VERSITA

Mathematica Slovaca

DOI: 10.2478/s12175-011-0049-7
Math. Slovaca **61** (2011), No. 5, 817–826

THE STABILITY OF AN ADDITIVE FUNCTIONAL EQUATION IN MENGER PROBABILISTIC φ-NORMED SPACES

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(*Communicated by Anatolij Dvurečenskij*)

ABSTRACT. We establish a stability result concerning the functional equation:

$$\sum_{i=1}^{m} f\left(mx_i + \sum_{j=1,\, j\neq i}^{m} x_j\right) + f\left(\sum_{i=1}^{m} x_i\right) = 2f\left(\sum_{i=1}^{m} mx_i\right)$$

in a large class of complete probabilistic normed spaces, via fixed point theory.

1. Introduction

The study of stability problems for functional equations is related to a question of Ulam [27] concerning the stability of group homomorphisms, affirmatively answered by Hyers [7] and further generalized by Aoki [1] for additive mappings and by Rassias [24] for linear mappings. The notion of the Hyers-Ulam-Rassias stability for a functional equation originates from the paper of Rassias [24]. There is a vast literature concerning the Hyers-Ulam-Rassias stability of functional equations. For some information on such problems the interested reader is referred to the papers [3, 5, 8, 9, 11, 25]. The notion of fuzzy stability of the functional equations has been introduced in [20], and developed in the recent papers [2], [13]–[22].

Radu [23] pointed out that a fixed point alternative method can be successfully used to solve the Ulam problem. The fixed point method in the study of

2010 Mathematics Subject Classification: Primary 54E40; Secondary 39B82, 46S50, 46S40.
Keywords: probabilistic normed space, Hyers-Ulam-Rassias stability, additive functional equation, fixed point theorem.

the probabilistic stability of functional equations firstly appears in [17]. In [14] and [15] a fixed point theorem of Luxemburg and Jung is used in order to prove the probabilistic stability of some functional equations. In this paper we apply the fixed point method to investigate the Hyers-Ulam-Rassias stability for the functional equation:

$$\sum_{i=1}^{m} f\left(mx_i + \sum_{j=1,\, j\neq i}^{m} x_j\right) + f\left(\sum_{i=1}^{m} x_i\right) = 2f\left(\sum_{i=1}^{m} mx_i\right), \tag{1.1}$$

where $m \geq 2$ is a given integer and the unknown is a mapping f from a real linear space to a Menger probabilistic φ-normed space (the notion of Menger probabilistic φ-normed space has been recently introduced in this journal by Golet [6]). We note that a mapping $f\colon X \to Y$ satisfies (1.1) if and only if it is additive ([4]).

A function $F\colon \mathbb{R} \to [0,1]$ is called a distribution function if it is non-decreasing and left continuous with $\sup_{t\in\mathbb{R}} F(t) = 1$ and $\inf_{t\in\mathbb{R}} F(t) = 0$. The class of all distribution functions F with $F(0) = 0$ is denoted by D^+. By ε_0 is denoted the distribution function defined through

$$\varepsilon_0(t) = \begin{cases} 0, & t \leq 0, \\ 1, & t > 0. \end{cases}$$

Definition 1.1. A mapping $T\colon [0,1] \times [0,1] \to [0,1]$ is a triangular norm (briefly, a t-norm) if T satisfies the following conditions:

(i) T is commutative and associative;
(ii) $T(a,1) = a$ for all $a \in [0,1]$;
(iii) $T(a,b) \leq T(c,d)$ whenever $a \leq c$ and $b \leq d$ $(a,b,c,d \in [0,1])$.

Two typical examples of continuous t-norms are $T_L(a,b) = \max(a+b-1,0)$ and $T_M(a,b) = \min(a,b)$.

The concept of Menger probabilistic φ-normed space has been introduced by Goleţ in [6].

Let φ be a function defined on the real field $\mathbb{R}$ into itself, with the following properties:

(a) $\varphi(-t) = \varphi(t)$ for every $t \in \mathbb{R}$;
(b) $\varphi(1) = 1$;
(c) φ is strictly increasing and continuous on $[0,\infty)$, $\varphi(0) = 0$ and $\lim_{\alpha\to\infty} \varphi(\alpha) = \infty$.

