

Local Heuristics and an Exact Formula for Abelian Surfaces Over Finite Fields

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Abstract. Consider a quartic q -Weil polynomial f . Motivated by equidistribution considerations, we define, for each prime ℓ , a local factor that measures the relative frequency with which $f \bmod \ell$ occurs as the characteristic polynomial of a symplectic similitude over \mathbb{F}_ℓ . For a certain class of polynomials, we show that the resulting infinite product calculates the number of principally polarized abelian surfaces over \mathbb{F}_q with Weil polynomial f .