Lipschitz retractions in Hadamard spaces via gradient flow semigroups

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Abstract. Let $X(n)$, for $n \in \mathbb{N}$, be the set of all subsets of a metric space (X, d) of cardinality at most n. The set $X(n)$ equipped with the Hausdorff metric is called a finite subset space. In this paper we are concerned with the existence of Lipschitz retractions $r \colon X(n) \to X(n-1)$ for $n \geq 2$. It is known that such retractions do not exist if X is the one-dimensional sphere. On the other hand L. Kovalev has recently established their existence in case X is a Hilbert space and he also posed a question as to whether or not such Lipschitz retractions exist for X being a Hadamard space. In the present paper we answer this question in the positive.