Examples of such functions are:

$$\varphi(t) = |t|; \qquad \varphi(t) = |t|^p, \quad p \in (0, \infty); \qquad \varphi(t) = \frac{2t^{2n}}{|t| + 1} \quad (n \in \mathbb{N}).$$

Definition 1.2. ([6]) A *Menger probabilistic φ-normed space* is a triple (X, ν, T), where X is a real vector space, T is a continuous t-norm, and ν is a mapping from X into D^+ such that the following conditions hold:

(PN1) $\nu_x(t) = \varepsilon_0(t)$ for all $t > 0$ if and only if $x = 0$;

(PN2) $\nu_{\alpha x}(t) = \nu_x\left(\frac{t}{\varphi(\alpha)}\right)$ for all x in X, $\alpha \neq 0$ and $t > 0$;

(PN3) $\nu_{x+y}(t + s) \geq T(\nu_x(t), \nu_y(s))$ for all $x, y \in X$ and $t, s \geq 0$.

From (PN2) it follows that $\nu_{-x}(t) = \nu_x(t)$ $(x \in X,\ t \geq 0)$.

We note that probabilistic φ-normed spaces include, in a natural way, p-normed spaces ([6]).

If (X, ν, T) is a Menger probabilistic φ-normed space under a continuous t-norm T such that $T \geq T_L$, then

$$\mathscr{V} = \{V(\varepsilon, \lambda) :\ \varepsilon > 0,\ \ \lambda \in (0, 1)\}, \qquad V(\varepsilon, \lambda) = \{x \in X :\ \nu_x(\varepsilon) > 1 - \lambda\}$$

is a complete system of neighborhoods of null vector for a linear topology on X generated by the φ-norm ν ([6]).

Definition 1.3. Let (X, ν, T) be a Menger probabilistic φ-normed space.

(i) A sequence $\{x_n\}$ in X is said to be *convergent* to x in X in the topology τ if for every $t > 0$ and $\varepsilon > 0$, there exists positive integer N such that $\nu_{x_n - x}(t) > 1 - \varepsilon$ whenever $n \geq N$.

(ii) A sequence $\{x_n\}$ in X is called *Cauchy* in the topology τ if, for every $t > 0$ and $\varepsilon > 0$, there exists positive integer N such that $\nu_{x_n - x_m}(t) > 1 - \varepsilon$ whenever $n \geq m \geq N$.

(iii) A Menger probabilistic φ-normed space (X, ν, T) is said to be *complete* if and only if every Cauchy sequence in X is convergent to a point in X.

The notion of generalized metric space has been introduced by Luxemburg in [12], by allowing the value $+\infty$ for the distance mapping. In the proof of our main theorem we use the following fixed point theorem in generalized metric spaces [10] (also see [14]).

Lemma 1.4 (**Luxemburg-Jung theorem**). ([10]) *Let (X, d) be a complete generalized metric space and $A\colon X \to X$ be a strict contraction with the Lipschitz constant $L \in (0,1)$, such that $d(x_0, A(x_0)) < +\infty$ for some $x_0 \in X$. Then A has a unique fixed point in the set $Y := \{y \in X : \ d(x_0, y) < \infty\}$ and the sequence $(A^n(x))_{n\in\mathbb{N}}$ converges to the fixed point x^* for every $x \in Y$. Moreover, $d(x_0, A(x_0)) \leq \delta$ implies $d(x^*, x_0) \leq \frac{\delta}{1-L}$.*

Let X be a linear space, (Y, ν, T_M) be a complete Menger probabilistic φ-normed space and G be a mapping from $X \times \mathbb{R}$ into $[0,1]$, such that $G(x, \cdot) \in D_+$ for all x. Consider the set $E := \{g\colon X \to Y : \ g(0) = 0\}$ and the mapping d_G defined on $E \times E$ through

$$d_G(g, h) = \inf\{a \in \mathbb{R}^+ : \ \nu_{g(x)-h(x)}(at) \geq G(x, t) \ \text{ for all } \ x \in X \ \text{ and } \ t > 0\} \tag{1.2}$$

where, as usual, $\inf \emptyset = +\infty$.

The following lemma can be proved as in [13]:

Lemma 1.5. (cf. [13, 17]) *d_G is a complete generalized metric on E.*

2. Probabilistic stability of the functional equation (1.1)

For convenience, we use the following abbreviation for a given mapping $f\colon X \to Y$:

$$Df(x_1, \dots, x_m) := \sum_{i=1}^{m} f\left(mx_i + \sum_{j=1,\, j\neq i}^{m} x_j\right) + f\left(\sum_{i=1}^{m} x_i\right) - 2f\left(\sum_{i=1}^{m} mx_i\right)$$

for all $x_j \in X$ $(1 \leq j \leq m)$.

