

Cover product and Betti polynomial of graphs

Aurora Llamas and José Martínez–Bernal

Abstract. For disjoint graphs G and H , with fixed vertex covers $C(G)$ and $C(H)$, their cover product is the graph $G \circledast H$ with vertex set $V(G) \cup V(H)$ and edge set $E(G) \cup E(H) \cup \{\{i, j\} : i \in C(G), j \in C(H)\}$. We describe the graded Betti numbers of $G \circledast H$ in terms of those of G and H . As applications we obtain: (i) For any positive integer k there exists a connected bipartite graph G such that $\text{reg } R/I(G) = \mu_S(G) + k$, where, $I(G)$ denotes the edge ideal of G , $\text{reg } R/I(G)$ is the Castelnuovo–Mumford regularity of $R/I(G)$ and $\mu_S(G)$ is the induced or strong matching number of G ; (ii) The graded Betti numbers of the complement of a tree only depends upon its number of vertices; (iii) The h -vector of $R/I(G \circledast H)$ is described in terms of the h -vectors of $R/I(G)$ and $R/I(H)$. Furthermore, in a different direction, we give a recursive formula for the graded Betti numbers of chordal bipartite graphs.