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Study the Effect of Variable Viscosity and Thermal Conductivity of Micropolar Fluid in a Porous Channel

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Abstract

To study the effect of thermal radiation on unsteady boundary layer flow with variable viscosity and thermal conductivity due to stretching sheet in porous channel in presence of magnetic field, a numerical model has been developed. The Rosseland diffusion approximation is used to describe the radiative heat flux in the energy equation. The governing equations reduced to similarity boundary layer equations using suitable transformations and then solved using shooting method. A parametric study illustrating the influence of the radiation R, variable viscosity ε, Darcy number Da, porous media inertia coefficient γ, thermal conductivity κ, unsteady A parameters on skin friction and Nusselt number and Magnetic field parameter M.

Key words: *unsteady flow, radiation, stretching sheet, variable viscosity, variable thermal conductivity, porous channel.*

1. Introduction

The heat transfer from a stretching surface is of interest in many practical applications. Specially in the manufacturing process of plastic and rubber sheets where it often necessary to blow a gaseous medium through the unsolidified material this situation arise. Extrusion processes, glass blowing, continuous coating and spinning of fibers also involve the flow due to a stretching surface.

Crane[12] was the first who studied the momentum boundary layer for linear stretching of sheet. The temperature field in the flow over stretching surface subject to a uniform heat flux was studied by Grubka and Bobba[13], while Elbashbeshy[6] considered the case of stretching surface with a variable surface heat flux. Elbashebeshy and Bazid[7] have found similarity solutions of the boundary layer equations, which describes the unsteady flow and heat transfer over a stretching sheet. Unsteady flow and heat transfer over a stretching sheet in viscous and incompressible fluid was also studied by Sharidan et al.[23]. Pop and Tsung[10] studied unsteady flow past a stretching sheet. Unsteady boundary layer flow in the region of the stagnation point on a stretching boundary layer flow in the region of the stagnation point on a stretching was studied by Nazar et al.[20]. Yurusoy[19] also investigated unsteady boundary layer flow of power law fluid on the stretching sheet surface. Effect of viscous dissipation on heat transfer in a non Newtonian liquid film over an unsteady stretching sheet was studied by Chen[5]. Dandapat et al.[4] investigated the effect of variable fluid properties and thermo capillarity on the flow of a thin film on an unsteady stretching sheet. Chiam[24] considered the effect of a variable thermal conductivity on the flow and heat transfer from a linearly stretching sheet.

Most of the previous studies were concerned with MHD fluid. Effects of variable viscosity and thermal conductivity on MHD flow and heat transfer in viscoelastic fluid over a stretching sheet studied by Salem[3]. Effect of variable viscosity and thermal conductivity on a unsteady two dimensional laminar flow of viscous incompressible conducting fluid past a semi infinite vertical porous moving plate taking into account the effect of a magnetic filed in the presence of variable suction was studied by Seddeek et al.[16]. Odda and Farhan[22] was studied the effects of variable viscosity and variable thermal conductivity on heat transfer from a stretching sheet by considering fluid viscosity and the thermal conductivity to vary as inverse linear functions of temperature. Natural convection with variable viscosity and thermal conductivity from a vertical wavy cone and steady two dimensional laminar forced flow and heat transfer of a viscous incompressible fluid having temperature dependent viscosity and thermal conductivity past a wedge with a uniform surface heat was studied by Hossain et al.[14,15]. Chamkha[2] has studied unsteady hydromagnetic flow and heat transfer from a non-isothermal stretching immersed in a porous media. Hassanien et al.[9] studied variable viscosity and thermal conductivity effects on combined heat and mass transfer in mixed convection over a UHF/UMF wedge in porous media in the entire regime. The effect of thermal radiation on mixed convection flow of two immiscible fluids in a vertical porous stratum was studied by Umavathi[11]. Considering the effect of thermal radiation on Forchheimer natural convection over vertical flat plate in a fluid saturated porous medium was studied by El-Amin et al.[17]. El-Kabeir et al.[21] have investigated the effect of thermal radiation and a transverse magnetic field with surface mass transfer in free convection on a vertical stretching surface with suction and blowing. The effect if thermal radiation on free convection flow with variable viscosity and uniform suction velocity along a uniformly heated vertical porous plate embedded in a porous medium in ht epresence of a uniform transverse magnetic field is analyzed by Modather and El-Kabeir[18]. The effect of thermal radiation on unsteady boundary layer flow with temperature dependent viscosity and thermal conductivity due to stretching sheet through porous media was studied by Abdou[1].

