The metric dimension of circulant graphs

Tomas Vetrik

Abstract. A subset W of the vertex set of a graph G is called a resolving set of G if for every pair of distinct vertices u, v of G, there is $w \in W$ such that the distance of w and u is different from the distance of w and v. The cardinality of a smallest resolving set is called the metric dimension of G, denoted by $dim(G)$. The circulant graph $C_n(1, 2, \ldots, t)$ consists of the vertices $v_0, v_1, \ldots, v_{n-1}$ and the edges $v_i v_{i+j}$, where $0 \le i \le n-1$, $1 \le j \le t$ ($2 \le t \le \lfloor \frac{n}{2} \rfloor$), the indices are taken modulo n. Grigorious et al. [On the metric dimension of circulant and Harary graphs, Applied Mathematics and Computation 248 (2014), 47–54] proved that $dim(C_n(1, 2, \ldots, t)) \ge t + 1$ for $t < \lfloor \frac{n}{2} \rfloor$, $n \ge 3$, and they presented a conjecture saying that $dim(C_n(1, 2, \ldots, t)) = t + p - 1$ for $n = 2tk + t + p$, where $3 \le p \le t + 1$. We disprove both statements. We show that if $t \ge 4$ is even, there exists an infinite set of values of n such that $dim(C_n(1, 2, \ldots, t)) = t$. We also prove that $dim(C_n(1, 2, \ldots, t)) \le t + \frac{p}{2}$ for $n = 2tk + t + p$, where t and p are even, $t \ge 4$, $2 \le p \le t$ and $k \ge 1$.