

# A Cohomological Property of $\pi$ -invariant Elements

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*Abstract.* Let  $A$  be a Banach algebra and  $\pi: A \longrightarrow \mathcal{L}(H)$  be a continuous representation of  $A$  on a separable Hilbert space  $H$  with  $\dim H = \mathfrak{m}$ . Let  $\pi_{ij}$  be the coordinate functions of  $\pi$  with respect to an orthonormal basis and suppose that for each  $1 \leq j \leq \mathfrak{m}$ ,  $C_j = \sum_{i=1}^{\mathfrak{m}} \|\pi_{ij}\|_{A^*} < \infty$  and  $\sup_j C_j < \infty$ . Under these conditions, we call an element  $\overline{\Phi} \in l^\infty(\mathfrak{m}, A^{**})$  left  $\pi$ -invariant if  $a \cdot \overline{\Phi} = {}^t\pi(a)\overline{\Phi}$  for all  $a \in A$ . In this paper we prove a link between the existence of left  $\pi$ -invariant elements and the vanishing of certain Hochschild cohomology groups of  $A$ . Our results extend an earlier result by Lau on  $F$ -algebras and recent results of Kaniuth–Lau–Pym and the second named author in the special case that  $\pi: A \longrightarrow \mathbf{C}$  is a non-zero character on  $A$ .