

GRUPPEN SECRET SHARING OR HOW TO SHARE SEVERAL SECRETS IF YOU MUST?

LÁSZLÓ CSIRMAZ

(Communicated by Stanislav Jakubec)

ABSTRACT. Each member of an n -person team has a secret, say a password. The k out of n *gruppen secret sharing* requires that any group of k members should be able to recover the secrets of the other $n - k$ members, while any group of $k - 1$ or less members should have no information on the secret of other team member *even if other secrets leak out*. We prove that when all secrets are chosen independently and have size s , then each team member must have a share of size at least $(n - k)s$, and we present a scheme which achieves this bound when s is large enough. This result shows a significant saving over n independent applications of Shamir's k out of $n - 1$ threshold schemes which assigns shares of size $(n - 1)s$ to each team member independently of k .

We also show how to set up such a scheme without any trusted dealer, and how the secrets can be recovered, possibly multiple times, without leaking information. We also discuss how our scheme fits to the much-investigated multiple secret sharing methods.

©2013
Mathematical Institute
Slovak Academy of Sciences

1. Introduction

A team has n members, and each member has a secret, say a password. As a safety caution, they want each secret to be distributed among other group members so that it could be recovered in the case any of them would forget it. Also, none of them trusts the others, thus they want their secrets to be independent of the information held by any group $k - 1$ or less team members

2010 Mathematics Subject Classification: Primary 94A62, 90C25; Secondary 05B35.
Keywords: multiple secret sharing, complexity, threshold scheme, secret sharing, interpolation.

This research was partially supported by the “Lendület Program” of the Hungarian Academy of Sciences and the grant TAMOP-4.2.2.C-11/1/KONV_2012-0001.

— even if other secrets leak out. This goal can be achieved by distributing all secrets using Shamir’s k out of $n - 1$ threshold secret sharing method, see [9]. Assuming that all secrets are s bit long, the total size of the information each team member must remember will be $n \cdot s$ bits: s bits for each team member plus her own password.

The question is: can we do better if the secrets are distributed simultaneously?

This question comes under the name of *multiple secret sharing*, which has two distinct flavors:

1. Different secrets are to be recovered by different access structures, usually only one of the secrets will ever be recovered; also known as *multiple secret-sharing*. A typical question is how much the information to be remembered by each member can be squeezed compared to the independent applications of traditional secret sharing. Results in this direction can be found in [1, 2, 6, 7].
2. A group recovers multiple secrets, this is *multiple-secret sharing*. In this case in order to decrease private information, unconditional security is traded for computational security. See, e.g., [5, 8, 10].

In both cases, *verifiable* schemes can also be considered, where participants can check whether shares provided by others are genuine or not, look at [5, 10].

Our problem, which we call *k out of n gruppen secret sharing* (see Section 2), belongs to the first flavor (which, in fact, is more general than the second one), as each secret is to be recovered by a different collection of team members. In Sections 3 and 4 we concentrate on the typical secret sharing question, and determine the amount of information each participant must receive by proving a lower bound, and giving a matching construction.

Section 5 looks at how one of the secrets can be recovered. This is an intricate issue in a multiple secret settings as — understandably — we do not want to compromise others secrets when recovering someone’s (allegedly) forgot password. This requirement is easy to overlook. Interestingly, the “private recovery process” our construction from Section 3 suggests still leaks out some information. We propose a perfect solution at the expense of increasing the round complexity of the protocol.

Secret sharing methods, just as our construction does, usually refer to a trusted *dealer* who knows all the secrets, distributes the shares privately, and disappears after doing her job keeping all secrets. The homomorphic property of our construction suggests another setup performed by the participants without

any trusted third party. In Section 6 we look at it in details, and conclude that it is, in fact, secure, and does not leak out information.

2. The “gruppen secret sharing”

Multiple secret sharing schemes are a natural generation of single secret sharing schemes, and been defined formally, among others, in [1]. Informally, in such a scheme we have a set P of *participants*, and n *access structures* $\mathcal{A}_1, \dots, \mathcal{A}_n$, that is, upward closed collections of subsets of the participants. The *dealer* picks (or receives) an n -tuple of secrets $\langle s_1, \dots, s_n \rangle$ from some finite domain with a given distribution (typically secrets are independent and uniformly distributed), and computes, using some randomness, the *shares* of the participants.

