Small prime solutions to cubic Diophantine equations II

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Abstract. Let $a_1, \cdots, a_9$ be non-zero integers and n any integer. Suppose that $a_1 + \cdots + a_9 \equiv n \pmod 2$ and $(a_i, a_j) = 1$ for $1 \le i < j \le 9$. In this paper we prove that
(i) if a_j are not all of the same sign, then the cubic equation $a_1 p_1^3 + \cdots + a_9 p_9^3 = n$ has prime solutions satisfying $p_j \ll |n|^{1/3} + \max\{|a_j|\}^{8+\varepsilon}$;
(ii) if all a_j are positive and $n \gg \max\{|a_j|\}^{25+\varepsilon}$, then $a_1 p_1^3 + \cdots + a_9 p_9^3 = n$ is soluble in primes p_j.
This results improve our previous results (Canad. Math. Bull., **56** (2013), 785-794) with the bounds $\max\{|a_j|\}^{14+\varepsilon}$ and $\max\{|a_j|\}^{43+\varepsilon}$ in place of $\max\{|a_j|\}^{8+\varepsilon}$ and $\max\{|a_j|\}^{25+\varepsilon}$ above, respectively.