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ANALYSIS OF BER DEGRADATION FOR TRANSMITTED DOWNLINK DSCDMA SIGNALS

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ABSTRACT

A predistorter-high power amplifier (PD-HPA) pair has become a common practice in wireless communication to compensate for nonlinear distortion due to HPA. However, the PD-HPA pair still produces severe signal distortion when the input signal exceeds the PD-HPA's saturation level. The effects of such distortion on bit error rate (BER) degradation in downlink direct sequence-code division multiple access signals (DS-CDMA) are analyzed. We establish which signal characteristics at the HPA input and the factors contributing to BER. Assuming that the baseband CDMA signal is characterized as a complex Gaussian process, we develop analytic expressions for the BER and the contributing factors to BER.

Keywords - CDMA, nonlinear distortion, predistortion.

I. INTRODUCTION

Downlink direct sequence-code division multiple access (DS-CDMA) signals typically exhibit large dynamic range since they represent the sum of signals of many users. Unfortunately, when passed through a high power amplifier (HPA), this large dynamic range results in distortion for components falling in the highly nonlinear regions of the HPA, which degrades the system bit error rate (BER) [1]. A common parameter for characterizing the dynamic range of a signal is the signal peak-to-average power ratio (PAR). PAR is often used as an indicator to how much harm the signal will suffer due to HPA nonlinearity, and also allows the system designer to determine the required amount of input back-off (IBO) to reduce nonlinear distortion effects. While PAR has its use, a signal's dynamic range in relation to an HPA only captures only one feature of the signal's interplay with the HPA. In this paper, we reformulate which signal characteristics to consider beyond dynamic range that can be linked directly with BER. In the analysis, we assume that the nonlinear amplifier chain includes a predistorter prior to the HPA, namely PDHPA. The PD-HPA has a zero AM-PM characteristic $\Phi[r\ (t)]$, and an AM-AM characteristic given by

$$G[r\ (t)] = \begin{cases} r\ (t) & 0 \leq r\ (t) \leq \zeta \\ \zeta & r\ (t) > \zeta \end{cases} \qquad (1)$$

where $r\ (t)$ is the input to the PD-HPA and ζ is the PD-HPA saturation (clipping) threshold. Assuming that the baseband CDMA signal is characterized as a band-limited complex Gaussian process, we establish analytical expressions for the signal characteristics, with respect to the IBO level, that lead to BER degradation. Moreover, we develop an analytic expression for the BER performance in presence of the considered nonlinear amplifier chain.

II. CDMA SYSTEM UNDER INVESTIGATION

The system under investigation is a downlink CDMA system in which the users' signals are synchronized and have equal power. The complex envelope of the CDMA signal for K active users is defined as [1], [2]

$$s(t) = \sum_{k=1}^{K} \sqrt{E_k} \sum_{n=-\infty}^{\infty} \sum_{\ell=0}^{L-1} a_n^{(k)} c_\ell^{(k)} h(t-\ell T_c - nT) = x(t) + jy(t) = r(t)e^{j\theta(t)} \tag{2}$$

where E_k is the kth user's signal energy per bit, T is the symbol duration, L is the spreading factor, $T_c = T/L$ is the chip duration, $c_\ell^{(k)}$ is the ℓth chip in the spreading code $\mathbf{c}^{(k)} = [c_0^{(k)}, \ldots, c_{L-1}^{(k)}]^T$, $h(t)$ is the impulse response of the transmit pulse shaping filter, and $a_n^{(k)} = (\pm 1 \pm j)/\sqrt{2}$ is the kth user's symbol data for QPSK modulation in the nth symbol duration. Moreover, the symbols $a_n^{(k)}$ are assumed to be independent with zero mean and variance of $E\{|a_n^{(k)}|^2\}$ [1]. For a large number of users and assuming the pulse shaping filter corresponds to a square-root raised cosine filter (SRRC) with small roll-off factor, $x(t)$ and $y(t)$ can be regarded as two uncorrelated zero-mean Gaussian processes with equal variances, that is $\sigma_x^2 = \sigma_y^2$ [3]. Hence, $s(t)$ can be regarded as a complex zero-mean Gaussian process with a variance of

