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A numerical investigation of viscous incompressible fluid flow in a porous square cavity with partially active vertical walls due to Magnetic convection

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ABSTRACT

A numerical study of two dimensional convection diffusion problem for incompressible viscous flows inside a porous cavity is presented. The flow is governed by the two mechanisms: (1) a viscous electrically driven convective flow due to magnetic field, and (2) partially thermally activated side walls. Extensive numerical results of the flow field governed by the Navier-Stokes equations are obtained over a wide range of physical parameters like Grashof number, Hartmann number and the Darcy number. The simulated results has a good agreement with the available numerical previously published results and the experimental observations. The total heat flux is computed both for high convection (due to magnetic field) and diffusive forces (due to temperature).

Keywords: Porous cavity; Magneto convection; Average Nusselt number.

1. Introduction

The study of Magnetic convection inside an enclosure with periodic thermal boundary conditions has wide range of applications in industry, crystal growth techniques, space applications, oceanography, astrophysics, vulcanology, drying chambers, material processing and metallurgy etc. It also occurs in many engineering applications, geothermal phenomena and the safe disposal of nuclear waste. In this flows, the non-linear convective terms are significantly suppressed by applying an external magnetic field. Among the few geometries that have been studied in detail is the study of cavity flow filled with a porous medium. Despite its simple geometry, flow in the cavity exhibits features of more complex geometry flows. Numerical investigation on the flow inside cavity was first initiated by Burggraf [1].

The general idea regarding the flow and energy transfer inside a porous driven cavity was studied by Al-Amiri [2]. He discussed the characteristics of a lid-driven flow for a stable thermally stratified water saturated porous medium. The effects of Darcy number and Richardson number over the flow field and temperature fields are detailed studied.

Extensive studies have been made in recent years to examine the effects of magnetic field on the flow structure. Lage and Bejan [3] studied both theoretically and numerically the natural convection in a two-dimensional square cavity with one side cold and isothermal, and the other side heated with pulsating heat flux. With the periodic heating at the side walls with high Rayleigh number, they observed that the buoyancy-driven flow has the tendency to resonate the periodic heating.

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Bilgen and Yedder [4] discussed about the natural convection in a cavity with sinusoidal heating and cooling system at one side wall and observed that the heat transfer is higher when the heated section is in the lower half of the cavity at high Rayleigh number.

Kladius and Prasad [5] have studied the influence of Darcy and Prandtl numbers for fluid flow in a porous medium. They have also investigated the effect of heat transfer and flow characteristics for a horizontal porous layer.

Natural convection flow of a viscous incompressible fluid in a rectangular porous cavity heated from below and inclined cold sidewalls was studied by Hossain & Rees [6]. They observed that the flow field is dominated by the Grashof numbers due to the effect of cold sidewalls.

Nithyadevi et al. [7] studied the magnetoconvection in a square cavity where the side walls are activated with partially supplied temperature combined with a time periodic boundary condition. The results presented for the case of clear fluid where various values of amplitude, period, Grashof number, Hartmann number and Prandtl number were considered. In their study they observed that the flow and the heat transfer rate are affected by the sinusoidal temperature profile and the magnetic field at lower values of Grashof number.

The present paper deals with the effect of magnetic field on the flow field and heat transport in side the porous cavity with partially thermally active vertical walls with time periodic boundary conditions. In our study, We have considered the flow is governed by two mechanisms; (1) mechanical force due to magnetic field, and (2) buoyancy forces due to horizontal thermal gradients. Our specific aim is to investigate the characteristics of the flow fields and heat transfer due to the variation of buoyancies (due to variation of Grashof number and Prandtl number). The pathlines, isotherms and the heat transfer rates are also investigated for various values of flow parameters.

2. Numerical model and equations

We consider the unsteady two dimensional magneto natural convective flow in a square porous cavity of length L filled with an electrically conducting fluid shown in Fig. 1. A portion (1/2 of the wall) of the right wall is kept at a constant temperature and a portion (1/2 of the wall) of the left wall temperature is maintained periodic in time. The remaining boundaries of the cavity are thermally insulated. The hot region is moving from top to bottom of the left wall and the cold region is moving from bottom to top of the opposite wall. But in the present study we have presented our result for a fixed hot and cold region at the left and right side of the wall(middle-middle). In the next phase of this work, the different combinations of the thermally active locations the heat transfer characteristics will be investigated. The gravity acts vertically downwards. The uniform external magnetic field B_0 is applied parallel to gravity. It is assumed that induced magnetic field is negligible in comparison to the applied magnetic field. Under these assumptions, the governing equations for the two dimensional viscous incompressible fluid flow for magneto natural convection in a square porous cavity are as follows.

