

# Co-Maximal Graphs of Subgroups of Groups

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*Abstract.* Let  $H$  be a group. The co-maximal graph of subgroups of  $H$ , denoted by  $\Gamma(H)$ , is a graph whose vertices are non-trivial and proper subgroups of  $H$  and two distinct vertices  $L$  and  $K$  are adjacent in  $\Gamma(H)$  if and only if  $H = LK$ . In this paper, we study the connectivity, diameter, clique number and vertex chromatic number of  $\Gamma(H)$ . For instance, we show that if  $\Gamma(H)$  has no isolated vertex, then  $\Gamma(H)$  is connected with diameter at most 3. Also, we characterize all finite groups whose co-maximal graphs are connected. Among other results, we show that if  $H$  is a finitely generated solvable group and  $\Gamma(H)$  is connected and moreover the degree of a maximal subgroup is finite, then  $H$  is finite. Furthermore, we show that the degree of each vertex in the co-maximal graph of a general linear group over an algebraically closed field is zero or infinite.