

# Analysis of Pulsatile Flow of Blood in a Porous Channel under Effect of Magnetic Field

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## Abstract

*To study the effect of magnetic field on pulsatile flow of blood in a porous channel a numerical model has been developed. An approximate solution is presented to the problem of pulsatile flow of blood in a porous channel in presence of transverse magnetic field. The blood is assumed to be an incompressible non Newtonian fluid. To reduce the equation of motion to an ordinary differential equation, a dimensionless variable is used. Numerical results were obtained for different values of the magnetic parameter, frequency parameter and Reynolds number. It is observed that when the Hartmann number increases, the fluid velocity as well as magnitude as well as magnitude of mass flux decrease.*

**Key words:** Pulsatile, Magnetic Field, Blood flow, Injection, Unsteady.

## 1. Introduction

Study of blood flows in the vessels of human circulatory system has become a matter of scientific research for quite long period. Mathematical

treatment of the problem has been subjected to constant changes and modifications to account for new evidence uncovered through improved experimental measurements. The most consistent treatment of the problem was given by Woomersley[121]. Later his analysis was extended by others to include the effect of initial stresses, per vascular tethering and orthotropic and visco-elastic behaviour of the arterial wall. A detailed comparison of this group of articles was given by Cox[30]. Woomersley's theory and its extensions are based on the linearised Navier-Stokes equations and small elastic deformations. Although they are shown to be satisfactory in describing certain aspects of the flow in small arteries, they fail to give an adequate representation of the flow field especially in large arteries(Ling [61]). Due to the large dynamic storage effect of these arteries, the non-linear convective acceleration terms of the Navier-Stokes equations are no longer negligible. Moreover the walls of arteries undergo large deformations. As a result of this, both the geometric and elastic non-linear effects come into play. To take these factors into account an approximate

numerical method has been developed by Ling and Atbek [62]. Pulse propagation phenomena in arteries are caused by the interaction of blood includes equations which govern the motion the motion of blood and the motion of the arterial wall, and also the relations (boundary conditions) which connect the two motions with each other. This set of equations and conditions make a formidable boundary value problem.

Streeter et. Al [105] studied pulsatile pressure and flow through distensible vessels. The behaviour of blood flow through narrow tubes has been studied experimentally by Bugliarello and Sevilla [14]. Lou [63] investigated the problem of blood flow in large elastic arteries in the mammalian circulatory system. The purpose of his investigation is to develop a theory for the problem of pulsatile blood flow in larger vessels such as the thoratic aorta. The theoretical studies of blood flow in arteriolar and venular bifurcations are studied by Popel et. al. [78]. Srivastava and Sexena [106] have investigated the effects of sedimentation of small red blood cell aggregates on blood flow in narrow horizontal tubes.

The application of Magneto hydrodynamics in physiological flow is of growing interest. The flow of blood can be controlled by applying appropriate magnetic field. Many researchers have shown that blood is an electrically conducting fluid (Kollin[58], Korchevskii and Marochnlk [59], Vardanyan [115]). The Lorentz's force will act on the constituent particles of blood and this force will oppose the motion of blood and thus reduces its velocity. This decelerated blood flow may help in the treatment of certain cardiovascular diseases and in the

diseases with accelerated blood circulation such as hypertension, hemorrhages etc. So, it is very essential to study the blood flow in presence of magnetic field. Many works have been done in this field by various investigators.

The pulsatile flow of blood with micro-organisms represented by two fluid model through vessels of small exponential divergence under the effect of magnetic filed has been studied by Rathod and Gayatri [81]. A similar problem on blood flow through a uniform pipe with sector of a circle as cross section in presence of transverse magnetic field has been studied by Rathod and Parveen [83]. Pulsatile blood flow through closed rectangular channel with micro-organisms has also been studied by Rathod and Mohesh [84]. Exponential representation of blood flow governing equation under external running pulse magnitude field has been studied by Jain, et. Al. [51]. Flow in a porous channel is important in transpiration cooling and gaseous diffusion process (Longwell [64]). Pulsatile flow in a porous channel, in particular is also important in the dialysis of blood in artificial kidney. Pulsatile flow in a porous channel has been investigated by Wang [116] without magnetic field. Pulsatile flow in a porous channel considering blood as Newtonian fluid investigated by B.C. Bhuyan and G.C. Hazarika [8] and considering as Non Newtonian fluid by Sut D.K. & Hazarika G.C.[107]. Actually due to simplification blood can be assumed as Newtonian fluid. But observing the behaviour of the blood anyone can speak blood is a Non Newtonian fluid. Here an attempt has been made in this analysis to study the pulsatile flow of blood in a porous

channel in presence of transverse magnetic field. Here blood is assumed to be an incompressible Non-Newtonian fluid.

## 2. Mathematical Formulation of the Problem

The real blood circulation system cannot of three-dimensional elastic tubes of varying cross-section and angle of bifurcation. For the sake of mathematical convenience, we discuss the magnetic effect on unsteady flow of blood through a two dimensional, non-conducting, parallel plate and equally branched channel such that one stream of blood from trunks is branched into two different streams.

For this analysis,, blood has been considered to be Non Newtonian, incompressible, homogeneous and viscous fluid. The Fahreus Lindquist effect is significant and when the vessel diameter is less than 1 mm unlike the case here. As such the Reynolds number does not vary much in the region of any one bifurcation. Thus the viscosity of the blood is treated as constant.

The static magnetic field  $B_0$  is applied in a direction perpendicular to the flow of blood. We make the

following assumptions for electromagnetic interactions:

(1)The induced magnetic field and the electromagnetic field produced by the motion

of blood are negligible.

(2)No external electric field is applied.

With the above assumptions, we consider a fluid driven by steady laminar flow of blood through an axially symmetric stenosed artery in presence of magnetic field. The axial coordinate and velocity are  $\hat{z}$  and  $\hat{u}$  respectively.

$$\frac{1}{\rho} \frac{\partial p}{\partial x} = A + B e^{i\omega t}$$

(1)

between two porous plates at  $y=0$  and  $y=h$ . Here  $A$  and  $B$  are known constants and  $\omega$  is the frequency. On one plate some fluid is injected with velocity  $v$  and it is sucked off at the opposite plate with same velocity. Due to continuity, the velocity component in the  $y$ -direction will be identically equal to  $v$  everywhere.  $B_0$  is the applied magnetic field in  $y$  direction.

