

## Adaptive filtering algorithms for channel equalization and echo cancellation

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### Abstract

*In this paper, we will be addressing the major concerns in telecommunication nowadays which are channel equalization and echo cancellation using different adaptive algorithms in order to identify the most efficient methodology. There are a number of conventional algorithms available in literature and every algorithm has its own properties, however the aim of every adaptive algorithm is to achieve minimum mean square error at a high rate of convergence and with less complexity. The experimental results prove that Least Mean Square algorithm (LMS) is the best for channel equalization and Recursive Least Square (RLS) is most efficient for echo cancellation. Moreover, LMS algorithms work efficiently in case of stochastic processes and on the contrary RLS is good for deterministic signals.*

### 1. Introduction

We have divided this paper in five different sections. The first section is an introduction to the concepts of adaptive filtering. The second section is basically an elaboration of the conventional algorithms that will be used for our system design. The third section emphasizes on the performance criteria's for the selection of an appropriate filtering technique. The fourth section comprises of the simulations of two different system applications i.e. channel equalization and echo cancellation along with their experimental results. The concluding section has been made in order to recommend the best possible algorithm for the selected applications and the loopholes that may ignite future scopes.

Adaptive Filtering is a specialized branch of Digital Signal Processing, dealing with adaptive filters and system design. They are used in a wide range of applications including system identification, noise cancellation, interference nullities, signal prediction, echo cancellation, beam forming and adaptive channel equalization.

Filtering is the extraction of information about a quantity of interest at time 't' by using data measured up to and including time t. If the inputs to the filter are stationary, the resulting solution of the filtering problem is known as the Wiener Filter, which is said to be optimum in mean square sense [10]. But it requires prior information about the statistics of the data to be processed. If the environment is unknown then another efficient method is to use an adaptive filter using recursive algorithm. The algorithm starts with some predetermined set of initial conditions, representing whatever we know about the environment.

Conventional frequency-selective digital filters with fixed coefficients are designed to have a given frequency response chosen to alter the spectrum of the input signal in a desired manner [11]. However, there are many practical application problems that cannot be successfully solved by using fixed digital filters because either we do not have sufficient information to design a digital filter with fixed coefficients or the design criteria changes during the normal operation of the filter. Most of these applications can be successfully solved by using special filters called adaptive filters. The distinguishing feature of adaptive filters is that they can modify their response to improve performance during operation without any intervention from the user.

### 2. Adaptive Filtering Algorithms

Least Mean Square (LMS) and Normalized Least Mean Square (NLMS) belong to an identical adaptive filtering family in comparison with Variable Step Size Least Mean Square (VLMS) which is a modified version of LMS. Recursive Least Square (RLS), however is different in design due to the usage of negative feedback and prior sample estimations.

## 2.1 Least Mean Square

The LMS algorithm is a member of the stochastic gradient algorithms, and because of its robustness and low computational complexity, it has been used in a wide range of applications [1]. Its iterative procedure involves computing the output of a Finite Impulse Response (FIR) filter produced by a set of filter coefficients, followed by the generation of an estimated error by comparing the output of the filter to a desired response and finally, adjusting the filter coefficients based on the estimation error. The following equations depict the aforementioned process.

$$y(n) = w^T(n)x(n) \quad \text{Filter Output} \quad (1)$$

$$e(n) = d(n) - y(n) \quad \text{Error} \quad (2)$$

$$w(n) = [w_0(n) \ w_1(n) \ \dots \ w_{M-1}(n)]^T \quad \text{Filter Coefficients at time } n$$

$$x(n) = [x(n) \ x(n-1) \ x(n-2) \ \dots \ x(n-M+1)]^T \quad \text{Input Data} \quad (4)$$

Where the filter coefficients are calculated using the equation

$$w(n+1) = w(n) + 2\mu e(n)x(n) \quad (5)$$

considering  $\mu$  as the step size. The algorithm at each iteration requires that  $x(n)$ ,  $d(n)$  and  $w(n)$  are known. As the step size decreases, the convergence speed to the optimal values is slower. This also implies that, the LMS algorithm is a stochastic gradient algorithm if the input signal is a stochastic process.

## 2.2 Normalized Least Mean Square

Consider the LMS recursion algorithm of equation (5) above where the step size  $\mu$  varies in time. Since the stability, convergence, and steady state behavior of the LMS algorithm are influenced by the filter length and the power of the signal, therefore, we may set

$$\mu(n) = \frac{1}{2x^T(n)x(n)} = \frac{1}{2\|x(n)\|^2} \quad (6)$$

And therefore,

$$w(n+1) = w(n) + \frac{\bar{\mu}}{\epsilon + x^T(n)x(n)} e(n)x(n) \quad (7)$$

Where  $\bar{\mu}$  and  $\epsilon$  are constants. The small constant  $\epsilon$  prevents division by a very small number of the data norm.

