\leq 0$, constant λ_1 and $0<\lambda_1<-a<\lambda_2$.

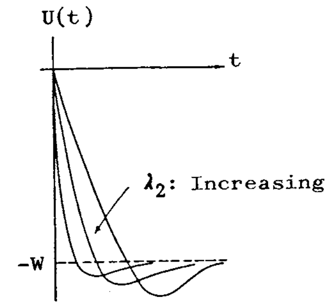

Fig. 2. Nonmonotonic behavior of control action with $a\leq 0$, constant λ_1 and λ_1, $\lambda_2>-a$.

1.3 Extension to load change

Consider that an unknown disturbance is introduced into Eq. (5) at $t=0$ as follows:

$$\dot{X}=AX+BU+W \tag{15}$$

where W is an n-vector of the unknown disturbance. It should be noted that limit values of the given constraints are not used here. The reason will be presented in the latter part of this section. To maintain the initial steady states in spite of the disturbance, one can prescribe the following form of noninteracting system equations with integral action.

$$\dot{X}=-(\lambda_1+\lambda_2)X-\lambda_1\lambda_2\int_0^t Xdt+W \tag{16}$$

where λ_1 and λ_2 are positive real constants. Then the following controller equation can be obtained from Eqs. (15) and (16):

$$U=-B^{-1}\left[AX+(\lambda_1+\lambda_2)X+\lambda_1\lambda_2\int_0^t Xdt\right]$$

$$u_i=-\sum_k \hat{b}_{ik}\left[\sum_j a_{kj}x_j+(\lambda_1+\lambda_2)x_k+\lambda_1\lambda_2\int_0^t x_k dt\right]$$

$$=-\sum_k \hat{b}_{ik}\left[\sum_{j\neq k} a_{kj}x_j+(a_{kk}+\lambda_1+\lambda_2)+\lambda_1\lambda_2\int_0^t x_k dt\right] \tag{17}$$

Now consider the first-order single-loop system with unknown constant disturbance at $t=0$ as follows:

$$\dot{x}=ax+u+w \tag{18}$$

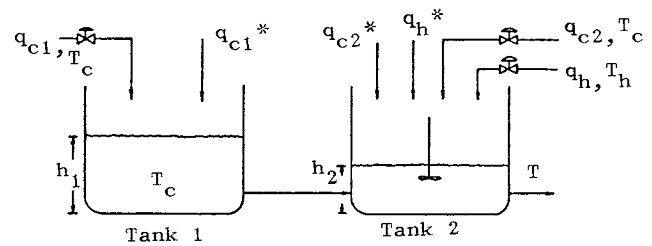

Fig. 3. Schematic diagram of the mixing process.

If PI controller is used to compensate the disturbance, the controller equation becomes

$$ax+u+w=-(\lambda_1+\lambda_2)x-\lambda_1\lambda_2\int_0^t xdt+w$$
$$u=-(a+\lambda_1+\lambda_2)x-\lambda_1\lambda_2\int_0^t xdt \quad (19)$$

From Eqs. (18) and (19) one can obtain

$$x(t)=\frac{w}{\lambda_2-\lambda_1}(e^{-\lambda_1 t}-e^{-\lambda_2 t})$$
$$u(t)=\frac{w}{\lambda_2-\lambda_1}[(-\lambda_1-a)e^{-\lambda_1 t}+(\lambda_2+a)e^{-\lambda_2 t}]-w \quad (20)$$

It can be shown easily that the larger the value of λ_2 with λ_1 being constant, the faster the state, $x(t)$, returns to the initial steady state. Now, let us consider the behavior of $u(t)$ for $a\le 0$ shown in **Figs. 1** and **2**. It can be observed that monotonic behavior of control action in the case of $0<\lambda_1<-a<\lambda_2$ seems more desirable than that in the case of $\lambda_1, \lambda_2>-a$ in order not to violate the given constraint. That is, after λ_1 being chosen such that $\lambda_1<-a$, the larger the value of λ_2, the faster the control action absorbs the disturbance.