The fluid is assumed to be Newtonian and its density is supposed to be constant, except in the gravitational force term in the Navier-Stokes equation, where it varies linearly with the local temperature fraction (Gebhart and Pera [8]) and is given by

$$\rho(\theta) \;=\; \rho_0[1-\beta_\theta(\theta^*-\theta^*_L)]. \tag{2.1}$$

ρ_0 is the density of the undisturbed fluid. The volumetric coefficient of the thermal expansion $\beta_T = -\frac{1}{\rho}\frac{\partial\rho}{\partial\theta^*} > 0$. The governing Navier-Stokes equations along with the heat transport equation in dimensional form with the Boussinesq-fluid assumption are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.2}$$

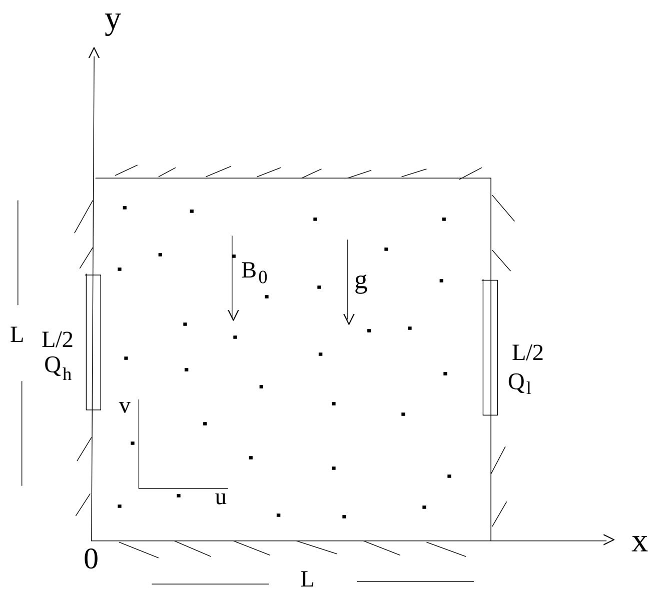

Figure 1: Schematic diagram of the 2-D cavity and computational domain.

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma B_0^2}{\rho_0}u-\frac{\nu}{K}u \tag{2.3}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y} = -\frac{1}{\rho_0}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\rho-\rho_0}{\rho_0}g-\frac{\nu}{K}v \tag{2.4}$$

$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y} = \kappa\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right). \tag{2.5}$$

Where u and v are the x and y components of the velocity field, K is the permeability of the porous medium, g is the acceleration due to gravity, κ is the thermal conductivity of the medium. The characteristic length scale and velocity scale are considered as L and $\frac{\nu}{L}$, respectively. Introducing the dimensionless variables as u^*, v^*, p^*, t^* and θ^* for the velocity, pressure, time and temperature, respectively, where

$$u^* = \frac{uL}{\nu},\ v^* = \frac{vL}{\nu},\ x^* = \frac{x}{L},\ y^* = \frac{y}{L} \tag{2.6}$$

$$t^* = \frac{t.\nu}{L^2},\ \theta^* = \frac{\theta-\theta_L}{\theta_H-\theta_L},\ p^* = \frac{pL^2}{\rho_0\nu^2}. \tag{2.7}$$

In non-dimensional form the equations can be represented as,

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \tag{2.8}$$

$$\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}\right) - (Ha^2 + \frac{1}{Kp})u^* \quad (2.9)$$

$$\frac{\partial v^*}{\partial t^*} + u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*} = -\frac{\partial p^*}{\partial y^*} + \left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}}\right) + Gr\theta^* - \frac{1}{Kp}v^* \quad (2.10)$$

$$\frac{\partial \theta^*}{\partial t^*} + u^*\frac{\partial \theta^*}{\partial x^*} + v^*\frac{\partial \theta^*}{\partial y^*} = \frac{1}{Pr}\left(\frac{\partial^2 \theta^*}{\partial x^{*2}} + \frac{\partial^2 \theta^*}{\partial y^{*2}}\right) \quad (2.11)$$