Theorem 2.1. *Let X be a real linear space, (Y, ν, T_M) be a complete Menger probabilistic φ-normed space and let $f\colon X \to Y$ be a Φ-approximate solution of the equation (1.1), in in the sense that*

$$\nu_{Df(x_1,\dots,x_m)}(t) \geq \Phi(x_1, \dots, x_m)(t) \qquad (x_1, \dots, x_m \in X, \ \ t > 0), \tag{2.1}$$

where Φ is a mapping from X^m to D^+.

If

$$\big(\exists\alpha \in (0, \varphi(m))\big)(\forall x \in X)(\forall t > 0)\big(\Phi(mx, 0, \dots, 0)(\alpha t) \geq \Phi(x, 0, \dots, 0)(t)\big) \tag{2.2}$$

and

$$\lim_{n\to\infty} \Phi(m^n x_1, \dots, m^n x_m)\left(\frac{t}{\varphi(\frac{1}{m^n})}\right) = 1 \qquad (x_1, \dots x_m \in X, \ \ t > 0) \tag{2.3}$$

then there is a unique additive mapping $g\colon X \to Y$ *such that*

$$\nu_{g(x)-f(x)}(t) \geq \Phi(x,0,\dots,0)\left((\varphi(m)-\alpha)t\right) \qquad (x \in X, \ \ t > 0). \tag{2.4}$$

Moreover,

$$g(x) = \lim_{n\to\infty} \frac{f(m^n x)}{m^n} \qquad (x \in X).$$

P r o o f. By setting $x_1 = x$ and $x_j = 0$ $(2 \leq j \leq m)$ in (2.1), we obtain

$$\nu_{mf(x)-f(mx)}(t) \geq \Phi(x,0,\dots,0)(t) \qquad (x \in X, \ \ t > 0),$$

whence

$$\nu_{m^{-1}f(mx)-f(x)}(t) \geq \Phi(x,0,\dots,0)(\varphi(m)t) \qquad (x \in X, \ \ t > 0).$$

Let $G(x,t) := \Phi(x,0,\dots,0)(\varphi(m)t)$. Consider the set $E := \big\{g\colon X \to Y : g(0) = 0\big\}$ and the mapping d_G defined on $E \times E$ by (1.2). By Lemma 1.5, (E, d_G) is a complete generalized metric space. Now, let us consider the linear mapping $J\colon E \to E$, $Jg(x) := \frac{1}{m} g(mx)$. It is easy to check that J is a strictly contractive self-mapping of E with the Lipschitz constant $L = \alpha/\varphi(m)$.

Indeed, let g, h in E be such that $d_G(g,h) < \varepsilon$. Then

$$\nu_{g(x)-h(x)}(\varepsilon t) \geq G(x,t) \qquad (x \in X, \ \ t > 0),$$

whence

$$\nu_{Jg(x)-Jh(x)}\left(\frac{\alpha}{\varphi(m)}\varepsilon t\right) = \nu_{g(mx)-h(mx)}(\alpha\varepsilon t) \geq G(mx,\alpha t) \qquad (x \in X, \ \ t > 0).$$

Since $G(mx,\alpha t) \geq G(x,t)$ for all $x \in X$ and $t > 0$, then $\nu_{Jg(x)-Jh(x)}\left(\frac{\alpha}{\varphi(m)}\varepsilon t\right) \geq G(x,t)$ $(x \in X,\ t > 0)$, that is, $d_G(g,h) < \varepsilon \implies d_G(Jg,Jh) \leq \frac{\alpha}{\varphi(m)}\varepsilon$. This means that

$$d_G(Jg,Jh) \leq \frac{\alpha}{\varphi(m)} d_G(g,h),$$

for all g, h in E.

Next, from

$$\nu_{f(x)-m^{-1}f(mx)}(t) \geq G(x,t) \qquad (x \in X, \ \ t > 0)$$

it follows that $d_G(f, Jf) \leq 1$. By using Lemma 1.4 we deduce the existence of a fixed point of J, that is, the existence of a mapping $g\colon X \to Y$ such that $g(mx) = mg(x)$, for all $x \in X$.