In the case of $a>0$, $u(t)$ has an extremum when

$$t=\left[\ln\frac{\lambda_2(\lambda_2+a)}{\lambda_1(\lambda_1+a)}\right]\Big/(\lambda_2-\lambda_1)$$

which approaches zero as λ_2 increases, and moreover the magnitude of the extremum decreases as λ_2 increases. In other words, $u(t)$ has a behavior similar to that obtainable when $\lambda_1, \lambda_2>-a$. Therefore, the larger the value of λ_2, the faster the control action absorbs the disturbance.

In the case of a multivariable system, the values of λ_1 and λ_2 in Eq. (16) may be chosen as follows: first, choose λ_1 such that $\lambda_1<-a_{ii}$ for every i such that $-a_{ii}>0$, and choose λ_2 as large as possible. Then the term $\sum_{j\neq k} a_{kj}x_j$ becomes smaller than the term $(a_{kk}+\lambda_1+\lambda_2)x_k$ in Eq. (17). Such a choice of λ_1 and λ_2 makes the transient behavior of control actions and states in a multivariable system similar to that of a single input-single output system. That is, the control inputs approach new steady states fast, showing nearly monotonic behavior. Therefore, in spite of the fast responses it is expected that the control inputs do not exceed new steady-state values more than by a negligible level. This is the reason why the limit value of the constraints on the control inputs is not used in the extension of the modified derivative decoupling control to load change.

2. Application to Mixing Process

To illustrate the modified derivative decoupling control method, a multivariable model for laboratory-scale mixing tanks in series is used where two liquid levels and the temperature of the second tank are to be controlled. A schematic diagram of the mixing process is given in **Fig. 3**. Assuming perfect mixing in tanks, the levels h_1, h_2 are set appropriately to avoid counterflow from tank 2 to tank 1, and mass and energy balances yield

$$A_1\frac{dh_1}{dt}=q_{c1}-k_1\sqrt{h_1-h_2} \quad (21)$$

$$A_2\frac{dh_2}{dt}=q_h+q_{c2}+k_1\sqrt{h_1-h_2}-k_2\sqrt{h_2} \quad (22)$$

$$\rho c_p\frac{d}{dt}[A_2(T-T_c)]=\rho c_p[q_h(T_h-T_c)-k_2\sqrt{h_2}\,(T-T_c)] \quad (23)$$

where h_1, h_2 and T are controlled variables; q_{c1}, q_{c2} and q_h are manipulated variables; q_{c1}^*, q_{c2}^*, and $q_h{}^*$ are unknown disturbances; values of k_1 and k_2 are 210 and 310 $cm^{2.5}$/min, respectively; cross-sectional area of tanks A_1 and A_2 is 283.53 cm^2; T_h and T_c are 41°C and 6°C, respectively. In designing the controller, the nonlinear Eqs. (21), (22) and (23) are linearized about the initial steady states as given below.

$$\frac{d}{dt}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=[B]\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+[C]\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} \quad (24)$$

where $x_1=h_1-h_1(0)$, $x_2=h_2-h_2(0)$, $x_3=T-T(0)$, $u_1=q_{c1}-q_{c1}(0)$, $u_2=q_{c2}-q_{c2}(0)$, and $u_3=q_h-q_h(0)$.

In experimental studies an on-line microcomputer, Cromenco System Three, was used for implementation of the modified derivative decoupling control algorithm and other control techniques, and the process interface was via a Cromenco D+7A I/O

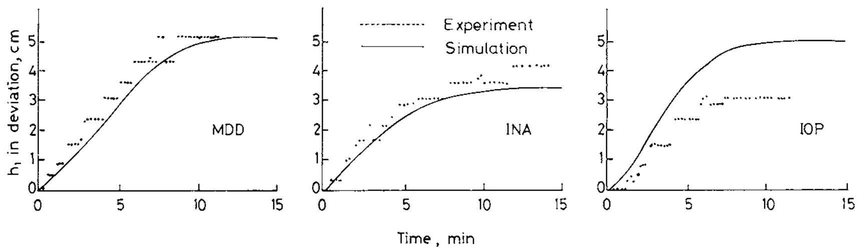

Fig. 4. Closed-loop responses of h_1 to a setpoint change.