$$\sigma_s^2 = 2\sigma_x^2 = \textstyle\sum_{k=1}^{K} E_k E\{|a_n^{(k)}|^2\} = KE_k \tag{3}$$

since $|c_\ell^{(k)}|$ =1.Envelope $r(t) = \sqrt{x^2(t) + y^2(t)}$ has a quasi- Rayleigh distribution [3], with probability density function [2]

$$f_r(r) = (r/\sigma_x^2)e^{-r^2/2\sigma_x^2}, \qquad r \geq 0. \tag{4}$$

Finally, the output from the PD-HPA can be expressed as

$$s_d(t) = r_d(t)e^{j\theta_d(t)} = G[r(t)]e^{j\theta(t)}. \tag{5}$$

III. BER PERFORMANCE ANALYSIS

Using the Bussgang theorem extension for a complex Gaussian input to a memory less nonlinear device, the output of the PD-HPA can be represented as the sum of two uncorrelated components: a scaled linear component and a nonlinear component, $s_{nl}(t)$, [1], [3], [4], that is

$$s_d(t) = \alpha_0 s(t) + s_{nl}(t) \tag{6}$$

where $E\{s(t)s_{nl}^*(t)\} = 0$ and α0 is the linear gain given by

$$\alpha_0 = \frac{E\{s_d^*(t)s(t)\}}{E\{|s(t)|^2\}} = \frac{E\{G[r]r\}}{2\sigma_x^2}. \tag{7}$$

Consequently, the variance of the distorted signal $s_d(t)$ is given by $\sigma_d^2 = |a_0|^2\sigma_s^2 + \sigma_{nl}^2$ where σ_{nl}^2 is the variance of the nonlinear component $s_{nl}(t)$. As far as $s(t)$ is considered as a zero-mean complex Gaussian process, σ_d^2 can be calculated as

$$\sigma_d^2 = E\{|s_d(t)|^2\} = E\{r_d^2(t)\} = \int_0^\infty r_d^2 f_r(r)dr = \int_0^\zeta r^2 f_r(r)dr + \zeta^2 \int_\zeta^\infty f_r(r)dr = 2\sigma_x^2(1 - e^{\frac{-\zeta^2}{2\sigma_x^2}}). \tag{8}$$

Evaluating the BER in the case of an optimum receiver is difficult and tedious; hence, the BER for a suboptimum receiver is used as an upper bound [2]. We adopt the decorrelating receiver, where multi-user interference is eliminated [2], [4] at the expense of noise enhancing. For the decorrelating receiver, the kth user BER is given by

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$$\mathrm{BER}_k = Q(\sqrt{\mathrm{SNR}_k}) \quad (9)$$

where SNR_k is the signal-to-noise ratio at the kth receiver & $Q(x) = \int_x^{\infty} e^{-\frac{u^2}{2}} du / \sqrt{2\pi}$. Including additive white Gaussian noise (AWGN), SNRk is given by

$$\mathrm{SNR}_k = \mathrm{SNR}_{\mathrm{AWGN},k} = \frac{\sigma^2_{s,k}}{\sigma^2_{n,k}} = \frac{E_k}{\varepsilon_k \sigma^2_n} \quad (10)$$

where $\mathrm{SNR}_{\mathrm{AWGN}}$ is the SNR due to AWGN only, σ2n is the variance of the AWGN and $\varepsilon_k = (\mathbb{R}_s^{-1})_{k,k}$ is the noise enhancement factor introduced by the decorrelating receiver [2], where Rs is the correlation matrix with elements $\rho_{jk}(0)$ defined as $\rho_{jk}(0) = [\mathbf{c}^{(j)}]^T[\mathbf{c}^{(k)}]$.