Velocity boundary layer equations for the steady two dimensional non-Newtonian flows are

$$\rho a_x = \frac{\partial}{\partial x} \tau_{xx} + \frac{\partial}{\partial y} \tau_{xy}$$

Here  $v$  is constant, so

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\Rightarrow \frac{\partial u}{\partial x} = 0$$

$$\text{i.e., } u = u(y, t)$$

$$\therefore a_x = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y}$$

$$\begin{aligned}
a_y &= \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = 0 \\
\therefore \tau_{xx} &= -p + 2\mu_1 \frac{\partial u}{\partial x} + 2\mu_2 \left[ \frac{\partial}{\partial x} a_x + \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 \right] + \mu_3 \left[ 4 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 \right] \\
&= -p + 2\mu_2 \frac{\partial}{\partial x} \left[ \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} \right] + \mu_3 \frac{\partial u}{\partial y} \\
&= -p + \mu_3 \frac{\partial u}{\partial y} \\
\tau_{xy} &= \mu_1 \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \mu_2 \left[ \frac{\partial}{\partial x} a_y + \frac{\partial}{\partial y} a_x + \left( \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \frac{\partial v}{\partial y} \right) \right] + \mu_3 \left[ 2 \frac{\partial u}{\partial x} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + 2 \frac{\partial v}{\partial y} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] \\
&= \mu_1 \frac{\partial u}{\partial y} + \mu_2 \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} \right) \\
\therefore \frac{\partial}{\partial x} \tau_{xx} &= -\frac{\partial p}{\partial x} \\
\frac{\partial}{\partial y} \tau_{xy} &= \mu_1 \frac{\partial^2 u}{\partial y^2} + \mu_2 \frac{\partial^2}{\partial y^2} \left( \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} \right) \\
\text{Now, } \rho a_x &= \frac{\partial}{\partial x} \tau_{xx} + \frac{\partial}{\partial y} \tau_{xy} + \bar{J} \times \bar{B} \\
\Rightarrow \rho \left( \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} \right) &= -\frac{\partial p}{\partial x} + \mu_1 \frac{\partial^2 u}{\partial y^2} + \mu_2 \frac{\partial^2}{\partial y^2} \left( \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} \right) - \sigma B_0^2 u \\
\Rightarrow \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \gamma_1 \frac{\partial^2 u}{\partial y^2} + \gamma_2 \frac{\partial^2}{\partial y^2} \left( \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} \right) - \frac{\sigma}{\rho} B_0^2 u \quad (2)
\end{aligned}$$

$$\text{and, } \rho a_y = \frac{\partial}{\partial x} \tau_{xy} + \frac{\partial}{\partial y} \tau_{yy}$$

$$\text{Since } a_y = 0$$

$$\begin{aligned}
\tau_{yy} &= -p + 2\mu_1 \frac{\partial v}{\partial y} + 2\mu_2 \left[ \frac{\partial a_y}{\partial y} + \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right] + \mu_3 \left[ 4 \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right] \\
&= -p + 2\mu_2 \left( \frac{\partial u}{\partial y} \right)^2 + \mu_3 \left( \frac{\partial u}{\partial y} \right)^2 \\
\frac{\partial}{\partial y} \tau_{yy} &= -\frac{\partial p}{\partial y} + 4\mu_2 \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} + 2\mu_3 \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} \\
&= -\frac{\partial p}{\partial y} + 2(2\mu_2 + \mu_3) \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} \\
\frac{\partial}{\partial x} \tau_{xy} &= 0 \\
\therefore \rho \cdot 0 &= 0 - \frac{\partial p}{\partial y} + 2(2\mu_2 + \mu_3) \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2}
\end{aligned}$$

$$\Rightarrow \frac{\partial p}{\partial y} = 2(2\mu_2 + \mu_3) \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} \quad (3)$$

$$\text{i.e.,} \quad \frac{1}{\rho} \frac{\partial p}{\partial x} = A + Be^{i\omega t} \quad (1)$$

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \gamma_1 \frac{\partial^2 u}{\partial y^2} + \gamma_2 \frac{\partial^2}{\partial y^2} \left( \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} \right) - \frac{\sigma}{\rho} B_o^2 u \quad (2)$$

$$\frac{\partial p}{\partial y} = 2(2\mu_2 + \mu_3) \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} \quad (3)$$

Using (1) in (2), we get

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = -A - Be^{i\omega t} + \gamma_1 \frac{\partial^2 u}{\partial y^2} + \gamma_2 \frac{\partial^2}{\partial y^2} \left( \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} \right) - \frac{\sigma}{\rho} B_o^2 u$$

We separate the above into a steady part denoted by tittle ( $\tilde{u}$ ) and unsteady part denoted by a bar ( $\bar{u}$ ).

$$v \frac{\partial \tilde{u}}{\partial t} = A + \gamma_1 \frac{\partial^2 \tilde{u}}{\partial y^2} + \gamma_2 \frac{\partial^2}{\partial y^2} \left( v \frac{\partial \tilde{u}}{\partial y} \right) - \frac{\sigma}{\rho} B_o^2 u \quad (4)$$

$$\frac{\partial \bar{u}}{\partial t} + v \frac{\partial \bar{u}}{\partial y} = -Be^{i\omega t} + \gamma_2 \frac{\partial^2}{\partial y^2} \left( \frac{\partial \bar{u}}{\partial t} + v \frac{\partial \bar{u}}{\partial y} \right) + \gamma_1 \frac{\partial^2 \bar{u}}{\partial y^2} \quad (5)$$

The boundary conditions are that both  $\tilde{u}$  and  $\bar{u}$  be zero at  $y=0$  and  $y=h$ .