Also, $d_G(f,g) \leq \frac{1}{1-L} d(f,Jf)$ implies the inequality $d_G(f,g) \leq \frac{1}{1-\frac{\alpha}{\varphi(m)}}$ from which it immediately follows $\nu_{g(x)-f(x)}\left(\frac{\varphi(m)}{\varphi(m)-\alpha}t\right) \geq G(x,t)$ $(x \in X,\ t > 0)$.

This means that

$$\nu_{g(x)-f(x)}(t) \geq G\left(x, \frac{\varphi(m)-\alpha}{\varphi(m)} t\right) \qquad (x \in X, \quad t > 0),$$

whence we obtain the estimation

$$\nu_{g(x)-f(x)}(t) \geq \Phi(x,0)\left((\varphi(m)-\alpha)t\right) \qquad (x \in X, \quad t > 0).$$

Since for any $x \in X$ and $t > 0$,

$$d_G(u,v) < \varepsilon \implies \nu_{u(x)-v(x)}(t) \geq G\left(x, \frac{t}{\varepsilon}\right),$$

from $d_G(J^n f, g) \to 0$, it follows $\lim_{n\to\infty} \frac{f(m^n x)}{m^n} = g(x)$ $(x \in X)$.

The additivity of g can be proven in the standard way, see [14], [21]. In fact, since T_M is a continuous t-norm, then $z \to \nu_z$ is continuous and thus, see [26, Chapter 12],

$$\begin{aligned} \nu_{Dg(x_1,\ldots,x_m)}(t) &= \lim_{n\to\infty} \nu_{\frac{Df(m^n x_1,\ldots,m^n x_m)}{m^n}}(t) \\ &= \lim_{n\to\infty} \nu_{Df(m^n x_1,\ldots,m^n x_m)}\left(\frac{t}{\varphi(\frac{1}{m^n})}\right) \\ &\geq \lim_{n\to\infty} \Phi(m^n x_1,\ldots,m^n x_m)\left(\frac{t}{\varphi(\frac{1}{m^n})}\right) \\ &= 1. \end{aligned}$$

We infer that $\nu_{Dg(x_1,\ldots,x_m)}(t) = 1$ for all $t > 0$, which implies

$$Dg(x_1,\ldots,x_m) = 0.$$

The uniqueness of g follows from the fact that g is the unique fixed point of J in the set $\{h \in E : \ g_G(f,h) < \infty\}$, that is, the only one with the property

$$\big(\exists C \in (0,\infty)\big)(\forall x \in X)(\forall t > 0)\big(\nu_{g(x)-f(x)}(Ct) \geq G(x,t)\big).$$

□

Corollary 2.2. *Let X be a real linear space, (Y, ν, T_M) be a complete Menger probabilistic φ-normed space and let $f\colon X \to Y$ be a Φ-approximate solution of the equation* (1.1)*. Suppose that*

$$\begin{aligned} &\big(\exists \alpha \in (0,\varphi(m))\big)(\forall x_1,\ldots,x_m \in X)(\forall t > 0) \\ &\qquad \big(\Phi(mx_1,\ldots,mx_m)(\alpha t) \geq \Phi(x_1,\ldots,x_m)(t)\big) \end{aligned} \tag{2.5}$$

and

$$\lim_{n\to\infty} \alpha^n \varphi\left(\frac{1}{m^n}\right) = 0. \tag{2.6}$$

Then there is a unique additive mapping $g\colon X \to Y$ such that

$$\nu_{g(x)-f(x)}(t) \geq \Phi(x,0,\dots,0)\left((\varphi(m)-\alpha)t\right) \qquad (x \in X, \quad t > 0). \tag{2.7}$$

Moreover,

$$g(x) = \lim_{n\to\infty} \frac{f(m^n x)}{m^n} \qquad (x \in X).$$

The proof immediately follows from the remark that

$$\lim_{n\to\infty} \Phi(m^n x_1, \dots m^n x_m)\left(\frac{t}{\varphi(\frac{1}{m^n})}\right) \geq \lim_{n\to\infty} \Phi(x_1,\dots,x_m)\left(\frac{t}{\alpha^n \varphi(\frac{1}{m^n})}\right) = 1.$$

We note that although the condition (2.5) is stronger than (2.2), it is more convenable to work with the condition (2.6) than with (2.3).

The next Corollary provides a Hyers-Ulam-Rassias stability result for the equation (1.1), similar to that in [4, Theorem 2.2].