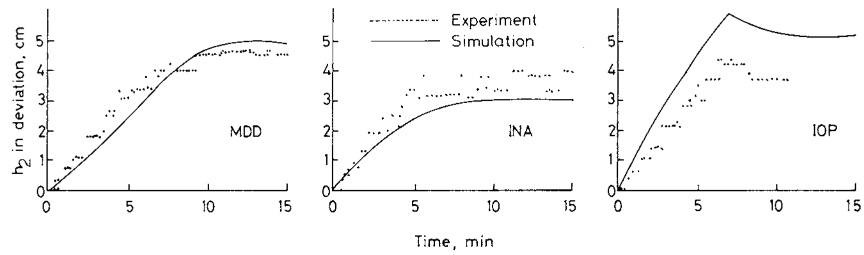

Fig. 5. Closed-loop responses of h_2 to a setpoint change.

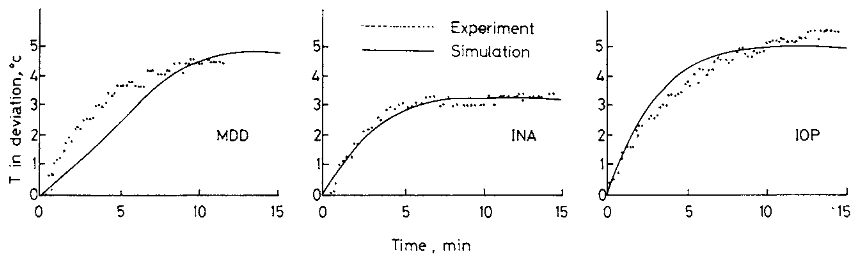

Fig. 6. Closed-loop responses of T to a setpoint change.

module which gives seven channels of 8-bit A/D conversion and seven channels of D/A conversion with a fast conversion time of 5.5 microseconds. All programming was done in Fortran. Signals out of the microcomputer were transduced by V/I and I/P converters. For control actions pneumatic control valves of equal percentage were used. Temperature of the second tank was measured by a Chromel–Alumel thermocouple connected to an amplifier. Changes in liquid levels were measured by a wound wire resistance, which is then converted to voltage to be sent to the microcomputer.

3. Results and Discussion

Control system performance was evaluated by experiments with a laboratory-scale mixing process under a controller designed using the modified derivative decoupling (MDD) control approach, and parameter sensitivity of the controller was estimated by comparing the results of the experiments with those of digital simulation. The simulation results were obtained from nonlinear model equations, Eqs. (21), (22) and (23), through controllers based on the linearized equation, Eq. (24).

3.1 Setpoint changes

Here setpoints are changed by 5°C, 5 cm and 5 cm, respectively corresponding to temperature (T), and two levels (h_1 and h_2) from the following initial steady-states: $h_1=21.5$ cm, $h_2=15.1$ cm, $T=6$°C, $q_{c1}=523\,\mathrm{cm^3/min}$, $q_{c2}=615\,\mathrm{cm^3/min}$ and $q_h=0\,\mathrm{cm^3/min}$. Constraints imposed on the manipulated variables are $0\leq u_1\leq 205$, $0\leq u_2\leq 105$ and $0\leq u_3\leq 240$.

In controller design by the inverse Nyquist array technique (INA), elements of the inverse of the open-loop transfer function matrix, $Q^{-1}(s)=G(0)G^{-1}(s)$, should satisfy the stability conditions with diagonal dominance, and thus the inverse of the matrix of the $G(0)$ premultiplied by a diagonal matrix with positive elements is taken as the controller, which means steady-state decoupling. That is, the controller matrix $K=G^{-1}(0)D$, where D is a diagonal matrix with positive elements. It is noted that $G(s)$ is the open-loop transfer function of a process. As the result of many trial-and-error runs by digital simulation, the elements 1.45, 2.9 and 3.9 of the diagonal matrix D were chosen, respectively corresponding to T, h_2 and h_1 in order to satisfy the given constraints on the manipulated variables. In the design of the IOP controller, equal weights were assigned to the state variables in the performance index. Once one of the three controlled variables reaches 95% of the setpoint change, the MDD controller is kept unchanged, and