### 3. Solution of the Problem

We introduce a new dimensionless variable  $\eta = \frac{y}{h}$ ,

$$\text{Taking } u = f(\eta), \quad M = \sqrt{\frac{\sigma}{\rho \gamma_1}} B_o h, \quad R = \frac{vh}{\gamma_1}$$

Equation (4) becomes

$$\begin{aligned} v f'(\eta) \cdot \frac{1}{h} &= -A + \gamma_1 \cdot f''(\eta) \cdot \frac{1}{h^2} + \gamma_2 \cdot v f'''(\eta) \cdot \frac{1}{h^3} - \frac{M^2 \gamma_1}{h^2} f(\eta) \\ \Rightarrow \frac{v}{h} f'(\eta) &= -A + \frac{\gamma_1}{h^2} f''(\eta) + \frac{\gamma_2 \cdot v}{h^3} f'''(\eta) - \frac{M^2 \gamma_1}{h^2} f(\eta) \\ \Rightarrow \frac{h^2}{\gamma_1} \frac{v}{h} f'(\eta) &= -A \frac{h^2}{\gamma_1} + \frac{h^2}{\gamma_1} \frac{\gamma_1}{h^2} f''(\eta) + \frac{h^2}{\gamma_1} \frac{\gamma_2 \cdot v}{h^3} f'''(\eta) - \frac{h^2}{\gamma_1} \frac{M^2 \gamma_1}{h^2} f(\eta) \\ \Rightarrow \frac{vh}{\gamma_1} f'(\eta) &= -\frac{Ah^2}{\gamma_1} + f''(\eta) + \frac{v\gamma_2}{\gamma_1 h} f'''(\eta) - M^2 f(\eta) \end{aligned}$$

$$\begin{aligned}
\Rightarrow Rf'(\eta) &= -\frac{Ah^2}{\gamma_1} + f''(\eta) + \frac{R\gamma_2}{h^2} f'''(\eta) - M^2 f(\eta) \\
\Rightarrow \frac{R\gamma_2}{h^2} f'''(\eta) + f''(\eta) - Rf'(\eta) - M^2 f(\eta) &= \frac{Ah^2}{\gamma_1} \\
\Rightarrow R_1 f'''(\eta) + f''(\eta) - Rf'(\eta) - M^2 f(\eta) &= \frac{Ah^2}{\gamma_1} \quad (6)
\end{aligned}$$

where  $R = \frac{\nu h}{\gamma}$ , the cross flow Reynolds number.

$$R_1 = \frac{R\gamma_2}{h^2},$$

$$M = \sqrt{\frac{\sigma}{\rho\nu}} B_o h, \text{ the Hartmann number.}$$

The boundary conditions are  $f=0$  at  $\eta=0$  and  $f=0$  at  $\eta=1$ .

Here dashes represent differentiation with respect to  $\eta$ . We are not interested in discussion of the steady part and so shall not go into details here.

The unsteady equation (5) can be reduced to an ordinary differential equation by introducing a non-dimensional variable.

$$\eta = \frac{y}{h}$$

$$\text{Substituting } u = f(\eta)e^{i\omega t}, R = \frac{\nu h}{\gamma_1}, M_1^2 = \frac{h^2}{\gamma_1} \omega$$

Equation (5) becomes

$$\begin{aligned}
&\frac{\partial}{\partial t}(f(\eta)e^{i\omega t}) + \nu \frac{\partial}{\partial y}(f(\eta)e^{i\omega t}) \\
&= -Be^{i\omega t} + \gamma_2 \frac{\partial^2}{\partial y^2} \left[ \frac{\partial}{\partial t}(f(\eta)e^{i\omega t}) + \nu \frac{\partial}{\partial y}(f(\eta)e^{i\omega t}) \right] - \frac{\sigma}{\rho} B_o^2 e^{i\omega t} f(\eta) + \gamma_1 \frac{\partial^2}{\partial y^2}(f(\eta)e^{i\omega t}) \\
&\Rightarrow f(\eta)e^{i\omega t} \cdot i\omega + \nu e^{i\omega t} f'(\eta) \cdot \frac{1}{h} \\
&= -Be^{i\omega t} + \gamma_2 \frac{\partial^2}{\partial y^2} \left[ f(\eta)e^{i\omega t} \cdot i\omega + \nu e^{i\omega t} f'(\eta) \cdot \frac{1}{h} \right] - \frac{\sigma}{\rho} B_o^2 e^{i\omega t} f(\eta) + \gamma_1 e^{i\omega t} f''(\eta) \cdot \frac{1}{h^2} \\
&\Rightarrow f(\eta) \cdot i\omega + \frac{\nu}{h} f'(\eta) = -B + \gamma_2 \frac{\partial^2}{\partial y^2} \left[ f(\eta) \cdot i\omega + \frac{\nu}{h} f'(\eta) \right] - \frac{\sigma}{\rho} B_o^2 f(\eta) + \frac{\gamma_1}{h^2} f''(\eta) \\
&\Rightarrow f(\eta) \cdot i\omega + \frac{\nu}{h} f'(\eta) = -B + \gamma_2 f''(\eta) \cdot \frac{1}{h^2} \cdot i\omega + \gamma_2 \frac{\nu}{h} f'''(\eta) \cdot \frac{1}{h^2} - \frac{\sigma}{\rho} B_o^2 f(\eta) + \frac{\gamma_1}{h^2} f''(\eta) \\
&\Rightarrow f(\eta) \cdot i\omega + \frac{\nu}{h} f'(\eta) = -B + \frac{\gamma_2 \nu}{h^2} i\omega f''(\eta) + \frac{\gamma_2 \nu}{h^3} f'''(\eta) - \frac{\sigma}{\rho} B_o^2 f(\eta) + \frac{\gamma_1}{h^2} f''(\eta)
\end{aligned}$$