DEFINITION 1. The multiple secret sharing scheme \mathcal{S} is *sound* if qualified subsets can recover the secret: whenever $A \in \mathcal{A}_i$, then members of A , using their private information only, can recover the secret s_i .

The scheme \mathcal{S} is *perfect*, if $B \subseteq P$ is not enabled to recover the secret s_i (that is, $B \notin \mathcal{A}_i$), then members in B , *even knowing all other secrets s_j for $j \neq i$* , have no more information on s_i than that already conveyed by the legally known values.

In particular, if the secrets are independently chosen, then the totality of shares of $B \notin \mathcal{A}_i$ should give no information on s_i whatsoever *even given all other secrets*.

A *gruppen secret sharing* scheme is a special, nevertheless interesting, case of multiple secret sharing schemes. Each participant has a secret drawn uniformly and independently from some finite domain S , say a password. Each password should be recoverable by any k out of the remaining $n - 1$ participants, but no coalition of $k - 1$ or less participants should know anything about the remaining secrets.

DEFINITION 2. In a k out of n *gruppen secret sharing scheme* there are n participants in P , participant $i \in P$ has a secret s_i drawn uniformly and independently from some domain, and the access structure \mathcal{A}_i — whose members should be able to recover s_i — consists of all subsets of $P - \{i\}$ with at least k members, that is the k out of $n - 1$ threshold structure on $P - \{i\}$.

For definiteness we assume that all secrets are s bit long (random) 0–1 sequences where s is large enough.

It is easy to construct a sound and perfect k out of n gruppen secret sharing scheme. For each participant $i \in P$, the dealer distributes the secret s_i to members of \mathcal{A}_i *independently* using Shamir's k out of $n - 1$ threshold scheme. As this latter scheme is also *perfect*, i.e., everyone gets a minimal size share (which is s bits), everyone receives $n - 1$ shares of s bit each, next to his s bit secret, which is a total $n \cdot s$ bits to remember. Is there any way to do it better? The next theorem answers this question.

THEOREM 3.

- a) *In a perfect and sound k out of n gruppen secret sharing scheme each participant must receive a share of at least $(n - k)s$ bits.*
- b) *For s large enough there is a perfect and sound k out of n gruppen secret sharing scheme where every participant receives exactly $(n - k)s$ bit share.*

We postpone the proof to Sections 3 and 4; here we illustrate the theorem for the case when $n = 3$ and $k = 2$. We have three participants whom we call Alice, Bob, and Cecil, having secrets a , b , and c , respectively. The lower bound on the share size is almost immediate. Bob has no information on Alice's and Cecil's secret. When Alice joins Bob, the two of them have enough information to determine both Alice's and Cecil's secret. This means $2s$ bits of information which should come from Alice. Her secret is s bit long, thus her share must also be at least s bit long to supply that much information.

As for the construction, the dealer should tell them shares which have size equal to that of the secrets, so that

- a) any two participant should be able to recover the secret of the third;
- b) no one should have any information on the others' secrets.

To satisfy the first requirement, our first attempt is to give Alice, Bob, and Cecil the shares c , a , and b , respectively. This way any two can recover the secret of the third one, but these shares definitely contradict the security requirement in Definition 1. So we "hide" these shares by xoring them with some value possessed by others: let the three shares be $c \oplus b$, $a \oplus c$, $b \oplus a$, respectively. Again, any two can recover the third's secret, for example Alice knows $c \oplus b$, Bob knows b , thus they can recover c . Unfortunately these shares also violate the security requirement. If Bob's secret leaks out, or Alice simply guesses it right, then Alice alone could recover Cecil's c . Also, if b and c are weak (and long) passwords, then it is a simply routine to recover both b and c from $c \oplus b$.

A solution could be using interpolating polynomials à la Shamir. Let r be a random polynomial which takes the secrets at its first three places:

$$r(0) = a, \quad r(1) = b, \quad \text{and} \quad r(2) = c.$$

Then let the shares be $r(3)$, $r(4)$, and $r(5)$, respectively. Any pair of participants knows r at four different places, thus if r has degree at most 3, then they can recover the polynomial r , thus the third participant's secret as well. The security requirement also holds: a single participant knows r 's value at two places. If one of the two other secrets leak out, then it is r 's value at an additional place. Given r 's value at (at most) three places provides no information about what values r can take at a fourth place, and this is what was required.