In the presence of a nonlinear amplifier, the SNR measured at the kth receiver input is the ratio of the transmitted distorted signal power $\sigma^2_{d,k} = |\alpha_0|^2\sigma^2_{s,k} + \sigma^2_{nl,k}$ to the noise power $\sigma^2_{nl,k}$

$$\mathrm{SNR}_k = \frac{\sigma^2_{d,k}}{\varepsilon_k\sigma^2_n} = \frac{\sigma^2_{s,k}(1 - e^{-\zeta^2/2\sigma^2_x})}{\varepsilon_k\sigma^2_n}$$
$$= \mathrm{SNR}_{\mathrm{AWGN},k}(1 - e^{-\zeta^2/2\sigma^2_x}). \quad (11)$$

Actually, once the PD-HPA is designed, its saturation threshold is fixed. However, the clipping threshold with respect to the input signal to the PD-HPA depends on the signal average power. Therefore, it is more convenient to relate the threshold ζ to IBO, where the IBO is the ratio of the input power at the PD-HPA saturation threshold to the signal average power. This relation allows the system operator to determine the optimum IBO required, according to the design demands. Hence, the threshold level can be defined thru the IBO, γ, multiplied by the signal average power P_{av}, that is

$$\zeta = \sqrt{P_{av} \times \gamma} = \sqrt{2\sigma^2_x \times \gamma}. \quad (12)$$

The term $e^{-\zeta^2/2\sigma^2_x}$ in (11) represents the probability that the signal envelope exceeds the threshold ζ, equivalently the threshold exceeding rate R_E, which using (12) is written as

$$R_E = e^{-\zeta^2/2\sigma^2_x} = e^{-\gamma} \quad (13)$$

Substituting (13) in (11) and then in (9), the BER in presence of the PD-HPA as a function of the IBO level has the form

$$\mathrm{BER}_k = Q\left(\sqrt{\mathrm{SNR}_{\mathrm{AWGN},k}(1 - R_E)}\right) \quad (14)$$

From (14), it is clear that the threshold exceeding rate *RE* is the main contributor to BER degradation; by minimizing R_E, SNR_k is maximized resulting in BER improvement, and vice versa. This result can be understood from the PD-HPA transfer function. When the input signal exceeds the saturation threshold ζ, an output signal value has a many-to-one mapping to the input values, where, discriminating between these input values at the receiver to recover the original transmitted data would be challenging, resulting in expected BER degradation. Effectively, errors are introduced in advance at the transmitter. Actually, R_E is not the only contributor to BER degradation since the BER in (14) is based on the SNR in (10). Such SNR is the *apparent* SNR since an increase in the nonlinear component variance $\sigma^2_{nl,k}$ as part of $\sigma^2_{d,k}$ in (11) increases the SNR, although effectively deteriorates the BER [4], [5]. While the *effective* SNR, where the non-linear signal s_{nl} (t) in (6) is assumed Gaussian distributed [1], is defined as [4]

$$\mathrm{SNR}_{\mathrm{eff},k} = \frac{|\alpha_0|^2\sigma^2_{s,k}}{\sigma^2_{n,k} + \sigma^2_{nl,k}} = \frac{|\alpha_0|^2\sigma^2_{s,k}}{\sigma^2_{n,k} + \sigma^2_{d,k} - |\alpha_0|^2\sigma^2_{s,k}}$$
$$= \frac{|\alpha_0|^2\sigma^2_{s,k}}{\sigma^2_{n,k} + \sigma^2_{s,k}(1 - R_E - |\alpha_0|^2)}. \quad (15)$$

Substituting (15) in (9), the BER in presence of the PD-HPA, according to SNReff,k, has the form

$$\mathrm{BER}_k = Q\left(\sqrt{\frac{|\alpha_0|^2}{1 - R_E - |\alpha_0|^2 + (\mathrm{SNR}_{\mathrm{AWGN},k})^{-1}}}\right). \quad (16)$$

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Looking to the linear gain α0, it can be expanded as

$$\alpha_0 = \frac{1}{2\sigma_x^2}\int_0^{\infty} r_d r f_r(r)dr$$
$$= \frac{1}{2\sigma_x^2}\left(\int_0^{\zeta} r^2 f_r(r)dr + \zeta\int_{\zeta}^{\infty} r f_r(r)dr\right)$$
$$= 1 - e^{-\zeta^2/2\sigma_x^2} + \frac{1}{2}\sqrt{\pi\zeta^2/2\sigma_x^2}\,\mathrm{erfc}(\zeta/\sqrt{2\sigma_x^2})$$
$$= 1 - R_E + \frac{\sqrt{\pi\gamma}}{2}\mathrm{erfc}(\sqrt{\gamma}). \qquad (17)$$