In the present work We propose to extend the work of Abdou[1] and study the effect of radiation on unsteady boundary layer flow with variable viscosity and thermal conductivity due to stretching sheet through porous channel in presence of magnetic field. The governing reduced to similarity boundary layer equations using suitable transformations and then solved using shooting method. Numerical results are presented in terms of local skin friction coefficient and rate of heat transfer for various values of the variable viscosity ε , Darcy number *Da*, porous media inertia coefficient γ, thermal conductivity κ, unsteady A parameters on skin friction and Nusselt number and Magnetic field parameter M. The effect of variation in ε, κ, *A*, *Da*, *R*, γ *and M* are presented graphically.

2. Mathematical formulation

We consider the unsteady two-dimensional laminar flow of a viscous incompressible micropolar fluid with variable viscosity and thermal conductivity past a semi infinite stretching sheet in the region *y>0* in presence of a magnetic field as shown in Fig. 1.

Keeping the origin fixed, the equal and opposite forces are suddenly applied along the x-axis, which results in stretching of the sheet and hence, flow is generated. At the same time, the wall temperature $T_w(x,t)$ of the sheet is suddenly raised from T_α to $T_w(t,x)(>T_\alpha)$. Under these assumptions, the basic unsteady boundary layer equations governing the flow and heat transfer due to stretching sheet are given by

The equation of continuity is

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \qquad (1)$$

The equation of momentum is

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\mu}{K}u-Cu^2-\frac{\sigma}{\rho}B_0{}^2u \qquad (2)$$

The equation of energy is

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}-q^r\right) \qquad (3)$$

In the above equations t is the time, u and v are the components of fluid velocity in the x and y directions respectively, ρ - the density of ambient fluid, T is the fluid temperature in the boundary layer region, B_0 is external magnetic field, μ and k are respectively the dynamic viscosity and the thermal conductivity, following are given as below:

$$\mu = \mu_\alpha \left[1 + \alpha_1 \frac{T - T_\alpha}{T_0 - T_\alpha}\right] \quad (4a)$$

and

$$k = k_\alpha \left[1 + \alpha_2 \frac{T - T_\alpha}{T_0 - T_\alpha}\right] \quad (4b)$$

In (4) μ_α is the viscosity and k_α is the thermal conductivity of the ambient fluid, T_α is the temperature of the ambient fluid, T_0 is some reference temperature and α_1, α_2 are constant. Clearly $\alpha_1 = 0$ and $\alpha_2 = 0$ represent that the dynamical viscosity and the thermal conductivity be uniform.

Solutions of the above equations have to satisfy the following boundary conditions [1]

$$t < 0; \quad u = v = 0; \quad T = T_\alpha \quad \text{for any } x, y$$

$$t \geq 0; \quad u = u_w(t,x) \quad v = 0 \quad T = T_w(t,x) \quad (5a)$$

$$u \to 0, \quad T \to T_\alpha \quad \text{as } y \to \alpha$$

We assume now that the velocity of the sheet $u_w(t,x)$ and the sheet temperature $T_w(t,x)$ have the following form:

$$u_w(t,x) = cx(1-\delta t)^{-1}, \quad T_w(t,x) = T_\alpha + \frac{c}{2vx^2}(1-\delta t)^{-\frac{3}{2}} \quad (5b)$$

where c is the stretching rate being a positive constant, δ is a positive constant, which measures the unsteadiness.