As usual, the polynomial r is over some finite field; the secrets are random elements from this field. The above scheme will work when the field has at least six distinct elements, thus we must have $s \geq 3$.

3. You cannot do better than ...

In this section we show that the amount of share every participant in a k out of n gruppen secret sharing scheme must have is at least $s \cdot (n - k)$ bits, where every secret is an (independent, uniformly random) s bit long 0–1 word. This proves the first part of Theorem 3.

First we give an informal reasoning, then we make it precise using the entropy method [3, 4]. Let a_1, \dots, a_{k-1} , and $b \in P$ be k different participants. We want to estimate the amount of private information b must have. By assumption, the totality of the private information the group $\{a_1, \dots, a_{k-1}, b\}$ has determines uniquely the secrets of the remaining $n - k$ participants, which amounts to $(n - k) \cdot s$ bits. By the security requirement, whatever $\{a_1, \dots, a_{k-1}\}$ know should be independent of those secrets *plus* the secret of b . Thus the additional $(n - k) \cdot s + s$ bits of information must be supplied by b . He has s bits of secret, thus must have a share of size at least $(n - k) \cdot s$ bits.

The above reasoning can be made precise using the so-called entropy method as described in, e.g., [3] or [4]. First of all, we consider the secrets and shares as random variables. The *size* of the value of a random variable ξ is its Shannon entropy $\mathbf{H}(\xi)$, which is (roughly) the number of necessary (independent) bits to define the value of ξ uniquely. Our assumption was that the secrets are independent s bit long 0–1 sequences, thus $\mathbf{H}(\xi\eta) = 2s$, where ξ and η are the secret values of two participants.

For any collection $\{\xi_i : i \in I\}$ of random variables define the real-valued function

$$f(I) = \mathbf{H}(\{\xi_i : i \in I\})$$

where the entropy is taken for the joint distribution of all indicated variables. For example, if a and b are (indices of) two participant's secrets, then $f(\{a\}) = s$, and $f(\{a, b\}) = 2s$ as we have seen above. Similarly, if j is (an index of) a share, then $f(\{j\})$ is the *size* of that share.

The function f is defined on all subsets of some finite set, and satisfies certain linear inequalities which follow from the so-called Shannon inequalities for the entropy function \mathbf{H} . The following claim collects those properties which will be used to prove the theorem. As usual, we write $f(XY)$ instead of $f(X \cup Y)$, and $f(x)$ and $f(xX)$ instead of $f(\{x\})$ and $f(\{x\} \cup X)$.

CLAIM 4. (See [4]) *For any subsets X and Y*

1. $f(X) \geq 0$ (*positivity*),
2. $f(X) \leq f(Y)$ if $X \subseteq Y$ (*monotonicity*),
3. $f(X) + f(Y) \geq f(XY)$ (*additivity*),
4. $f(XY) = f(X)$ if (the variables in) X determines the values of (the variables in) Y ;
5. $f(XY) = f(X) + f(Y)$ if X and Y are statistically independent.

The entropy method can be rephrased in a few words as follows. Let j be the (index) of any share. Suppose for *any function* f satisfying properties enlisted in Claim 4 there are (indices) a_1, \dots, a_ℓ of secrets such that

$$f(j) \geq f(a_1) + \dots + f(a_\ell).$$

The the size of share j must be at least ℓ times the size of the secrets.

LEMMA 5. *Suppose G is a group of participant with $k - 1$ members, and $a, \bar{b} = \langle b_1, \dots, b_{n-k} \rangle$ are the (indices of the) secrets of participants not in G and (the indices of) their shares are j and $\bar{j} = \langle j_1, \dots, j_{n-k} \rangle$, respectively. Then*

$$f(a) + f(j) \geq f(a\bar{b}).$$

Proof. Let us denote the total data (secret plus share) held by G by G as well. By assumption, G together with a and j determines all the secrets b_i , that is $f(ajG) = f(a\bar{b}jG)$. Also, G should have no information on the secrets a

and b_i or on their combinations, thus $f(a\bar{b}G) = f(a\bar{b}) + f(G)$. Using these, the additivity and monotonicity property of f , we have

$$f(a) + f(j) + f(G) \geq f(ajG) = f(a\bar{b}jG) \geq f(a\bar{b}G) = f(a\bar{b}) + f(G).$$

Comparing the first and last tag gives the claim of the Lemma. □

From this lemma we can easily deduct the required lower bound on the size of the share each participant receives.