$$\begin{aligned}
&\Rightarrow f(\eta).i\omega + \frac{\nu}{h} f'(\eta) = -B + \frac{\gamma_2 i\omega}{h^2} f''(\eta) + \frac{\gamma_2 \nu}{h^3} f'''(\eta) - \frac{M^2 \sigma}{\rho} B_o^2 f(\eta) + \frac{\gamma_1}{h^2} f''(\eta) \\
&\Rightarrow \frac{h^2}{\gamma_1} f(\eta).i\omega + \frac{h^2 \nu}{\gamma_1 h} f'(\eta) = -B \frac{h^2}{\gamma_1} + \frac{h^2 \gamma_2 i\omega}{\gamma_1 h^2} f''(\eta) + \frac{h^2 \gamma_2 \nu}{\gamma_1 h^3} f'''(\eta) - \frac{h^2 M^2 \sigma}{\gamma_1 \rho} B_o^2 f(\eta) + f''(\eta) \\
&\Rightarrow \frac{h^2}{\gamma_1} f(\eta).i\omega + R f'(\eta) = -\frac{B h^2}{\gamma_1} + \frac{\gamma_2 i\omega}{\gamma_1} f''(\eta) + \frac{R \gamma_2}{h^2} f'''(\eta) - M^2 f(\eta) + f''(\eta) \\
&\Rightarrow M_1^2 i f(\eta) + R f'(\eta) = -\frac{B h^2}{\gamma_1} + \frac{\gamma_2 i\omega}{\gamma_1} f''(\eta) + \frac{R \gamma_2}{h^2} f'''(\eta) - M^2 f(\eta) + f''(\eta) \\
&\Rightarrow \frac{R \gamma_2}{h^2} f'''(\eta) + f''(\eta) + \frac{\gamma_2 i\omega}{\gamma_1} f''(\eta) - R f'(\eta) - (M^2 + M_1^2 i) f(\eta) = \frac{h^2 B}{\gamma_1} \quad (7)
\end{aligned}$$

We put

$$\begin{aligned}
f &= \bar{u} e^{-i\omega t} = (\bar{u}_1 + i\bar{u}_2) e^{-i\omega t} \\
f' &= \bar{u}' e^{-i\omega t} = (\bar{u}'_1 + i\bar{u}'_2) e^{-i\omega t} \\
f'' &= \bar{u}'' e^{-i\omega t} = (\bar{u}''_1 + i\bar{u}''_2) e^{-i\omega t} \\
f''' &= \bar{u}''' e^{-i\omega t} = (\bar{u}'''_1 + i\bar{u}'''_2) e^{-i\omega t}
\end{aligned}$$

where  $\bar{u} = \bar{u}_1 + i\bar{u}_2$ ,  $\bar{u}' = \bar{u}'_1 + i\bar{u}'_2$ ,  $\bar{u}'' = \bar{u}''_1 + i\bar{u}''_2$ ,  $\bar{u}''' = \bar{u}'''_1 + i\bar{u}'''_2$

On putting the values of  $f'$ ,  $f''$ ,  $f'''$  in (7), equating real and imaginary parts and after a few steps of calculation we get the following ordinary differential equations

$$\begin{aligned}
\frac{R \gamma_2}{h^2} \bar{u}'''_1 + \bar{u}''_1 - \frac{\gamma_2 \omega}{\gamma_1} \bar{u}''_2 - R \bar{u}'_1 - (M^2 \bar{u}'_1 - M_1^2 \bar{u}_2) &= \frac{h^2 B}{\gamma_1} \cos \omega t \\
\frac{R \gamma_2}{h^2} \bar{u}'''_2 + \bar{u}''_2 + \frac{\gamma_2 \omega}{\gamma_1} \bar{u}''_1 - R \bar{u}'_2 - (M^2 \bar{u}'_2 + M_1^2 \bar{u}_1) &= \frac{h^2 B}{\gamma_1} \sin \omega t
\end{aligned}$$

$\Rightarrow$

$$\begin{aligned}
\frac{R \gamma_2}{h^2} \bar{u}'''_1 + \bar{u}''_1 - \frac{\gamma_2 \omega}{\gamma_1} \bar{u}''_2 - R \bar{u}'_1 - M^2 \bar{u}'_1 + M_1^2 \bar{u}_2 &= \frac{h^2 B}{\gamma_1} \cos \omega t \\
\frac{R \gamma_2}{h^2} \bar{u}'''_2 + \bar{u}''_2 + \frac{\gamma_2 \omega}{\gamma_1} \bar{u}''_1 - R \bar{u}'_2 - M^2 \bar{u}'_2 - M_1^2 \bar{u}_1 &= \frac{h^2 B}{\gamma_1} \sin \omega t
\end{aligned}$$

$\Rightarrow$

$$R_1 \bar{u}'''_1 + \bar{u}''_1 - R_2 \bar{u}''_2 - R \bar{u}'_1 - M^2 \bar{u}'_1 + M_1^2 \bar{u}_2 = \frac{h^2 B}{\gamma_1} \cos \omega t \quad (8)$$

$$R_1 \bar{u}'''_2 + \bar{u}''_2 + R_2 \bar{u}''_1 - R \bar{u}'_2 - M^2 \bar{u}'_2 - M_1^2 \bar{u}_1 = \frac{h^2 B}{\gamma_1} \sin \omega t \quad (9)$$

where

$$R_1 = \frac{R \gamma_2}{h^2}, \quad R_2 = \frac{\gamma_2 \omega}{\gamma_1},$$

Boundary conditions are

$$u_1 = 0, u_2 = 0 \text{ at } \eta = 0 \text{ and } 1$$

Equations (8) and (9) are solved numerically using shooting method for  $u_1, u_2$  and consequently the real part of (1) can be computed.

#### 4. Results and Discussions

The problem under investigation is dominated mainly by  $R$  the cross flow Reynolds number,  $R_1$ ,  $R_2$ ,  $M_1$  the frequency parameter and  $M$  the Hartmann number. Our interest is to study investigate the roll of magnetic parameter  $M$  on the velocity field.

When the frequency parameter  $M_1$  is small (i.e.,  $M_1=1$  and  $M=0$ ,  $R=0$ ,  $R_1=0.1$ ,  $R_2=0.1$ ) the velocity profile is almost parabolic (Fig. 1). For large value of the frequency  $M_1$  (i.e.,  $M_1=10$  and  $R=0$ ,  $R_1=0.1$ ,  $R_2=0.1$ ), the maxima of the velocity is shifted to the boundary layer near the wall for  $M=0, 5$  at  $R=0$  (Fig. 2 & 3) and velocity profiles are almost equally distributed over the boundary layer region when  $wt$  changes from  $0^\circ$  to  $360^\circ$ .

