Derivations on Toeplitz Algebras

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Abstract. Let $H^2(\Omega)$ be the Hardy space on a strictly pseudoconvex domain $\Omega \subset \mathbb{C}^n$, and let $A \subset L^\infty(\partial\Omega)$ denote the subalgebra of all L^∞-functions f with compact Hankel operator H_f. Given any closed subalgebra $B \subset A$ containing $C(\partial\Omega)$, we describe the first Hochschild cohomology group of the corresponding Toeplitz algebra $\mathcal{T}(B) \subset B(H^2(\Omega))$. In particular, we show that every derivation on $\mathcal{T}(A)$ is inner. These results are new even for $n = 1$, where it follows that every derivation on $\mathcal{T}(H^\infty + C)$ is inner, while there are non-inner derivations on $\mathcal{T}(H^\infty + C(\partial\mathbb{B}_n))$ over the unit ball $\mathbb{B}_n$ in dimension $n > 1$.