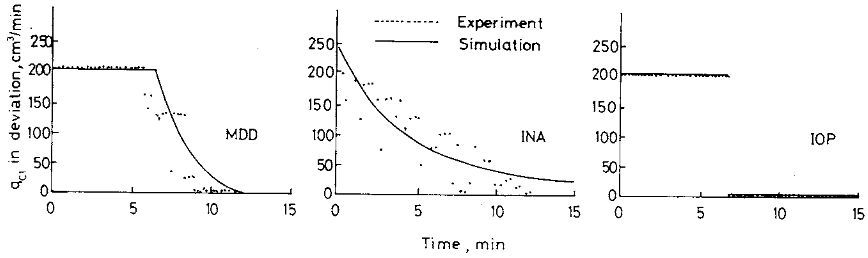

Fig. 7. Behavior of q_{c1} to a setpoint change.

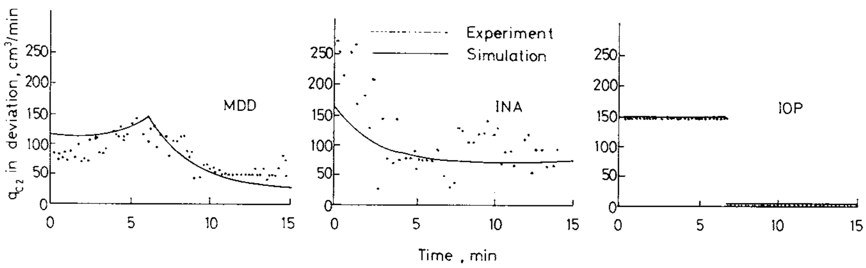

Fig. 8. Behavior to q_{c2} to a setpoint change.

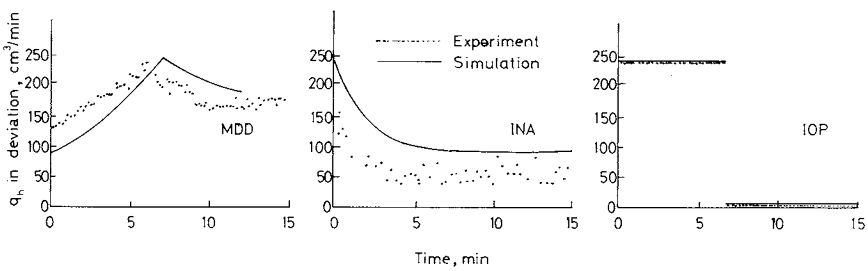

Fig. 9. Behavior of q_h to a setpoint change.

the control actions of IOP are switched to new steady-state values at setpoints.

Comparisons of MDD, INA and IOP controllers with the setpoint changes are given in **Figs. 4, 5** and **6**. As can be expected, the simulated responses of all three controlled variables are seen to be critically damped with the MDD controller. In spite of the absence of integral action, the steady-state offset caused by the MDD controller is very small because λ_j in the controlled dynamics of Eq. (8) becomes larger as time passes. On the contrary, steady-state offsets are considerable in both simulated and experimental results with INA controller due to the use of proportional action only, and also in experimental results with the IOP controller because of model error and hysteresis of control valves. In the INA controller, an integral action can be added to reduce the offset, which, however, adds more complexity in the controller design to satisfy the stability conditions in addition to many trial and error runs required due to additional controller parameters.

With MDD and INA, the experimental behavior of the states is found to be similar to that of simulation, but not with IOP as illustrated in Figs. 4 through 6. This means that the MDD control system is as insensitive as the INA system to model error and hysteresis of control valves, and is less sensitive than the IOP. The control actions required to achieve these performances are compared in **Figs. 7, 8** and **9**. From Figs. 7 through 9 the behavior of control actions required in the experiments is seen to be similar to simulated results.

3.2 Load changes

Results from extension of the MDD controller to load changes are shown in **Figs. 10** and **11**, where the performance of the INA controller is also compared. Experimental conditions used are as follows: $h_1 = 19.9\,\mathrm{cm}$, $h_2 = 15.45\,\mathrm{cm}$, $T = 10°\mathrm{C}$, $q_{c1}^* = -128\,\mathrm{cm^3/min}$ and $q_{c2}^* = 82\,\mathrm{cm^3/min}$ and $q_h{}^* = -170\,\mathrm{cm^3/min}$ for INA; $h_1 = 21.4\,\mathrm{cm}$, $h_2 = 16.88\,\mathrm{cm}$, $T = 10°\mathrm{C}$, $q_{c1}^* = -55\,\mathrm{cm^3/min}$, $q_{c2}^* = 50\,\mathrm{cm^3/min}$ and $q_h{}^* = -168\,\mathrm{cm^3/min}$ for MDD. Differences in load changes for each case have no meaning except that it was not easy to introduce disturbances of equal magnitude in each case.