Fig. 8 show the velocity profiles with effect of magnetic field for various values of  $M_1$ ,  $R$ ,  $R_1$ ,  $R_2$  and  $wt$ . It is seen that the fluid velocity, decreases as the magnetic parameter  $M$  increases. The maxima of the velocity is shifted to the boundary layer in the region from  $\eta=.5$  to 1 (Fig. 4 & 5) for all values of  $M$  when  $R=0$  and  $R=10$  at  $M_1=1$ .

From Fig. 5, it is observed that the velocity profiles are symmetrically

distributed over almost from the half of the boundary layer for  $M_1=1$ ,  $R=0$ ,  $R_1=0.1$ ,  $R_2=0.1$  and  $wt=45^\circ$ . Here it is also observed that the fluid velocity decreases with the increase of magnetic parameter  $M$ .

Fig. 9 shows that the fluid velocity decreases as the cross flow Reynolds number increases at  $M=.5$ ,  $M_1=1$ ,  $R_1=0.1$ ,  $R_2=0.1$  and  $wt=45^\circ$ .

For large value of  $M_1$  and  $R$  (i.e.,  $M_1=10$ ,  $R=10$ ), the velocity decreases at the beginning and then decreases as the Hartmann number increases at  $wt=45^\circ$ . (Fig. 8) When  $M_1=1$  and  $R=10$  at  $M=0$ , the velocity profiles are shifted to near the boundary layer in the region from  $\eta=.5$  to 1 (Fig. 10)

Here it is seen that the fluid velocity is greatly affected due to the presence of the magnetic field. When the magnetic parameter, the Hartmann number increase, the fluid velocity decreases. Also the magnitude of mass flux is dominated by the magnetic field. The mathematical expressions may help medical practitioners to control the blood flow of a patient whose blood pressure is very high by applying certain magnetic field.



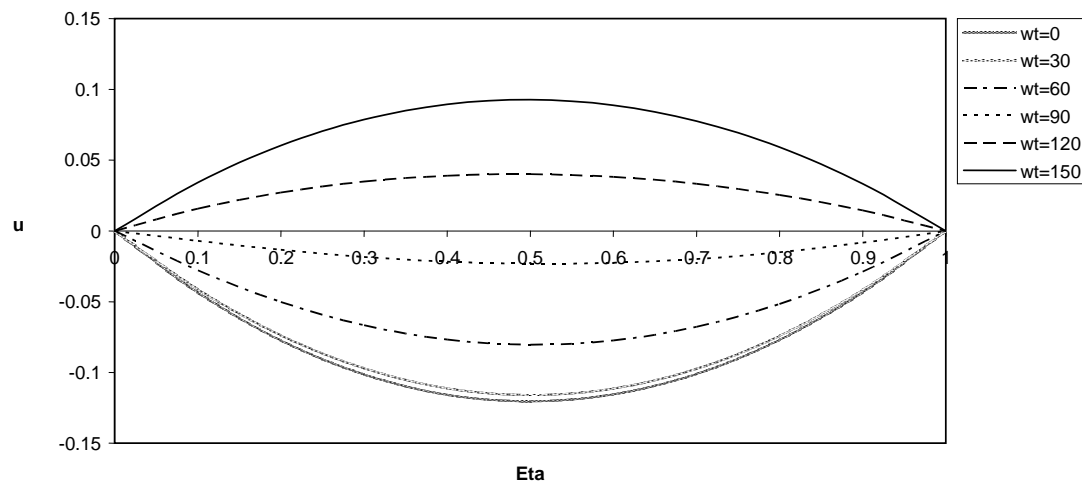


Fig. 1. Instantaneous velocity profiles for different values of  $wt$  at  $M=0$ ,  $M_1=1.0$ ,  $R=0$ ,  $R=0.10$ ,  $R_2=0.10$

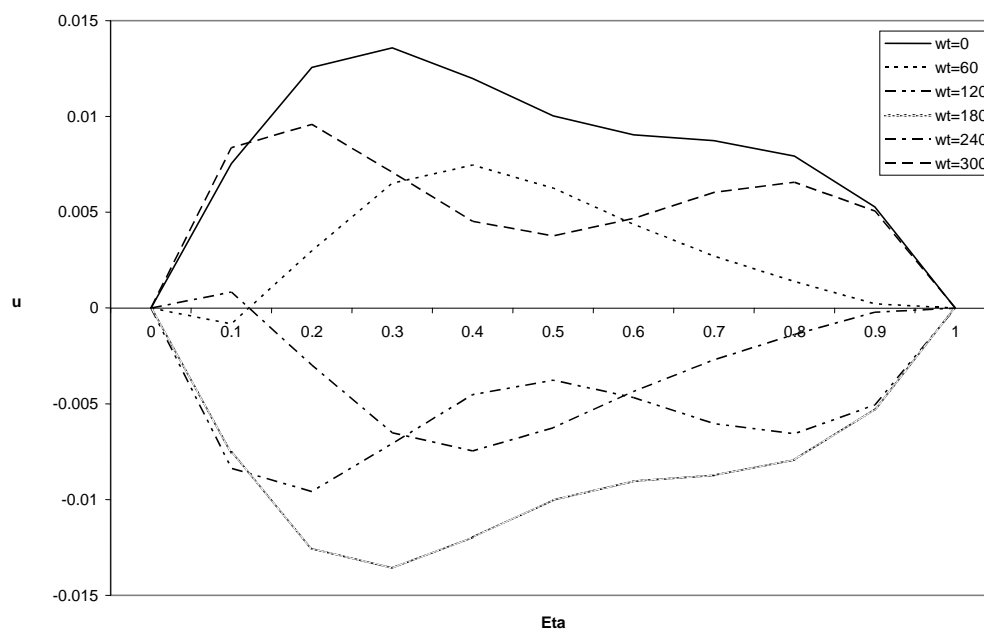


Fig. 2. Instantaneous velocity profiles for different values of  $wt$  at  $M_1=10$ ,  $R=0$ ,  $R_1=0.10$ ,  $R_2=0.10$ ,  $M=0$