The output of the PD-HPA can be written as

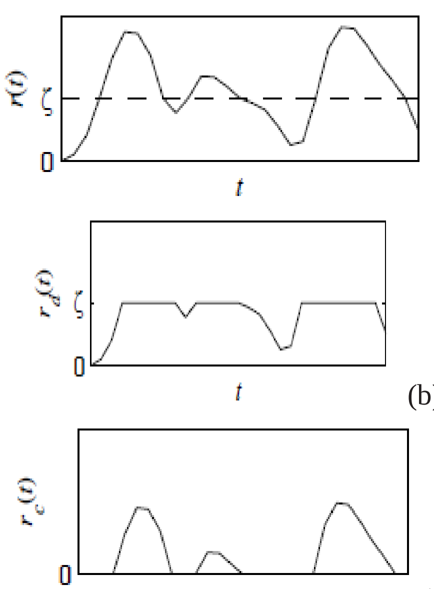
$$s_d(t) = r_d(t)e^{j\theta(t)} = (r(t) - r_c(t))e^{j\theta(t)} = s(t) - s_c(t) \quad (18)$$

where $s_c(t) = r_c(t)e^{j\theta(t)}$ is the signal portion that is clipped from $s(t)$, $E\{s_c(t)\} = 0$, and its envelope $r_c(t)$ has the form

$$r_c(t) = r(t) - r_d(t) = \begin{cases} 0, & r(t) \le \zeta, \\ r(t) - \zeta, & r(t) > \zeta. \end{cases} \quad (19)$$

Examples of $r(t)$, $r_d(t)$, and $r_c(t)$ are shown in Figure 1(a), Figure 1(b), and Figure 1(c), respectively. The variance σ_c^2 of $s_c(t)$ can be derived using the definition of the envelope of the clipped signal portion in (19) as

$$\sigma_c^2 = E\{|s_c(t)|^2\} = E\{r_c^2(t)\} = \int_{\zeta}^{\infty}(r-\zeta)^2 f_r(r)dr$$
$$= 2\sigma_x^2 e^{-\zeta^2/2\sigma_x^2} - \sqrt{2\pi\sigma_x^2}\,\zeta\,\mathrm{erfc}(\zeta/\sqrt{2\sigma_x^2})$$
$$= 2\sigma_x^2(e^{-\gamma} - \sqrt{\pi\gamma}\,\mathrm{erfc}(\sqrt{\gamma})). \qquad (20)$$



(a)
(b)
(c)

Figure 1. (a) Input envelope $r(t)$ to PD-HPA, (b) output envelope $r_d(t)$ from PD-HPA, and (c) clipped envelope portion $r_c(t)$.

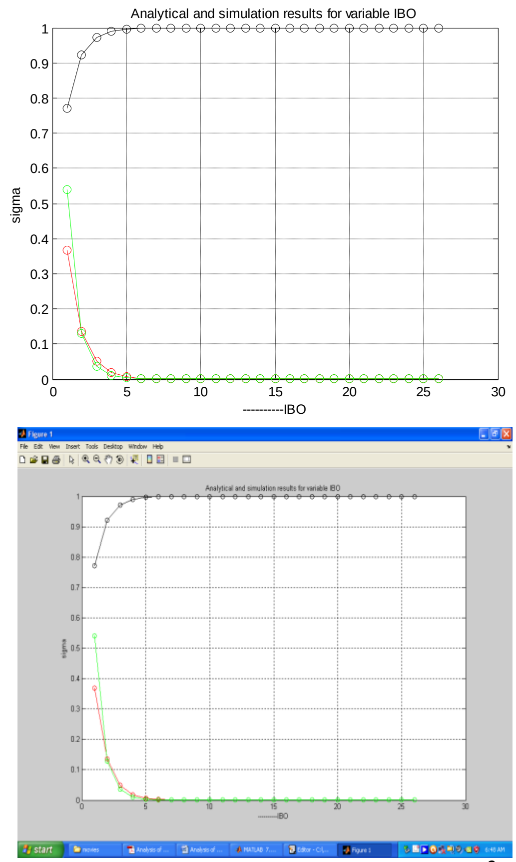


Figure 2. Analytic and simulated plots of RE, σ_c^2 and α_0 over a range of IBOs.

