Existence of multiple solutions for a p-Laplacian system in $\mathbf{R}^N$ with sign-changing weight functions

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Abstract. In this paper, we consider the quasi-linear elliptic problem

$$\begin{cases} -M\left(\int_{\mathbb{R}^N} |x|^{-ap}|\nabla u|^p dx\right) \operatorname{div}\left(|x|^{-ap}|\nabla u|^{p-2}\nabla u\right) \\ \qquad = \dfrac{\alpha}{\alpha+\beta} H(x)|u|^{\alpha-2}u|v|^{\beta} + \lambda h_1(x)|u|^{q-2}u, \\ -M\left(\int_{\mathbb{R}^N} |x|^{-ap}|\nabla v|^p dx\right) \operatorname{div}\left(|x|^{-ap}|\nabla v|^{p-2}\nabla v\right) \\ \qquad = \dfrac{\beta}{\alpha+\beta} H(x)|v|^{\beta-2}v|u|^{\alpha} + \mu h_2(x)|v|^{q-2}v, \\ u(x) > 0,\ v(x) > 0,\ x \in \mathbb{R}^N \end{cases}$$

where $\lambda, \mu > 0$, $1 < p < N$, $1 < q < p < p(\tau+1) < \alpha+\beta < p^* = \frac{Np}{N-p}$, $0 \le a < \frac{N-p}{p}$, $a \le b < a + d = a + 1 - b > 0$, $M(s) = k + ls^{\tau}$, $k > 0$, $l, \tau \ge 0$ and the weight $H(x)$, $h_1(x)$, $h_2(x)$ are continuous functions whic change sign in $\mathbb{R}^N$. We will prove that the problem has at least two positive solutions by using the Nehari manifo and the fibering maps associated with the Euler functional for this problem.