## 2.3 Variable Step Size LMS

The VLMS algorithm overcomes the conflicting requirements of the step size parameter i.e. a large step size is needed for faster convergence and a small step size is needed to reduce misadjustment [6]. When the adaptation begins and  $w(n)$  is far from its optimum value, the step size parameter should be large in order for the convergence to be rapid. However, as the filter coefficients  $w(n)$  approaches the steady state solution, the

step size should decrease in order to reduce the excess MSE.

In order to accomplish the variations in step size, each filter coefficient is given a separate time varying step size parameter such that the LMS algorithm takes the form

$$w_i(n+1) = w_i(n) + 2\mu_i(n)e(n)x(n-i) \quad (8)$$

Where  $i=0,1,\dots,M-1$   
And  $w_i(n)$  is the  $i^{\text{th}}$  coefficient of  $w(n)$  at iteration  $n$  and  $\mu_i(n)$  is its associated step size. The step sizes are determined in an ad-hoc manner based on the monitoring sign changes in the instantaneous gradient estimate  $e(n)x(n-i)$ .

## 2.4 Recursive Least Square

The least square algorithms require all the past samples of the input signal as well as the desired output at every iteration. The RLS algorithm however is based on the least square estimate of the filter coefficients  $w(n-1)$  at iteration  $n-1$ , by computing its estimate at iteration  $n$  using the newly arrived data. This algorithm is a special case of the Kalman Filter [3].

In RLS the computation begins with known initial conditions and further updates the old estimates based on the information contained in the new data samples. Thereafter, the cost function  $J(n)$  is minimized, contrary to the conventional minimization of MSE in case of LMS, NLMS and VLMS, where  $n$  is the variable length of the observed data.

$$J(n) = \sum_{k=1}^n \eta_n(k) e^2(k) \quad (9)$$

where  $\eta_n(k)$  = weighting factor. The filter coefficients are fixed during the observation time  $1 \leq k \leq n$  during which the cost function  $J(n)$  is defined. The weighting factor is chosen to have the exponential form

$$\eta_n(k) = \lambda^{n-k} \quad k=1,2,\dots,n \quad (10)$$

Where the value of  $\lambda$  is less than one and is known as the forgetting factor since it emphasize the recent data and tends to forget the past. This gives the RLS algorithm tracking capabilities.

## 3. Performance parameters

The prime factors [17] that are used in this paper to judge the performance of an adaptive algorithm are listed as follows.

### 3.1 Convergence Rate

The convergence rate is defined as the number of iterations required for the algorithm to converge to its steady state mean square error. The steady state MSE is also known as the Mean asymptotic square error or MASE.

### 3.2 Computation Time

This is an important parameter from a practical view point. The computation time depends on the number of operations required for one complete iteration of the algorithm along with the amount of memory needed to store the mathematical calculations.

### 3.3 Stability

An algorithm is said to be stable if the mean-squared error converges to a finite value.

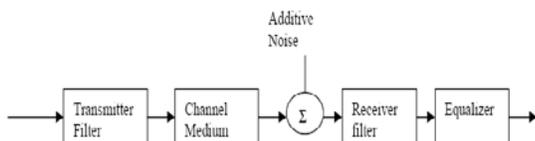
Ideally, applications prefer computationally simple and numerically robust adaptive filters with high rate of convergence.

## 4. System design and analysis

Implementing adaptive algorithms in practical applications like echo cancellation and channel equalization, require specialized designing as per the applications requirement. The following sections emphasize the system configurations and setups for comparison of the above algorithms.

### 4.1 Channel Equalization

Telecommunication channels such as telephone, wireless and optical channels are susceptible to inter symbol interference (ISI). Channel equalization is a simple way of mitigating the detrimental effects caused by a frequency-selective and/or dispersive communication link between sender and receiver, hence enabling higher data rates. A typical communication system is depicted in Figure 1, where the equalizer is incorporated within the receiver and the channel introduces inter symbol interference.



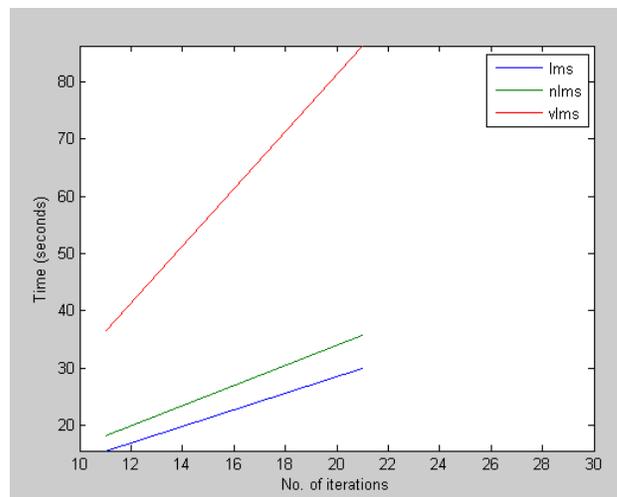
**Figure 1: System design for channel equalization.**

For our experiment, the equalizer shown in the above figure is designed to be adaptive. The computed values of MSE using the three conventional adaptive algorithms, namely LMS, NLMS and VLMS, are obtained by using additive white Gaussian noise (AWGN) of different variance to a sample speech of 11220 samples.