The quantity q^r on the right-hand side of equation (3) represents the radiative heat flux in the y direction. In order to reduce the complexity of the problem and to provide a means of comparison with future studies, a more detailed representation for radiative heat flux, for optically thick radiation limit, is considered in the present analysis. Thus the radiative heat flux term in the energy equation is simplified by utilizing the Rosseland diffusion approximation (Sparrow and Cess [8]) for an optically thick boundary layer as follows:

$$q^r = \frac{4\sigma}{3\chi(a_0 + \sigma_s)} \frac{\partial T^4}{\partial y} \quad (5c)$$

Where σ is the Stefan-Boltzman constant, χ is the mean absorption coefficient, a_0 is the Rosseland mean absorption coefficient and σ_s is the scattering coefficient. This approximation is valid at point optically far from the bounding surface layer. If temperature differences withing the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature, then the Taylors series for T^4 about T_α, after neglecting higher order term s, is given by : [1]

$$T^4 \approx 4TT_\alpha^{\;3} - 3T_\alpha^{\;4}$$

We introduce now the following variables

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$$

$$\eta = y\sqrt{\frac{c}{v_\alpha(1-\delta t)}}y, \psi = \sqrt{\frac{cv_\alpha}{1-\delta t}}xf(\eta), T = T_\alpha + \frac{c}{2v_\alpha x^2}(1-\delta t)^{-3/2}\theta(\eta) \quad (6)$$

where ψ is the stream function satisfying the continuity equation, $f(\eta)$ is the dimensionless stream function, η is the pseudo-similarity variable, $\theta(\eta)$ is the dimensionless temperature of the fluid in the boundary layer region, $v_\alpha = \mu_\alpha / \rho$ is the stream kinematics viscosity.

Substituting the transformation given in (6) into (2), (3) one obtains the following non-similar system equations governing the flow:

$$(1+\varepsilon\theta)f''' + (f + \varepsilon\theta')f'' - (1+\gamma)f'^2 - A\left(f' + \frac{1}{2}\eta f''\right) - \left(\frac{1+\varepsilon\theta}{Da} + M_1\right)f' = 0 \quad (7)$$

$$\frac{1}{\Pr}\left\{\left(1 + \frac{4}{3}R\right)(1+k\theta)\theta'' + k\theta'^2\right\} + f\theta' + 2\theta f' - \frac{A}{2}(3\theta + \eta\theta') = 0 \quad (8)$$

The transform boundary conditions are

$$f(0) = 0, \quad f'(0) = 1 \quad \theta(0) = 1 \quad (9)$$

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$$f'(\alpha)=0 \quad \theta(\alpha)=0$$

Where

$$A=\frac{\delta}{c},\quad \Pr=\frac{\mu_\alpha c_p}{k},\quad \varepsilon=\alpha_1\frac{T-T_\alpha}{T_0-T_\alpha},\quad k=\alpha_2\frac{T-T_\alpha}{T_0-T_\alpha}$$

$$M_1=\frac{\sigma B_0^{\ 2}}{\rho c},\quad R=R=\frac{\rho c_p k v}{4\sigma T_\alpha^{\ 3}},\quad Da=\frac{kc}{\mu_\alpha v}$$

Where ε is the viscosity variation, k is the thermal conductivity parameter, A is a non dimensional constant which measures the flow and heat transfer unsteadiness and Pr is the Prandtl number, M_1 is the magnetic parameter, R is the radiation parameter, Da is the Darcy number.

The quantities of physical interested, namely, the local skin friction C_f and the rate of heat transfer in terms of local Nussel number Nu_x are prescribed by:

$$C_f=\frac{\tau_w}{(1/2)\rho u_\alpha^{\ 2}}$$

$$Nu_x=\frac{q_w x}{k_\alpha(T_w-T_\alpha)}$$

Where τ_w is the skin friction and q_w is the heat transfer from the sheet are given by:

$$\tau_w=\mu_\alpha\left(\frac{\partial u}{\partial y}\right)_{y=0}=\mu_\alpha f''(0),\qquad q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

$$C_f R_e^{\ 1/2}=f''(0),\qquad Nu_x/\mathrm{Re}_x^{\ 1/2}=-\theta'(0)$$

Where $\mathrm{Re}_x=u_w x/v_\alpha$ is the local Reynolds number.