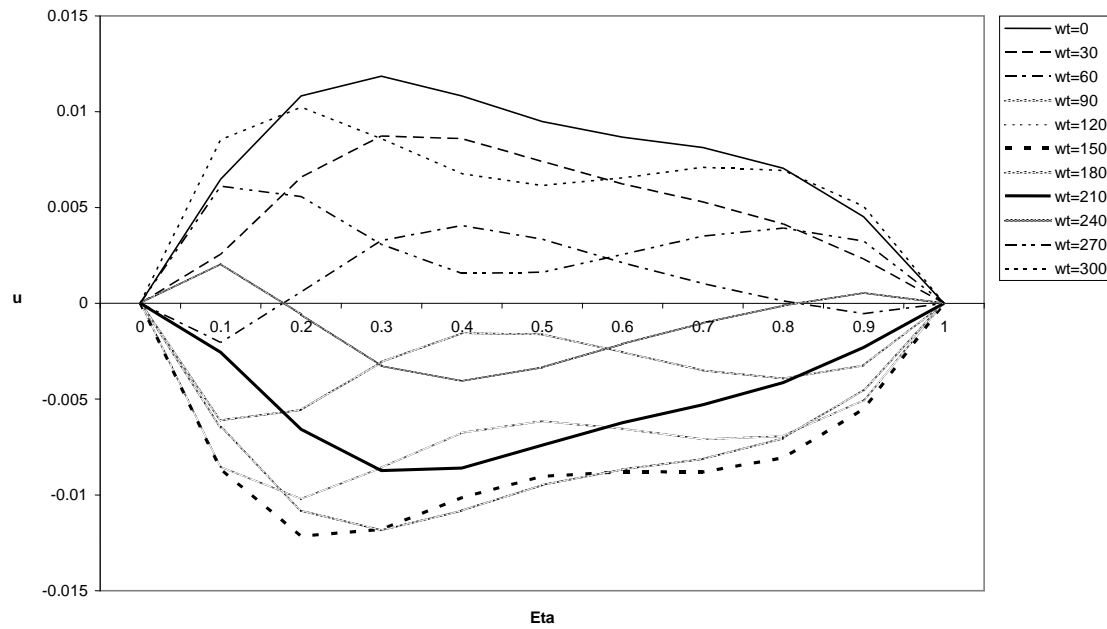


Fig. 3 Comparison of velocity profile for different values of  $wt$  at  $M=5$ ,  $M_1=10$ ,  $R=0$ ,  $R_1=0.10$ ,  $R_2=0.10$

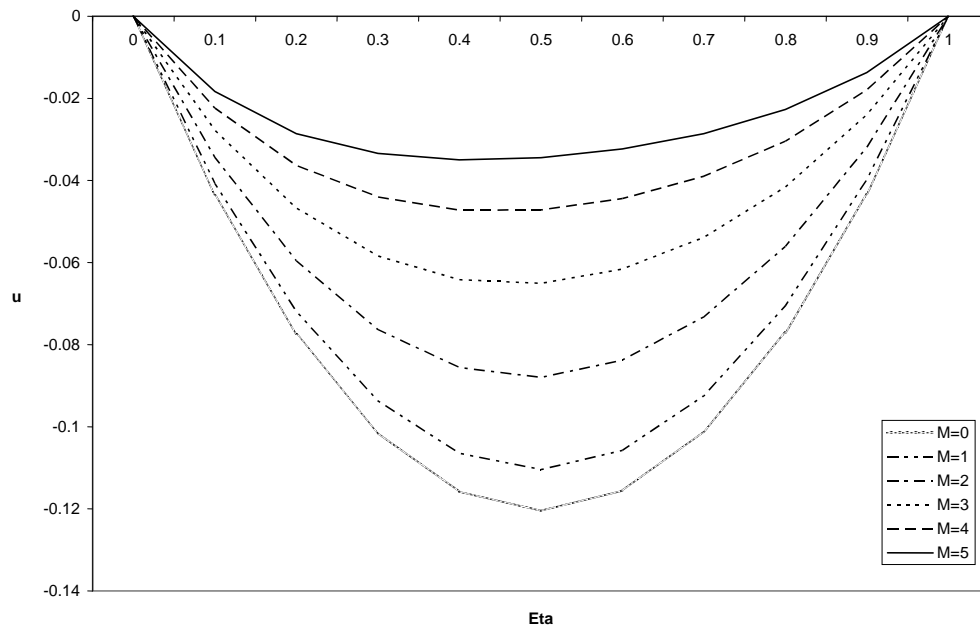


Fig. 4 Comparison of velocity profile for different Hartmann number at  $M_1=1$ ,  $R=0$ ,  $R_1=0.10$ ,  $R_2=0.10$