**Table 1. Experimental results**

Algorit hm	Variance	Filter Coeff	Iteratio ns	MSE	Time (secon ds)
LMS	0.0001	100	21	0.0817	30.036
NLMS	0.0001	100	21	0.0821	35.734
VLMS	0.0001	100	21	0.0819	86.356
LMS	0.01	50	11	0.1425	15.545
NLMS	0.01	50	11	0.1421	18.183
VLMS	0.01	50	11	0.1418	36.455

The lesser the error, the more accurate the algorithm is. The above table clearly projects LMS to be the most accurate algorithm for equalization with better performance and lesser complexity. RLS algorithm however, failed in this application due to its unstable nature, thereby making the equalizer unstable for the given noise variance and iteration set.

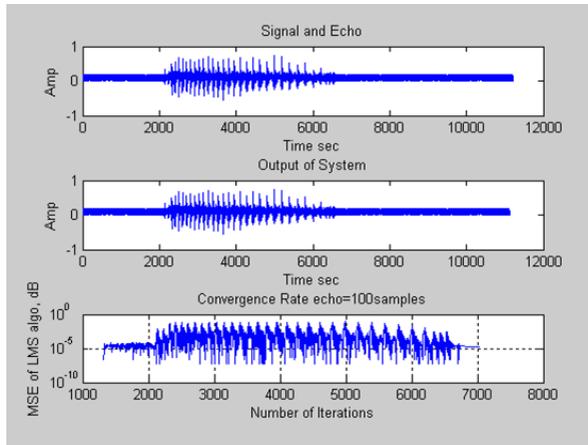


**Figure 2: Comparisons of computation time for LMS, NLMS and VLMS.**

### 4.2 Echo Cancellation

Echo cancellation is used in telephony to remove echo from voice communications in order to improve voice quality during a telephone call. In addition to improving the voice quality, this process also increases the capacity by preventing echo from traveling across a network. The issues faced by echo cancellers, generally, are the convergence time and the degree of cancellation. Here, convergence time means the time taken to reach an acceptable level of residual echo and the degree of cancellation is the amount of echo cancelled [14].

The system design of an echo canceller comprises of an echo estimator and a subtractor. The echo estimator monitors the received path and dynamically builds a mathematical model, based on an adaptive algorithm, of the channel that creates the echo. The model of the line is then convolved with the voice stream from the receiver. This generates an estimate of the echo, which is applied to the subtractor. The subtractor finally, eliminates the linear part of the echo from the transmitter. The figure below displays the input and output signals of the echo canceller.



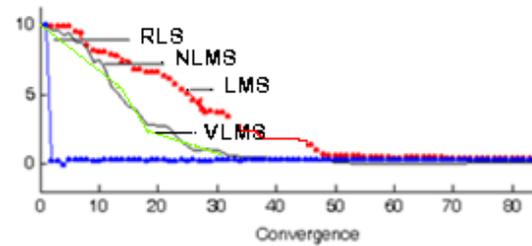
**Figure 3: Input and output signals of echo canceller.**

In our experiment, the echo signal was generated by overlapping the last 100 samples of a speech signal with the initial 100 values. The experimental results are given below

**Table 2. Experimental results**

Algorithm	Filter Coeff	Iterations	MSE	Time (seconds)
LMS	100	50	0.0713	0.269
NLMS	100	50	0.0016	0.318
VLMS	100	50	0.0005	0.528
RLS	100	50	0.0002	0.799

Another important factor in judging the appropriate algorithm for an application is its convergence rate. This is defined as the number of iterations required for the algorithm to converge to its steady state mean square error or MSE.



**Figure 4: Comparisons of convergence rates for LMS, NLMS and RLS**

Figure 4 above, shows that the convergence rate of the RLS algorithm is much greater than that of LMS and NLMS. Also the mean square error is least for RLS thereby increasing the filtering-accuracy.

## 5. Conclusion

The experimental results prove that LMS algorithm is the best for channel equalization and RLS is most efficient for echo cancellation. However, the complexity of RLS algorithm prevents its usage and thereby we recommend NLMS/VLMS instead for echo cancellation depending upon the application priority. In case the echo canceller is to be used for a high quality application, using VLMS would give best results and for a quick echo canceller, NLMS can be used instead. The instability of RLS algorithm makes it unsuitable for equalization purposes also. Varying step sizes and iteration patterns in adaptive algorithms for smaller speech signals increases the computation time with minor variations in accuracy of estimations.

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