3. Results and Discussion

Equations (7), (8) with the boundary conditions (9) are solved numerically using shooting method. Calculations were carried out for the value of Prandtl number 1.0, the viscosity variation parameter ε ranged from 0.0 to 1.0, unsteady parameter A ranged from 0.0 to 2.0, radiation parameter R ranged from 0.0 to 4.0, Darcy number Da range from 1.0 to α and porous media inertia coefficient γ range from 0.0 to 1.0.

Figs. 1-9 display the dimensionless of velocity f' and temperature θ profiles for various values of Darcy number parameter Da, unsteady parameter A, the thermal conductivity variation parameter k, and porous media inertia coefficient γ, magnetic parameter M_1, radiation parameter R, viscosity parameter ε respectively, while the others parameters are fixed. It can be seen that velocity and temperature of the fluid increases as Darcy number parameter Da, thermal conductivity variation parameter k and magnetic parameter M_1 increase, while the opposite trend is observed for the effect of A and porous media inertia coefficient γ. This behaviour is in agreement with the results of Elbashbeshy and Bazid [7] and Sharidan et al. [23]. The same result was found by Abdou [1] in non magnetic cases. Moreover, the boundary layer thickness decreases with an increase in A which in turn increases the skin friction coefficient and Nussel number. In addition, the effect of k on the boundary layer separation is not very pronounced compared to the effects of A.

4. Conclusion

In the present work we consider the effect of thermal radiation on unsteady boundary layer flow with variable viscosity and thermal conductivity due to a stretching sheet through porous media in presence of external magnetic field. The governing equations reduced to similarity boundary layer equations using suitable transformations and then solved using the shooting method. Numerical results are presented in terms of Darcy number parameter Da, unsteady parameter A, the thermal conductivity variation parameter k, and porous media inertia coefficient γ, magnetic parameter M_1, radiation parameter R, viscosity parameter ε respectively. We notice the values of Nusselt number increases with increasing of radiation parameter R and thermal conductivity parameter k, while the skin fiction has the opposite behaviour, on the other hand as porous media inertia coefficient γ increases the shear stress decreases and Nusselt number increases.

REFERENCES

1. Abdou, M.M.M., *Effect of radiation with temperature dependent viscosity and thermal conductivity on unsteady a stretching sheet through porous media, Nonlinear Analysis*:

Modeling and Control, Vol. 15, No. 3, pp. 257-270 (2010)
2. A.J. Chamkha, *Unsteady hydromagnetic flow and heat transfer from a non-isothermal stretching sheet immersed in a porous medium*, Int. Commun. Heat Mass, Vol. 25 (6), pp. 899-906 (1998).
3. A. M. Salem, *Variable viscosity and thermal conductivity effects on MHD flow and heat transfer in viscoelastic fluid over a stretching sheet*, Phys. Lett. A, Vol. 369 (4), pp. 315-322 (2007).
4. B. S. Dandapat, B. Santra, K. Vajravelu, *The effects of variable fluid properties and thermocapilliarity on the flow of a thin film on an unsteady stretching sheet*, Int. J. Heat Mass Tran, Vol. 50 (5-6), pp. 991-996 (2007).
5. Ch. H. Chen, *Effect of viscous dissipation on heat transfer in a non-Newtonian liquid film over an unsteady stretching sheet*, J. Non-Newton. Fluid, Vol. 135(2-3), pp. 128-135, (2006).
6. E. M. A. Elbashebeshy, *Heat transfer over a stretching surface with variable surface heat flux*, J. Phys. D Appl. Phys., Vol. 31, pp. 1951-1954 (1998)
7. E. M. A. Elbashebeshy, M. A. A. Bazid, *Heat transfer over a unsteady stretching surface*, Heat Mass Transfer, Vol. 41, pp. 1-4 (2004)
8. E.M. Sparrow, D.R. Cess, *Radiation Heat Transfer Augmented Edition*, *Hemisphere*, Publishing Corporation, Washington, D.C. (1978).
9. I.A. Hossanien, A.H. Essawy, N.M. Moursy, *Variable viscosity and thermal conductivity effects on combined heat and mass transfer in mixed convection over a UHF/UMF wedge in porous media: the entire regime*, Appl. Math. Comput. Vol. 145(2-3), pp. 667-682 (2003).
10. I. Pop, T. Y. Na, *Unsteady flow past a stretching sheet*, Mech. Res. Commun, Vol. 23(4), pp. 413-422 (1996)
11. J.C. Umavathi, A. J. Chamkha, M. H. Manjula, A. AI-Mudhaf, *Radiative heat transfer of a two fluid flow in a vertical porous Stratum*, J. Fluid Mechanics Research, Vol. 35(6), pp. 510-543 (2008).
12. L. J. Crane, *Flow past a stretching plate*, Z. Angew. Math. Phys., Vol. 21, pp. 645-647 (1970)
13. L. J. Grubka, K. M. Bobba, *Heat transfer characteristic of a continuous stretching surface with variable temperature*, J. Heat Transf., Vol. 107, pp. 248-250 (1985)
14. M. A. Hossain, M. S. Munir, D.A.S. Rees, *Flow of viscous incompressible fluid with temperature dependent viscosity and thermal conductivity past a permeable wedge with uniform surface heat flux*, Int. J. Therm. Sci., Vol. 39, pp. 635-644 (2000).
15. M. A. Hossain, M. S. Munir, I. Pop, *Natural convection with variable viscosity and thermal conductivity from a vertical wavy cone*, Int. J. Therm. Sci., Vol. 40, pp. 437-443 (2001).
16. M. A. Seddeek, F.A. Salama, *The effects of temperature dependent viscosity and thermal conductivity on unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction*, Comp. Mater. Sci., Vol. 40(2), pp. 186-192 (2007).
17. M. F. EI-Amin, I. Abbas, R. S. R. Gorla, *Effect of thermal radiation on a natural convection in a porous medium*, Int. J. Fluid Mechanics Research, Vol. 34(2), pp. 129-144 (2007).
18. M. Modather, M. Abdou, S.M.M. EL-Kabeir, *Magneto hydrodynamics and radiative effect on free convection flow of fluid with variable viscosity from a vertical plate through a porous medium*, J. Porous Media, Vol. 10(5), pp. 503-556 (2007).
19. M. Yurusoy, *Unsteady boundary layer flow of power-law fluid on stretching sheet surface*, Int. J. Eng. Sci., Vol. 44 (5-6), pp. 325-332 (2006).
20. R. Nazar, N. Amin, D. Filip, I. Pop, *Unsteady boundary layer flow in the region of the stagnation point on a stretching sheet*, Int. J. Eng. Sci., Vol. 42(11-12), pp. 1241-1253 (2004)
21. S.M.M. EL-Kabeir, M. Modather, M. Abdou, R.S.R. Gorla, *Hydromagnetic combined convection from radiating vertical stretching sheet with surface mass transfer*, Int. J. Fluid

ISSN:2229-6093

Mechanics Research, Vol. 35(6), pp. 544-556 (2008).

22. S. N. Odda, A.M. Farhan, *Chebyshev finite difference method for the effects of variable viscosity and variable thermal conductivity on heat transfer to a micro-polar fluid from a non-isothermal stretching sheet with suction and blowing*, Chaos Solution Fract., Vol. 30(4), pp. 851-858 (2006).
23. S. Sharidan, T. Mahmood, I. Pop, *Similarity solution for the unsteady boundary layer flow and heat transfer due to a streatching sheet*, Int. J. Appl. Mech. Mech. Eng., Vol. 11(3), pp. 647-654 (2006).
24. T.C. Chiam, *Heat transfer in a fluid with variable thermal conductivity over a linearly stretching sheet*, Acta Meh., Vol. 129, pp. 63-72 (1998).