Proof of first part of Theorem 3. Use notations from Lemma 5, in particular let a and j respectively be (indices of) the secret and the share of participant a . All secrets have the same size, thus

$$f(a) = f(b_1) = \dots = f(b_{n-k}).$$

By assumption the secrets are totally independent, which means

$$f(a\bar{b}) = f(ab_1 \dots b_{n-k}) = f(a) + f(b_1) + \dots + f(b_{n-k}) = (n - k + 1)f(a).$$

From Lemma 5 we know that $f(j) \geq f(a\bar{b}) - f(a) = (n - k)f(a)$, which proves part a) of Theorem 3. □

4. An (optimal) protocol

Our k out of n gruppen secret sharing scheme, whose complexity matches the bound given in Section 3, is a straightforward generalization of the one sketched in Section 2. Let \mathbb{F} be a finite field with more than $n(n - k + 1)$ elements. Secrets will be chosen uniformly and independently from \mathbb{F} , which means that if secrets are s bit long 0–1 sequences, then \mathbb{F} can be chosen to be the field of characteristic 2 on 2^s elements. To give a scheme means to describe how the dealer computes (determines) the shares given the randomly and uniformly chosen secrets; or, equivalently, how the dealer can distribute the shares *and the secrets* simultaneously as long as the secrets come from the appropriate distribution. We will choose this latter approach, and hint how to modify the scheme when the secrets are given in advance.

Let p_i for $1 \leq i \leq n$ denote the participants. The dealer chooses different field elements $x_{i,j}$ for $1 \leq i \leq n$ and $0 \leq j \leq n - k$, and picks a random polynomial $r(x)$ over \mathbb{F} of degree less than $k(n - k + 1)$.

The *secret* of participant p_i will be the value of r at $x_{i,0}$. (When given the secrets in advance, the same distribution can be achieved by simply choosing r

randomly from among those polynomials which give the secret of p_i at $r(x_{i,0})$.) As for the share, the dealer gives participant p_i all field elements $r(x_{i,1})$ up to $r(x_{i,n-k})$. Observe that secrets are uniform random elements from the field, thus the “size” (entropy) of every secret is the same, namely $\log_2(|\mathbb{F}|)$. Similarly, all participants receive $(n - k)$ field elements as share, therefore the size of the share is exactly $(n - k)$ times that of the secret.

We claim that any k participants can determine the secret value of the remaining $n - k$ participants. This is clear, as the k participants know the value of r at $k(n - k + 1)$ different places, while r has smaller degree, thus they can determine r , and its value at $x_{p,0}$ for any participant p .

Next, we claim that the total information of $k - 1$ participants is statistically independent of the secrets of the other $n - k + 1$ participants. This is true as r is a random polynomial of degree below $k(n - k + 1)$, and $k - 1$ participants know the value of this polynomial at $(k - 1)(n - k + 1)$ places, thus the polynomial can take all the possibilities with equal probability at any $n - k + 1$ predetermined places — in particular at $x_{p,0}$ where p runs over the missing $n - k + 1$ participants. Consequently, all private information of $k - 1$ participants, *plus* the secret of all but one remaining participants is statistically independent of the secret of the last participant. This is exactly the security requirement which proves the second part of Theorem 3.

5. Secret recovery

The method outlined in the previous Section to recover the secret of $p \in P$ was that k participants, using their private values, recover the polynomial r , and then compute r 's value at $x_{p,0}$. This recovery process has the drawback that once r is known, all secrets are revealed, not only the secret of p . How can we achieve that they recover the value of r at $x_{p,0}$ only and not the whole polynomial r ? Let $B \subseteq \{1, \dots, n\}$ be the subset of size k which wants to recover the secret of $p \notin B$. As the values $x_{i,j}$ are publicly known, everyone can compute the constants $\lambda_{i,j} \in \mathbb{F}$ using, e.g., the Lagrange interpolation formula such that

$$r(x_{p,0}) = \sum_{i \in B} \sum_{j=0}^{n-k} \lambda_{i,j} r(x_{i,j})$$

independently what the values $r(x_{i,j})$ are. Consequently to recover p 's secret, participant $i \in B$ should only compute the sum

$$t_{i,p} = \sum_{j=0}^{n-k} \lambda_{i,j} r(x_{i,j}) \tag{1}$$

and send it privately to p , rather than revealing all the $r(x_{i,j})$ values. p receives the k values (1) from participants in B , he simply adds them up to recover his secret.