Corollary 2.3. *Let X be a real linear space, let f be a mapping from X into a Banach p-normed space $(Y, \|\cdot\|_p)$ ($p \in (0,1]$) and let $\Psi\colon X^m \to \mathbb{R}^+$ be a mapping with the property*

$$\big(\exists \alpha \in (0, m^p)\big)(\forall x_1,\dots,x_m \in X)\big(\Psi(mx_1,\dots,mx_m) \leq \alpha\Psi(x_1,\dots,x_m)\big).$$

If

$$\|Df(x_1,\dots,x_m)\|_p \leq \Psi(x_1,\dots,x_m) \qquad (x_1,\dots,x_m \in X),$$

then there is a unique additive mapping $g\colon X \to Y$ such that

$$\|g(x) - f(x)\|_p \leq \frac{1}{|m|^p - \alpha}\Psi(x,0,\dots,0) \qquad (x \in X).$$

Moreover,

$$g(x) = \lim_{n\to\infty} \frac{f(m^n x)}{m^n} \qquad (x \in X).$$

Proof. Recall that a *p-normed space* ($0 < p \leq 1$) is a pair $(Y, \|\cdot\|_p)$, where Y is real linear space and $\|\cdot\|_p$ is a real valued function on Y (called a p-norm) satisfying the following conditions:

(i) $\|x\|_p \geq 0$ for all $x \in Y$ and $\|x\|_p = 0$ if and only if $x = 0$;

(ii) $\|\lambda x\|_p = |\lambda|^p \|x\|_p$ for all $x \in Y$ and $\lambda \in \mathbb{R}$;

(iii) $\|x + y\|_p \leq \|x\|_p + \|y\|_p$ for all $x, y \in Y$.

A p-normed space $(Y, \|\cdot\|_p)$ induces a Menger probabilistic φ-normed space (Y, ν, T_M), through

$$\nu_x(t) = \frac{t}{t + \|x\|_p}$$

and $\varphi(t) = |t|^p$.

Indeed, (PN1) is obviously verified and (PN2) follows from

$$\nu_{\alpha x}(t)=\frac{t}{t+\|\alpha x\|_p}=\frac{t}{t+|\alpha|^p\|x\|_p}=\frac{t/|\alpha|^p}{t/|\alpha|^p+\|x\|_p}$$
$$=\nu_x\left(\frac{t}{|\alpha|^p}\right)=\nu_x\left(\frac{t}{\varphi(\alpha)}\right).$$

Finally, if $\frac{t}{t+\|x\|_p}\le\frac{s}{s+\|y\|_p}$, then the inequality

$$\frac{t+s}{t+s+\|x+y\|_p}\ge\frac{t}{t+\|x\|_p}$$

follows from $t\|x\|_p+s\|x\|_p\ge t\|x\|_p+t\|y\|_p\ge t\|x+y\|_p$.

If we consider the induced Menger probabilistic φ-normed space (Y,ν,T_M) and the mapping $\Phi\colon X^m\to D^+$, $\Phi(x_1,\dots,x_m)(t)=\frac{t}{t+\Psi(x_1,\dots,x_m)}$, then the condition

$$\Phi(mx_1,\dots,mx_m)(\alpha t)\ge\Phi(x_1,\dots,x_m)(t)\qquad(x_1,\dots,x_m\in X,\quad t>0)$$

is equivalent to

$$\Psi(mx_1,\dots,mx_m)\le\alpha\Psi(x_1,\dots,x_m)\qquad(x_1,\dots,x_m\in X)$$

and the condition

$$\lim_{n\to\infty}\alpha^n\varphi\left(\frac{1}{m^n}\right)=0$$

reduces to

$$\lim_{n\to\infty}\left(\frac{\alpha}{m^p}\right)^n=0.$$

Now the conclusion follows from Corollary 2.2. □

Conclusion

We established the stability of the functional equation

$$\sum_{i=1}^m f\left(mx_i+\sum_{j=1,\,j\ne i}^m x_j\right)+f\left(\sum_{i=1}^m x_i\right)=2f\left(\sum_{i=1}^m mx_i\right)\qquad(m\in\mathbb{N},\quad m\ge 2),$$

where the unknown f is a mapping with values in a Menger probabilistic φ-normed space. As a particular case we obtained a Hyers-Ulam-Rassias stability result for this equation when Y is a p-Banach space, similar to that in the recent paper [4].

Acknowledgement. The authors are grateful to the Managing Editor and the reviewer for their valuable comments and suggestions.

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Received 3. 3. 2009
Accepted 19. 3. 2009

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