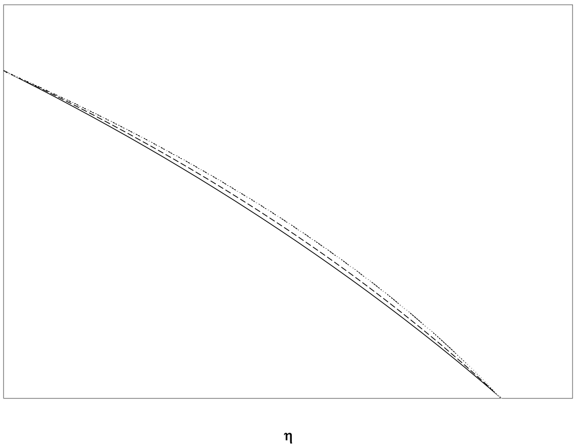

Fig. 1. Velocity profiles for various values of the unsteady parameter A

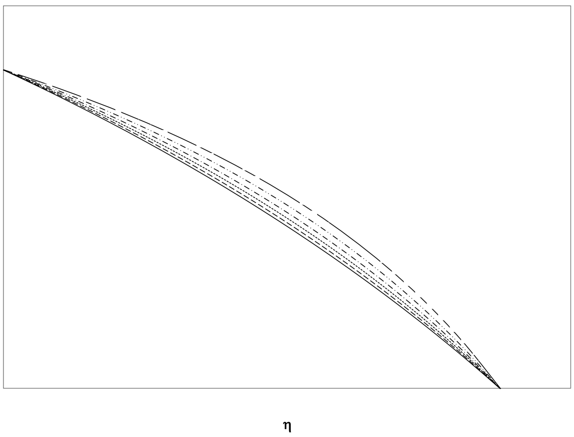

Fig. 2. Velocity profiles for various values of the thermal conductivity parameter k.

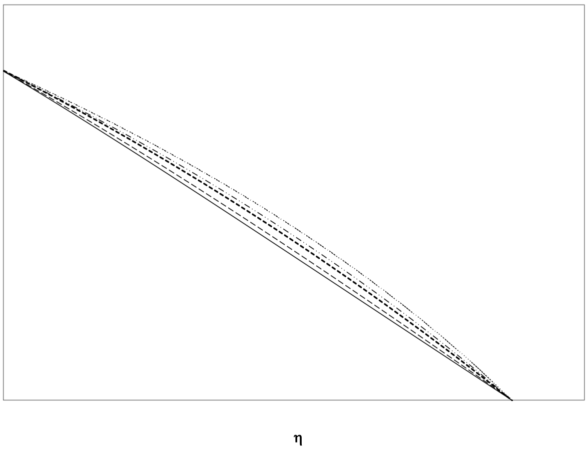

Fig. 3. Velocity profiles for various values of the unsteady parameter Da

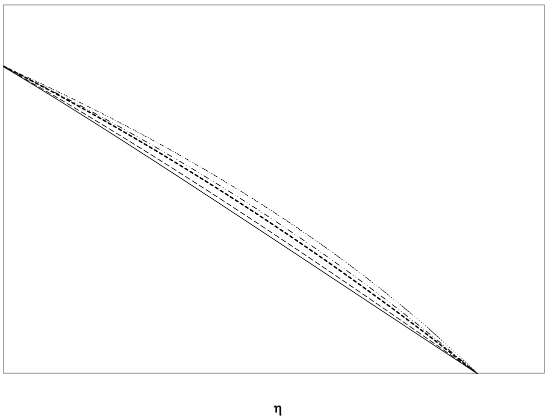

Fig. 4. Velocity profiles for various values of the magnetic parameter M_1.

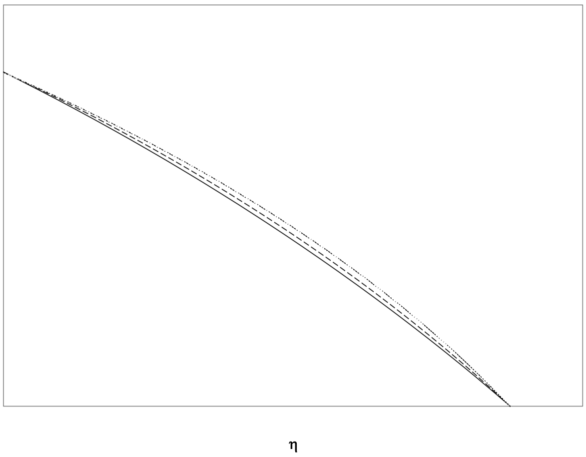

Fig. 5. Velocity profiles for various values of permeability parameter γ.