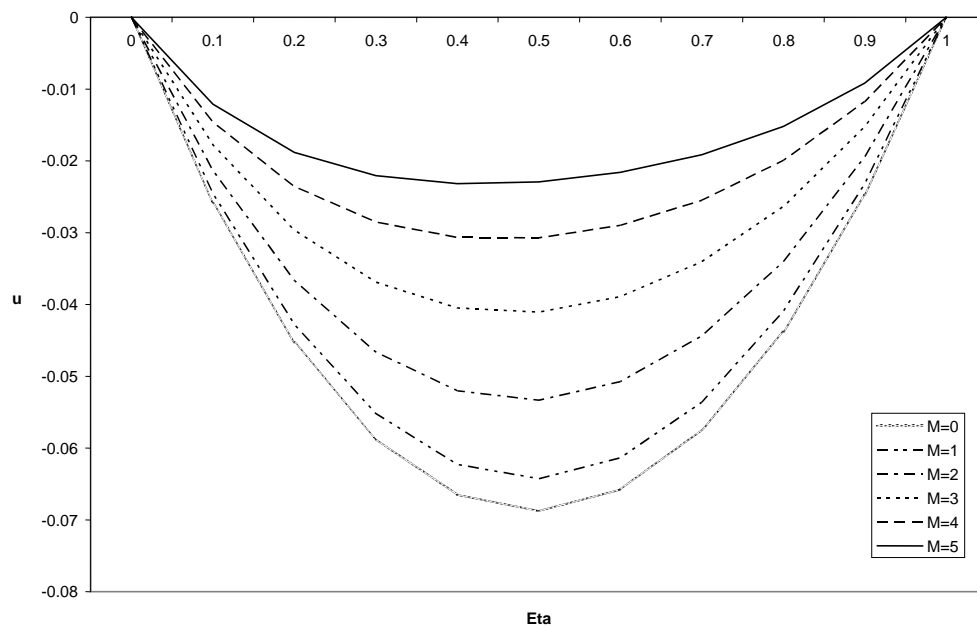


Fig. 5. Comparison of velocity profile for different Hartmann number at  $M_1=1$ ,  $R=0$ ,  $R_1=0.10$ ,  $R_2=0.10$

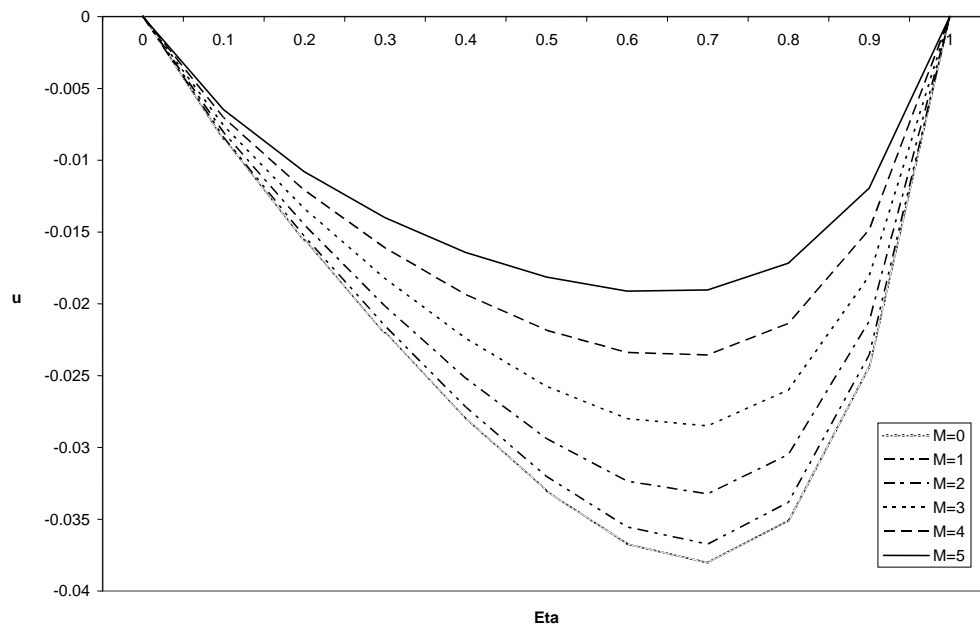


Fig. 6. Comparison of velocity profiles for different Hartmann number at  $M_1=1$ ,  $R=10$ ,  $R_1=0.10$ ,  $R_2=0.10$

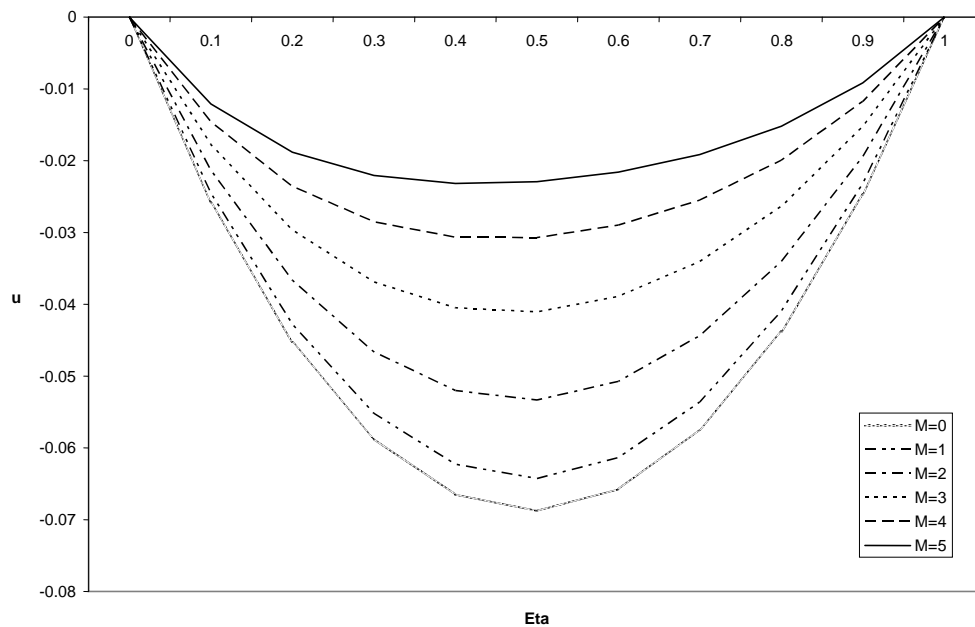


Fig. 7. Comparison of velocity profile for different Hartmann number at  $M_1=1$ ,  $R=0$ ,  $R_1=0.10$ ,  $R_2=0.10$

