

On Automorphisms and Commutativity in Semiprime Rings

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Abstract. Let R be a semiprime ring with center $Z(R)$. For $x, y \in R$, we denote by $[x, y] = xy - yx$ the commutator of x and y . If σ is a non-identity automorphism of R such that

$$\left[\left[\dots \left[\left[\sigma(x^{n_0}), x^{n_1} \right], x^{n_2} \right], \dots \right], x^{n_k} \right] = 0$$

for all $x \in R$, where $n_0, n_1, n_2, \dots, n_k$ are fixed positive integers, then there exists a map $\mu: R \rightarrow Z(R)$ such that $\sigma(x) = x + \mu(x)$ for all $x \in R$. In particular, when R is a prime ring, R is commutative.