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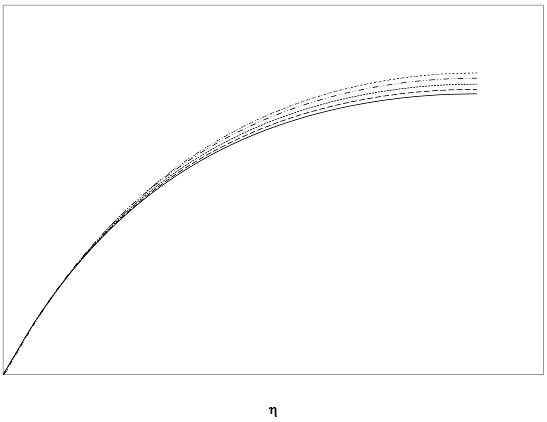

Fig. 6. Temperature profiles for various values of the unsteady parameter A

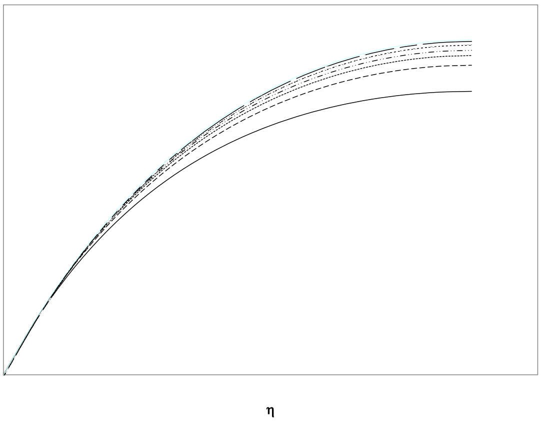

Fig. 7. Temperature profiles for various values of magnetic parameter M_1.

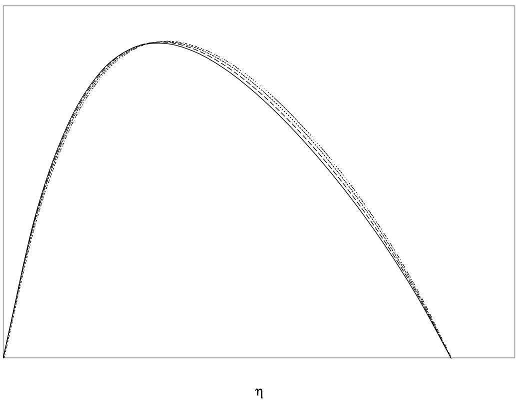

Fig. 8. Temperature profiles for various values of viscosity parameter ε.

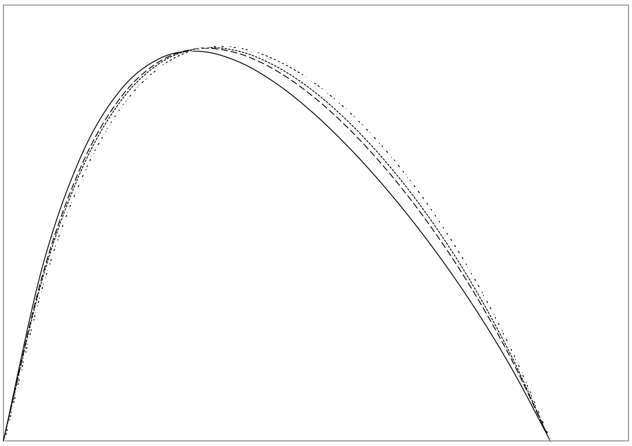

Fig. 9. Temperature profiles for various values of the radiation parameter R.