It can be seen from the equations that the flow and heat transfer are characterized by three dimensionless parameters: (i) Hartmann number Ha, (ii) Grashof number Gr, (iii) Prandtl number Pr , which are given by

$$Ha^2 = \frac{{B_0}^2 L^2 \sigma_e}{\mu}, \quad Gr = \frac{g\beta_T(\theta_H^* - \theta_L^*)L^3}{\nu^2}, \quad Pr = \frac{\nu}{\kappa}, \quad Kp = \frac{K}{L^2}. \quad (2.12)$$

$$Q_h = 1 - Asin(\pi\theta^*/\delta), \qquad Q_l = 0 \quad (2.13)$$

2δ is the period and A is the amplitude factor.

The initial conditions for time $t^* = 0$ are given by

$$u^* = v^* = 0, \quad \theta^* = 0 \qquad \text{for } 0 \le x^* \le 1, \quad 0 \le y^* \le 1 \quad (2.14)$$

The boundary conditions for time $t^* > 0$ are given by

$$u^* = v^* = 0, \qquad \frac{\partial \theta^*}{\partial y} = 0 \qquad \text{at } y^* = 0 \text{ and } 1, \quad 0 \le x^* \le 1 \quad (2.15)$$

$$u^* = v^* = 0, \quad \frac{\partial \theta^*}{\partial x} = 0, \qquad \text{at } x^* = 0 \text{ and } 1, \quad 0 \le y^* \le 0.25 \quad (2.16)$$

$$u^* = v^* = 0, \quad \frac{\partial \theta^*}{\partial x} = 0, \qquad \text{at } x^* = 0 \text{ and } 1, \quad 0.75 \le y^* \le 1 \quad (2.17)$$

$$u^* = v^* = 0, \quad \theta^* = Q_h, \qquad \text{at } x^* = 0\ , \quad 0.25 \le y^* \le 0.75 \quad (2.18)$$

$$u^* = v^* = 0, \quad \theta^* = Q_l, \qquad \text{at } x^* = 1\ , \quad 0.25 \le y^* \le 0.75 \quad (2.19)$$

The heat flux at the left wall are written in non-dimensional terms by Nusselt number as

$$Nu = \left.\frac{\partial \theta^*}{\partial x^*}\right]_{y^*=0} \quad (2.20)$$

and average Nusselt number

$$\overline{Nu} = \int_h Nu dy^*. \quad (2.21)$$

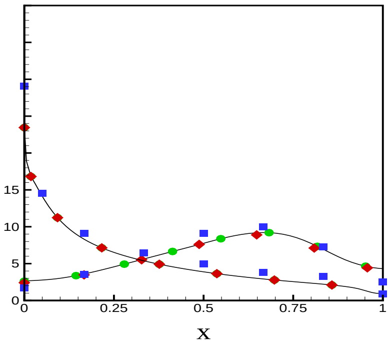

Figure 2: Influence of grid sizes on the average heat flux profile, at upper and lower lids with Moallemi and Jang [10] when Pr=1.0, B=0.0. Grashof number is 10^5.

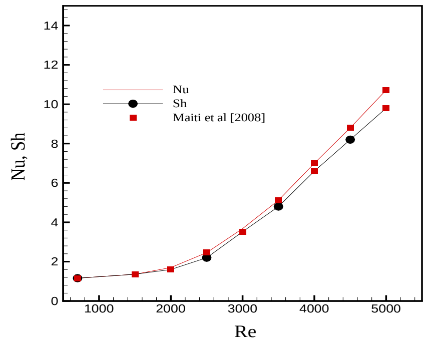

Figure 3: Comparison of present 2-D average Nusselt number with Maiti et al. [9], when B=5.0 and Grashof number is -10^6 for different Re values. Solid lines represents the present result and • represents the result due to Maiti et al.[9].

3. Numerical Method

We used the pressure correction based iterative SIMPLE algorithm for solving those governing equations with boundary conditions specified previously. This procedure is based on a cyclic series of guess-and-correct operations to solve the governing equations. A third level fully implicit scheme is used for discritization of time derivatives. Also a third order QUICK is employed to discretize the convective terms in the Navier-Stokes equations (Maiti et al. [9]). The velocity components are first calculated from the momentum equations using a guessed pressure field. The pressure and velocities are then corrected so as to satisfy continuity. This process continues until the solution converges.