The INA controller for load changes is designed in a similar manner to setpoint changes with diagonal matrix whose elements are 2, 2 and 4. In the MDD controller, values of λ_1 and λ_2 in Eq. (16) are chosen to be 1.0 and 0.1 respectively in order to make the settling time of the control actions of the two controllers nearly equal and not to saturate before steady

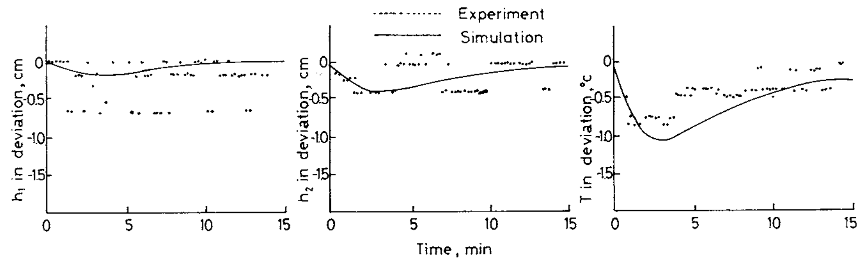

Fig. 10. Responses to load changes under MDD.

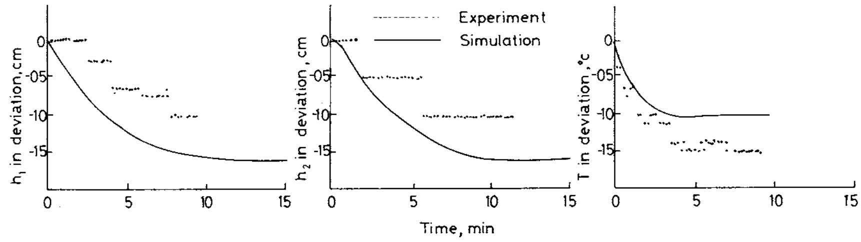

Fig. 11. Responses to load changes by INA.

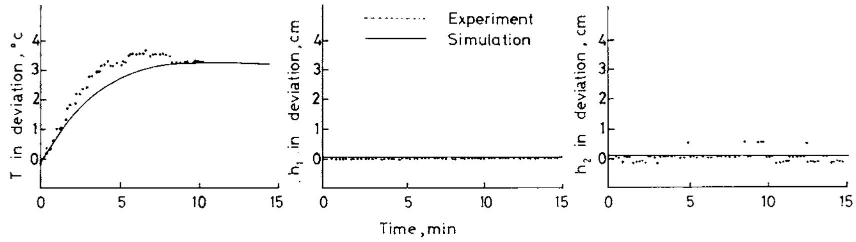

Fig. 12. Noninteracting responses to a setpoint change in T under MDD.

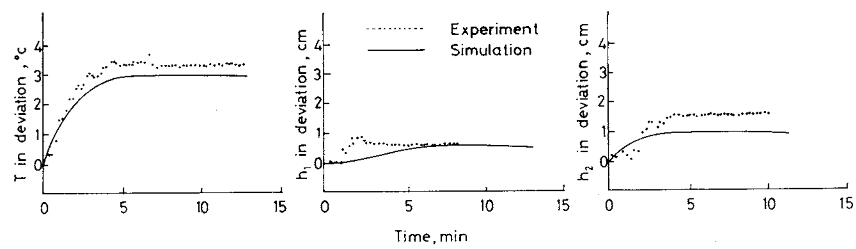

Fig. 13. Interacting responses to a setpoint change in T under PC.

state is attained. As illustrated in Figs. 10 and 11, both temperature and the two liquid levels are found to be held closely to the initial steady states with the MDD controller, while large steady-state offsets are shown with the INA controller.

