

# Note on the Grothendieck Group of Subspaces of Rational Functions and Shokurov's Cartier $b$ -divisors

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*Abstract.* In a previous paper the authors developed an intersection theory for subspaces of rational functions on an algebraic variety  $X$  over  $\mathbf{k} = \mathbb{C}$ . In this short note, we first extend this intersection theory to an arbitrary algebraically closed ground field  $\mathbf{k}$ . Secondly we give an isomorphism between the group of Cartier  $b$ -divisors on the birational class of  $X$  and the Grothendieck group of the semigroup of subspaces of rational functions on  $X$ . The constructed isomorphism moreover preserves the intersection numbers. This provides an alternative point of view on Cartier  $b$ -divisors and their intersection theory.