3.1 *Grid Consideration and Algorithm Testing*

First we tried to compare our simulated results with some validated previously published numerical results for solid cavity case and then we compared our 2-D porous simulated results with some of the 2-D published results considering the same geometry.

To test the accuracy of our numerical algorithm we have considered the two dimensional lid driven cavity flow without porosity. For this structure the non-dimensional governing equation involves the parameters Reynolds, Grashof, and Prandtl numbers. The grid independent tests were performed by varying the grids between 61×61 to 161×161. Fig. 2 presents the effect of grid size on the average Nusselt number. We found that the changes in solution due to halving the grid size occur on the third decimal place. The grids 81×81 were found to be optimal. This figure also shows the comparison of local Nusselt number along the top (y=1) and bottom lids (y=0) for the case of no species diffusion when $Re = 500$, $Gr = 10^5$. The maximum percentage difference of Nusselt number on upper lid from the result due to Moallemi and Jang [10] is 5.5%.

To validate the accuracy of our 2-D results, we have compared our results with the average Nusselt number with Maiti et al. [9]. They studied the double-diffusive convection in a square cavity with a sliding top lid in the presence of combined vertical temperature and concentration gradients. The bottom lid and other two walls are kept fixed. The side walls are adiabatic and impermeable to solute while the top and bottom lids are kept at constant but distinct temperature and concentration. We have shown our comparison in Fig. 3, when buoyancy ratio B=5.0 and Grashof number as -10^6 for different Re

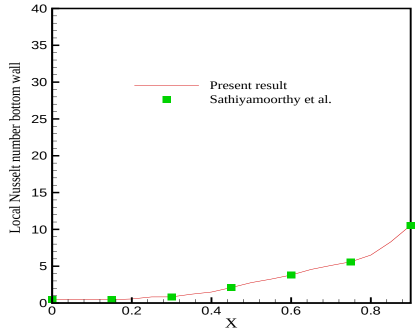

Figure 4: Comparison of present 2-D average Nusselt number with Sathiyamoorthy et al. [11], when Pr=0.7 and Darcy number Da is 10^{-4}. Solid lines represents the present result and square represents the result due to Sathiyamoorthy et al.[11].

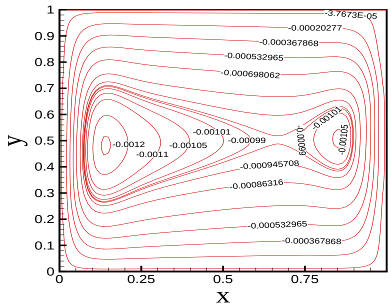

Figure 5: Streamlines for middle-middle active walls for $Gr = 10^5$, Pr= 0.054, Ha = 10, A = 0.4, $\Omega = 3$.

values. It is seen that the agreement between the published data by Maiti et al. [9] and our numerical result is very good. .

A comparison of our results for local temperature flux profiles with the results due to Satiyamoorthy et al [11] is presented in Fig. 4. Sathiyamoorthy et al. [11] considered a steady natural convection flow in a square cavity filled with a porous medium for linearly heated side walls. The results has been presented for local Nusselt number due to the uniformly heated wall where the parameters are considered as $Pr = 0.7$, $Da = 10^{-4}$.

3.2 *Results and discussion*

The magnetoconvection of an electrically conducting fluid in a square cavity filled with a porous medium (Porosity for this investigation is 1mm) is studied numerically for middle-middle thermally active locations. The flow and heat transfer characteristics are governed by the parameters Pr, Gr and Da. The computations are carried out for various values of the Grashof numbers from 10^4 to 10^6, Hartmann numbers from 10 to 50, Prandtl number 0.054, amplitude 0.4 and period 3, of the time periodic hot zone.

In Fig.5 and Fig.6 we have presented the streamlines and isothermal lines for middle-middle active walls for $Gr = 10^5$, Pr = 0.054, Ha = 10, A = 0.4, $\Omega = 3$. Two recirculation eddies are formed closed to two side walls, where the active thermal gradients are present. The flow lines shows a linear variation close to the center of the cavity. The magnetic convection plays a dominant role and the recirculation flow is mostly generated by thermal gradients. It is seen from Fig.5 that the recirculation is clockwise close to left and right walls and some perturbations are seen in streamlines far from the wall due to impingement of fluid at the horizontal wall.