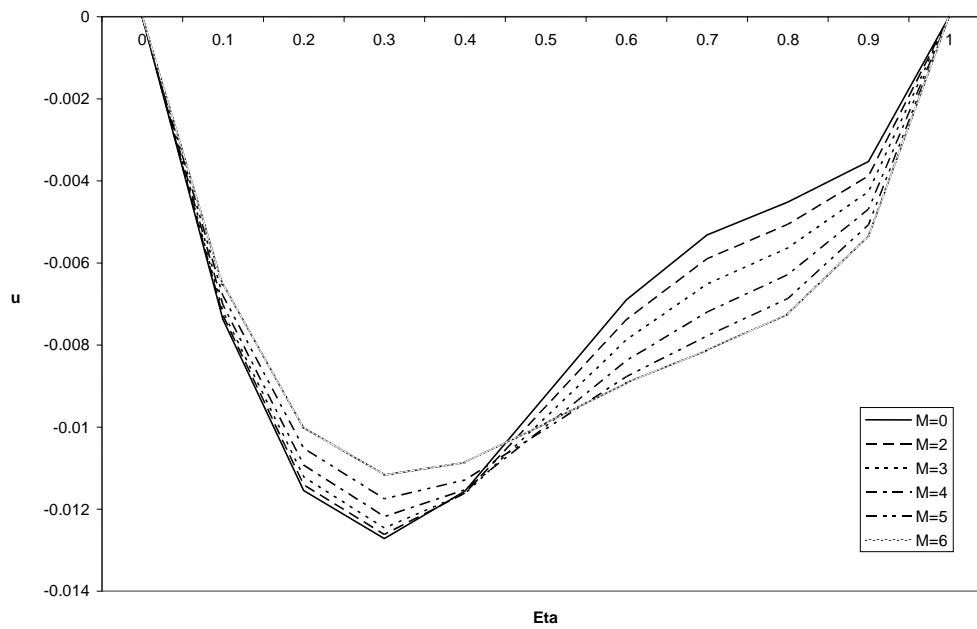


Fig. 8. Comparison of velocity profile for different Hartmann numbers at  $M_1=10$ ,  $R=10$ ,  $R_1=0.10$ ,  $R_2=0.10$

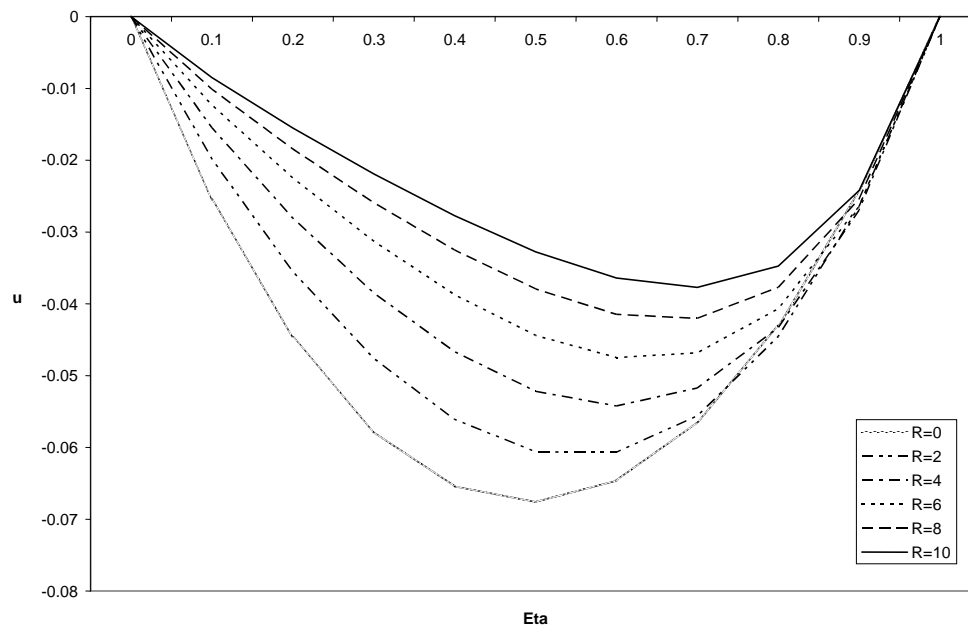


Fig.9. Comparison of velocity profile for different Reynolds numbers at  $M=0.5$ ,  $M_1=1$ ,  $R_1=0.10$ ,  $R_2=0.10$

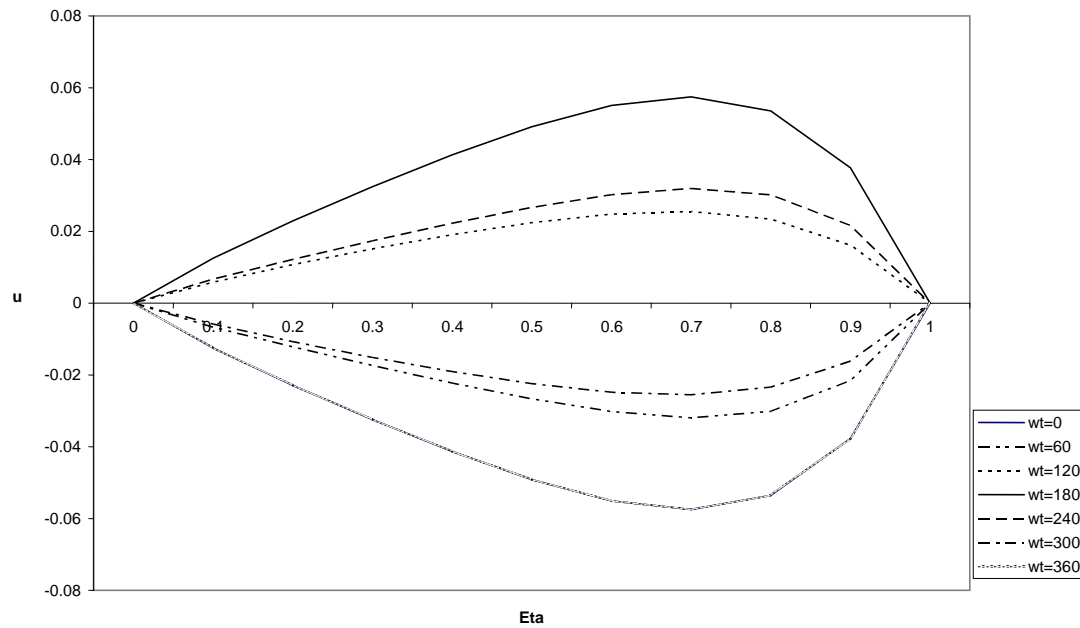


Fig. 10. Comparison of velocity profiles for different values of  $wt$  at  $R=10$ ,  $R_1=0.10$ ,  $R_2=0.10$ ,  $M=0$ ,  $M_1=1$