Rearranging (20) and substituting into (17), α_0 is given by

$$\alpha_0 = 1 - \frac{1}{2}e^{-\gamma} - \frac{1}{4\sigma_x^2}\sigma_c^2 = 1 - \frac{1}{2}R_E - \frac{1}{4\sigma_x^2}\sigma_c^2. \qquad (21)$$

From (21) as part of (14), it is clear that RE and σ_c^2 are the main contributors to the BER performance; decreasing R_E and σ_c^2, increases α_0, increases SNR_k, and in turn improves BER.

IV. ANALYSIS VALIDATION

To assess the validity of the derived expressions for the factors contributing to BER degradation, R_E and σ_c^2, and the linear gain α_0, a 64-user CDMA signal filtered using an SRRC filter with Walsh codes of length $L = 64$ is simulated. R_E, σ_c^2 and α_0 are

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measured for the simulated signal and compared against the analytic expressions in (13), (20), and (21), respectively. The relations of such factors versus IBO are shown in Figure 2, where the agreement between the analytic curves and simulated points confirms the analytic derivations. Finally, the analytically derived BER is computed from (14) and compared against the BER measured from simulation.

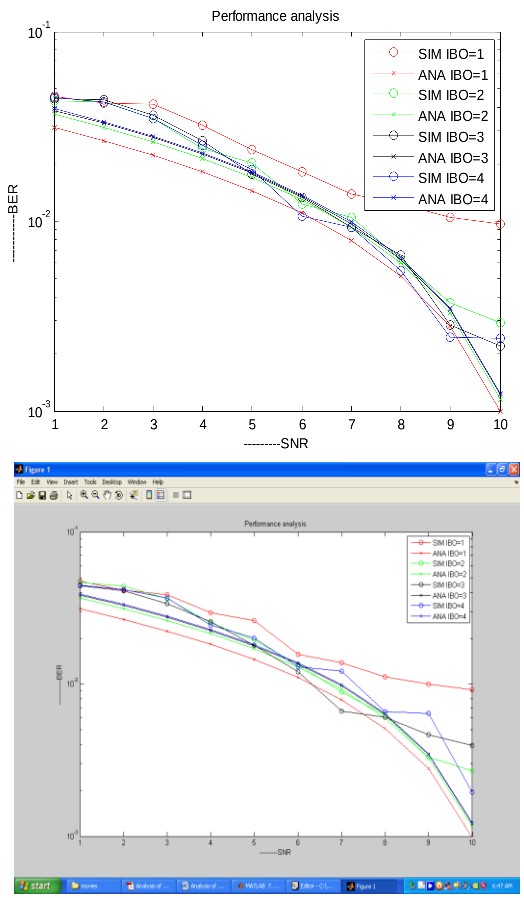

Figure 3. Analytic and simulated plots of BER at different IBO levels.

Figure 3 shows the BER curves, where it is evident that good agreement exists between the analytic curves and the measured points for all IBO levels.

V. CONCLUSIONS

In this paper, how nonlinear distortion due to PDHPA affects BER degradation for transmitted downlink DSCDMA signals has been explained and established analytically how the threshold exceeding rate R_E and the variance of the clipped portion of the signal σ^2_c contribute to BER degradation. The motivation of this work is to provide system designers/operators with efficient tools that provide potential insight into the interactions between CDMA signals and the nonlinear PD-HPA, leading to better understanding of the impact of the PD-HPA on system BER. Moreover, establishing these characteristics for the input signal in relation to the PD-HPA characteristics opens new avenues for research to minimize the effect of nonlinear distortion before the signal even hits the amplifier. For instance, in search and optimization techniques such as the selected mapping technique, where many representations of the same signal are generated and the one that achieves the minimum of these characteristics is selected for transmission.

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