It is also observed that the temperature has a large variation near the active thermal part of the wall. Since the distance increases from the wall, the isothermal contours shows a little variation, as the temperature flux variation is less along the core of the cavity. The isothermal contours shown in Fig.6 exhibit strong dependence on wall temperature. This indicates that the thermal boundary layers over the left and right walls are thicken compared to core of cavity.

The variation in the average Nusselt number for the increase in Prandtl number are shown in Fig.7 for Ha = 10, Gr = 10^5, $\delta = 3$ and A = 0.4. The rate of increase in average heat flux is very high in the range of $Pr = 0.054$ to 0.71. The comparison with Sathiyamoorthy et al. [11] without porosity shows a close

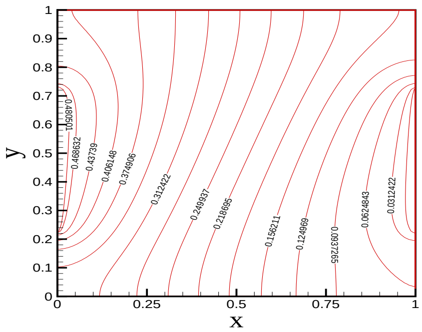

Figure 6: Isotherms profile for middlemiddle active walls for $Gr = 10^5$, Pr = 0.054, Ha = 10, A = 0.4, Ω =3.

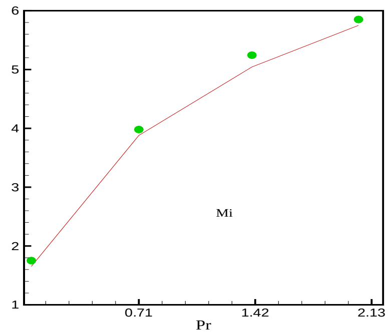

Figure 7: Average Nu vs Pr for middle–middle active walls, Gr = 10^5, A =0.4, δ = 3 and Ha = 10.

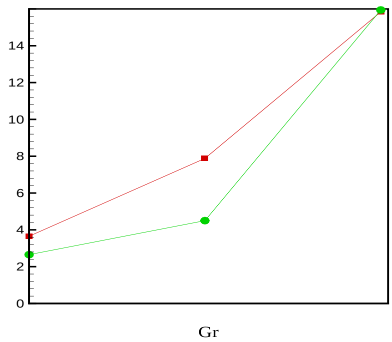

Figure 8: Average Nu vs Gr for middle-middle active walls, $Pr = 0.054$, $A = 0.4$, $\delta = 3$, $Ha = 10$ and 50 respectively.

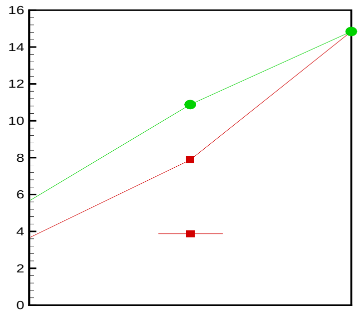

Figure 9: Average Nu vs Da for middle-middle active walls, $Pr = 0.054$, $A = 0.4$, $\delta = 3$, $Gr = 10^4$ and 10^6 respectively.

agreement with porous, that in the middle-middle active locations the heat transfer rate is maximum.

With the increase of the value of Hartmann number, the rate of heat transfer is sharply reduced. Fig.8 indicates the variations of the average Nusselt number with Grashof number for several values of Ha. The effect of magnetic field on the average Nusselt number is more in the lower Grashof number region. At higher Grashof number values, convection is dominant the reduction in heat transfer due to magnetic field is not significant.

The effects of the Darcy number and thermal behaviors of the porous cavity are investigated in Fig.9. The Darcy number, which is directly proportional to the permeability of the porous medium, was set to vary between 0.001 to 0.1. As Grashof number increases, the average heat transfer flux reduces sharply. For high porosity values the rate of heat transfer is almost same for two different Grashof numbers.

4. Conclusion

In our study we have presented the numerical results of magneto convection in a square cavity filled with a porous medium, where the side walls are inclined with a periodic temperature boundary conditions. Liquid metals (Pr = 0.054) is used as a coolant in nuclear reactors for thermodynamics systems[7]. The heat transfer rate is maximized in the middle portion of the cavity when the thermally active locations are placed in middle position of the side walls. The average Nusselt number increases as Prandtl number is increased and also the same trend is observed when the Grashof number is also increased for a fixed Darcy- and Hartmann number.

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