Unfortunately this process leaks out information, and cannot be repeated indefinitely. To see why this is the case, we go back to the 2 out of 3 gruppen secret sharing scheme as discussed in Section 2. Alice, Bob, and Cecil have secrets $r(0)$, $r(1)$, and $r(2)$, and have shares $r(3)$, $r(4)$, and $r(5)$, respectively. Now Alice announces that she lost her secret. Lagrange says that

$$r(0) = \frac{10}{3}r(1) + \frac{5}{3}r(4) - \frac{10}{3}f(2) - \frac{2}{3}r(5),$$

thus Bob sends Alice the value

$$t_b = \frac{10}{3}r(1) + \frac{5}{3}r(4),$$

and Cecil sends

$$t_c = -\frac{10}{3}f(2) - \frac{2}{3}r(5).$$

The Alice could recover her secret as $t_b + t_c$. However, Alice could be cheating, as she still have her share $r(3)$. Again by Lagrange

$$r(3) = -\frac{1}{6}r(1) + \frac{2}{3}r(4) + \frac{2}{3}r(2) - \frac{1}{6}r(5).$$

Alice can eliminate $r(4)$ and $r(5)$ using the values she received from Bob and Cecil, thus she knows

$$\frac{2}{3}\left(r(3) - \frac{2}{5}t_b - \frac{1}{4}t_c\right) = -r(1) + r(2)$$

a nontrivial combination of Bob's and Cecil's secrets. Thus if any of those two secrets leak out, or Alice could successfully guess it, she'll know the other secret immediately.

Switching to linear algebra from polynomial interpolation, a random polynomial of degree less than $k(n - k + 1)$ can be considered as a random vector if the $k(n - k + 1)$ -dimensional space. Knowing the value at a certain place amounts to know a (fixed) linear combination of the coefficients of the random vector. Initially every participant knows $(n - k + 1)$ such linear combinations. Thus the codimension of $k - 1$ participants private information is $n - k + 1$, it is just the linear space where the remaining $n - k + 1$ participants have their secrets.

During the recovery procedure p receives k further linear combinations (1). As these add up to his secret, the number of new linear combinations he knows is $(k - 1)$ more. Thus if this process p is joined by $k - 2$ other participants, the

codimension of their information is $n - 2(k - 1)$, thus must leak some information about the other's secrets.

A possible remedy is to keep track the codimension of the total information of any $k' \leq k$ participants, and let run the recovery process until it is large enough. Also, if p announces that he lost his secret, then p should be excluded in any further recovery stage.

Another remedy is to use freshly generated random values which hide the exact values of the sums (1) from p . This solution has the drawback that it increases the communication overhead, requires some further (trivial) communication. However it has further advantages:

- anyone's secret can be recovered arbitrary number of times without affecting the security level;
- not only the secret, but also the shares can be recovered without significant increase in the communication, thus recovering the "full state" after a break down.

As before, let B be the k -element set of participants who want to recover p 's secret. Each $i \in B$ generates k random and independent elements from \mathbb{F} , say $r_{i,j}$, $j \in B$. Then i sends $r_{i,j}$ to j . After this step i will know all $r_{i,j}$ (as he generated those numbers), and $r_{j,i}$ (as he received them from the others). After this i sends p the obfuscated element

$$t'_{i,p} = t_{i,p} + \sum_{j \in B} (r_{i,j} - r_{j,i}). \quad (2)$$

After receiving all sums in (2), p simply adds them up and recovers his secret.

Rather than interpolating the polynomial r at the place $x_{p,0}$ only, participants in B can interpolate r at every $x_{p,j}$ and compute the sums similar to (1). In the obfuscating step everyone generates $k(n - k + 1)$ random elements (k for each j) independently, and then sends the $(n - k + 1)$ obfuscated interpolation sums to p , who can recover his secret plus all the shares. This way no private information is leaked out, and the whole process can be repeated indefinitely.