From the results shown in Figs. 10 through 11, it is evident that steady-state offsets are unavoidable in the INA controller with constant gains, and as would be expected the control actions of the MDD controller are seen to be critically damped. In consideration of experimental error, the responses of the MDD controller show similar behavior to that of the simulated results indicating insensitivity to model error.

3.3 Noninteraction behavior of MDD

A setpoint of temperature only is changed from the initial steady states to investigate noninteracting behavior of the MDD control system. Results are compared with those of three single conventional proportional controllers. In the conventional proportional control system (PC), h_1 is paired with q_{c1}, h_2 with q_{c2}, and T with q_h in consideration of the physical meaning of the process, and the gains chosen are 70, 70 and 50, respectively corresponding to h_1, and h_2 and T in order to satisfy the constraints on the manipulated variables. Initial steady states are: $q_1 = 530\,\mathrm{cm^3/min}$, $q_{c2} = 680\,\mathrm{cm^3/min}$, $q_h = 0\,\mathrm{cm^3/min}$, $h_1 = 21.2\,\mathrm{cm}$, $h_2 = 15.2\,\mathrm{cm}$, and $T = 6°\mathrm{C}$. Constraints on the manipulated variables are as follows: $-115 \leq u_1 \leq 95$, $-150 \leq u_2 \leq 0$ and $0 \leq u_3 \leq 240$.

A setpoint of temperature is changed by 3°C when the MDD controller is applied, and by 5°C when PC is applied in order to make the new steady state of temperature obtained by PC equal to the setpoint of temperature when the MDD controller is applied in consideration of the steady-state offset caused by PC. As would be expected, the responses of levels by the MDD controller are kept nearly unchanged, but with PC considerable offsets are caused by interaction. Thus it is shown that the MDD control system reduces the interaction to a negligible level, as shown in **Figs. 12** and **13**.

4. Conclusion

This paper presents a design method that facilitates on-line design of a noninteracting controller to handle constraints on the control variables based on the derivative decoupling control approach. This proposed modified derivative decoupling control method is extended to load changes (unknown disturbances).

A controller is designed that exhibits both good simulated and experimental performance for both setpoint and load changes for a three-input, three-output process, although the model used for the controller design is based on linearization about the initial steady states.

It is demonstrated in this work that the modified derivative decoupling controller does not require off-line calculations to handle constraints on the control variables. Further, both the simulated and the experimental performance of the controller are found to be better than those obtained with a controller based on the inverse Nyquist array technique.

Nomenclature

A	= $n \times n$ constant matrix in a linearized form of Eq. (5)	
A_1, A_2	= cross-sectional areas of mixing tanks 1 and 2, respectively	[cm^2]
a_i	= proportional coefficients defined in Eq. (4)	
a_{ij}	= i-j element of A	
B	= $n \times n$ constant matrix in a linearized form of Eq. (5)	
$\hat{b}_{ij}$	= i-j element of B^{-1}	
c_p	= heat capacity of water	[J/g-°C]
d_i	= ith element of X_d	
E	= n-dimensional error defined by R-X	
e_i	= ith element of E	
F	= n-dimensional column vector of nonlinear functions defined in Eq. (1)	
G	= n-dimensional column vector defined by Eq. (12)	
h_1, h_2	= liquid levels of mixing tanks 1 and 2, respectively	[cm]
h_{ij}	= i-j element of $-B^{-1}A$ of Eq. (7)	
k_1, k_2	= contraction constants in Eqs. (21) and (22)	[cm$^{2.5}$/min]
q	= liquid flow rates	[cm^3/min]
R	= n-dimensional setpoint	
s	= Laplace transform variable	
T	= temperatures of liquids	[°C]
U	= n-dimensional manipulated variable	
u_i	= ith element of U	
X	= n-dimensional state variable	
X_d	= setpoint change	
x_i	= ith element of X	
W	= n-dimensional disturbance	
λ_i	= positive real constant defined in Eq. (8)	
ρ	= density of liquid	[g/cm^3]

⟨Superscripts⟩

min	= lower bound
max	= upper bound
*	= constant disturbances

⟨Subscripts⟩

c, c_1, c_2	= cold streams
h	= hot stream

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