6. How to distribute the shares

Any secret sharing scheme relies on a trusted *dealer* to set up the scheme, who collects the secrets from the participants, generates the shares, and tells the every participant her share privately, and then disappears without leaking out

GRUPPEN SECRET SHARING

any information. Such a trusted entity is quite hard to find, protocols not relying on trusted party are preferable to ones which use one. Fortunately, the scheme described in Section 4 has the *homomorphic property*: if a scheme distributes secrets s_i and with shares h_i for $i \in P$, another scheme distributes secrets s'_i and has shares h'_i , then for the secrets $s_i + s'_i$ the shares $h_i + h'_i$ are correct ones, and have the appropriate distribution. Here the addition is the addition in the field \mathbb{F} .

Using this homomorphic property, a gruppen secret sharing scheme can be set up by the participants as follows. Suppose participant $i \in B$ has the secret s_i . He computes, as a dealer, the shares of the k out of n gruppen secret sharing scheme as described in Section 4, where all secrets are zero, except his own, which is s_i . He generates the shares $h_{i,j}$ for $j \in P$, and then sends $h_{i,j}$ to participant j .

Each participant receives shares from everyone else (including himself), and his share in the final scheme is just the sum of all shares received. As a consequence of the homomorphic property, in this way the participants achieved a correct scheme which distributes their secrets. In an ideal scheme the participant $p \in P$ receives only his share from the dealer; now every participant receives an $(n - k)$ -dimensional vector of \mathbb{F} from the others such that his share is the sum of the vectors received. Thus it might happen that a coalition of $k - 1$ participants could extract extra information from the values they received. We claim that this is not the case, this set-up is, in fact, $k - 1$ -secure.

Let us fix a coalition B of $k - 1$ participants, and look at what they receive from $p \notin B$. p generates a random polynomial of degree $< k(n - k + 1)$ which takes zero at $x_{i,0}$, $i \neq p$, and p 's secret at $x_{p,0}$. To make the polynomial random, its value should be given *randomly and independently* at further $k(n - k + 1) - n = (k - 1)(n - k)$ places. Now participants in the coalition B receive the polynomial's value at exactly $(k - 1)(n - k)$ places, thus the random polynomial can be set up by choosing its value at these places randomly and independently. This also means that members of B can extract no information from the values they receive from p .

REFERENCES

- [1] BLUNDO, C.—MASUCCI, B.: *Randomness in multi-secret sharing schemes*, J.UCS **5** (1999), 367–389
- [2] BLUDO, C. — SANTIS, A. DE — CRESCENZO, G. DI — GAGGIA, A. G. — VACCARO, U.: *Multi-secret sharing schemes*. In: *Advances in Cryptology – CRYPTO'94* (Y. G. Desmedt, ed.). Lecture Notes in Comput. Sci. 839, Springer, Berlin, 1994, pp. 150–163.

LÁSZLÓ CSIRMAZ

- [3] CAPOCELLI, R. M.—SANTIS, A. DE—GARGANO, L.—VACCARO, U.: *On the size of shares of secret sharing schemes*, J. Cryptology **6** (1993), 157–168.
- [4] CSIRMAZ, L.: *Secret sharing schemes on graphs*, Studia Sci. Math. Hungar. **44** (2007) 297–306; Available as IACR preprint <http://eprint.iacr.org/2005/059>
- [5] DAS, A.—ADHIKARI, A.: *An efficient multi-use multi-secret sharing scheme based on hash function*, Appl. Math. Lett. **23** (2010) 993–996.
- [6] FERRAS, O.—GRACIA, I.—MARTIN, S.—PADRO, C.: *Linear threshold multiset secret sharing schemes*, In: ICITS 2009, pp. 110–126.
- [7] JACKSON, W.-A.—MARTIN, K. M.—O’KEEFE, C. M.: *Multiset secret threshold schemes*. In: Advances in Cryptology – CRYPTO’93 (D. R. Stinson, ed.). Lecture Notes in Comput. Sci. 773, Springer, Berlin, 1994, pp. 126–135.
- [8] HAN-YO LINE—YI-SHIUNG YEH: *Dynamic multi-secret sharing scheme*, Int. J. Contemp. Math. Sci. **3** (2008), 37–42.
- [9] SHAMIR, A.: *How to share a secret*, Commun. ACM **22** (1979), 612–613.
- [10] ZHAO, J.—ZHANG, J.—ZHAO, R.: *A practical verifiable multi-secret sharing scheme*, Comput. Standards Interfaces **29** (2007) 138–141.

Received 15. 9. 2011

Accepted 13. 12. 2012

Central European University

Budapest

Rényi Institute

Budapest

HUNGARY

E-mail: csirmaz@renyi.hu