

Solving The (3+1)-Dimensional Kadomtsev-Petviashvili Equation By $(\frac{G'}{G})$ -Expansion Method

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Abstract:

In this paper, the $(\frac{G'}{G})$ -Expansion Method is used for construct explicit the travelling wave solutions involving parameters of the (3+1)-dimensional equation. The travelling wave solutions are expressed by the hyperbolic functions, the trigonometric functions and the rational functions.

Keywords: (G'/G) -expansion method, (3+1)-Dimensional Kadomtsev-Petviashvili, Homogeneous balance.

1 Introduction

In recent years, searching for explicit solutions of nonlinear evolution equations by using various methods has become the main goal for many authors such as the homogeneous balance method [1,2], the hyperbolic tangent expansion method [3,4], the trial function method [5], the tanh-method [6,7,8,9,10], the non-linear transform method [11], the inverse scattering transform [12]. The objective of this paper is to use a new method which is called the $(\frac{G'}{G})$ -expansion method [13]. The paper is arranged as follows. In Section 2, we describe briefly the $(\frac{G'}{G})$ -expansion method. where $G = G(\xi)$ satisfies the second order linear ordinary differential equation $G'' + \lambda G' + \mu G = 0$, where $\xi = sx + ly + kz - \omega t$, where s, l, k and ω are constants. The degree of this polynomial can be determined by considering the homogeneous balance between the highest order derivatives and the non-linear terms appearing in the given non-linear equations. In Sections 3, we apply this method to (3+1)-Dimensional Kadomtsev-Petviashvili equation. In section 4 some conclusions are given.

2 The $\frac{G'}{G}$ -expansion method

Suppose that a nonlinear equation, say in two independent variables x and t , is given by

$$P(u, u_x, u_t, u_{tt}, u_{xt}, u_{xx}, \dots) = 0 \quad (2.1)$$

where $u = u(x, t)$ is an unknown function, P is a polynomial in $u = u(x, t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following we give the main steps of the $\frac{G'}{G}$ -expansion method.

step 1:Combining the independent variables x and t into one variable $\xi = x - vt$, we suppose that

$$u(x, t) = u(\xi), \quad \xi = x - vt \quad (2.2)$$

The travelling wave variable (2.2) permits us to reduce Eq.(2.1) to an ODE for $u = u(\xi)$, namely

$$P(u, -vu', u', v^2u'', -vu'', u'', \dots) = 0 \quad (2.3)$$

step 2:Suppose that the solution of ODE (2.3) can be expressed by a polynomial in $\frac{G'}{G}$ as follows

$$u(\xi) = \alpha_m \left(\frac{G'}{G} \right)^m + \dots, \quad (2.4)$$

where $G = G(\xi)$ satisfies the second order LODE in the form

$$G'' + \lambda G' + \mu G = 0 \quad (2.5)$$

α_m, \dots, λ and μ are constants to be determined later, $\alpha_m \neq 0$, the unwritten part in (2.4) is also a polynomial in $\frac{G'}{G}$, but the degree of which is generally equal to or less than $m - 1$, the positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in ODE (2.3).

step 3:By substituting (2.4) into Eq.(2.3) and using the second order linear ODE (2.5), collecting all terms with the same order of $\frac{G'}{G}$ together, the left-hand side of Eq. (2.3) is converted into another polynomial in $\frac{G'}{G}$. Equating each coefficient of this polynomial to zero yields a set of algebraic equations for α_m, \dots, λ and μ .

step 4:Assuming that the constants α_m, \dots, λ and μ can be obtained by solving the algebraic equations in Step 3, since the general solutions of the second order LODE (2.5) have been well known for us, then substituting α_m, \dots, v and the general solutions of Eq. (2.5) into (2.4) we have more travelling wave solutions of the nonlinear evolution equation (2.1).

3 (3+1)-Dimensional Kadomtsev-Petviashvili Equation

In this section we consider the (3+1)-Dimensional Kadomtsev-Petviashvili Equation in the form

$$(u_t + 6uu_x + u_{xxx})_x - 3(u_{yy} + u_{zz}) = 0 \quad (3.1)$$

Equation (3.1) is not integrable by inverse scattering transformation unless $\partial_z = 0$, but it passes the Painleve property [13]. In what follows, we study the travelling wave solutions to Eq. (3.1). Substituting $u = u(\xi)$, $\xi = sx + ly + kz - \omega t$ into Eq. (3.1) and integrating twice, we have

$$s^4 u'' + 3l^2 u^2 - (\omega s + 3l^2 + 3k^2)u = c \quad (3.2)$$

where C is the integration constant, and the first integrating constant is taken to zero. Suppose that the solutions of the O.D.E (3.2) can be expressed by a polynomial in $\frac{G'}{G}$ as follows:

$$u(\xi) = \alpha_m \left(\frac{G'}{G}\right)^m + \dots, \quad (3.3)$$

Where $G = G(\xi)$ satisfies the second order LODE in the form

$$G'' + \lambda G' + \mu G = 0 \quad (3.4)$$

By using (3.3) and (3.4) and considering the homogeneous balance between u'' and u^2 in Eq. (3.2) we required that $2m = m + 2$ then $m = 2$. So we can write (3.3) as

$$u(\xi) = \alpha_2 \left(\frac{G'}{G}\right) + \alpha_1 \left(\frac{G'}{G}\right) + \alpha_0 \quad (3.5)$$

So

$$u^2 = \alpha_2^2 \left(\frac{G'}{G}\right)^4 + 2\alpha_2\alpha_1 \left(\frac{G'}{G}\right)^3 + (2\alpha_2\alpha_0 + \alpha_1^2) \left(\frac{G'}{G}\right)^2 + 2\alpha_1\alpha_0 \left(\frac{G'}{G}\right) + \alpha_0^2 \quad (3.6)$$

By using (3.4) and (3.5) it is derived that

$$\begin{aligned} u'' &= 6\alpha_2 \left(\frac{G'}{G}\right)^4 + (2\alpha_1 + 10\alpha_2\lambda) \left(\frac{G'}{G}\right)^3 + (8\alpha_2\mu + 3\alpha_1\lambda + 4\alpha_2\lambda^2) \left(\frac{G'}{G}\right)^2 \\ &+ (6\alpha_2\lambda\mu + 2\alpha_1\mu + \alpha_1\lambda^2) \left(\frac{G'}{G}\right) + 2\alpha_2\mu^2 + \alpha_1\lambda\mu \end{aligned} \quad (3.7)$$

On substituting (3.5) – (3.7) into (3.2), collecting all terms with the same powers of $\frac{G'}{G}$ and setting each coefficient to zero, we obtain the following system of algebraic equations:

$$\begin{aligned} s^4(2\alpha_2\mu^2 + \alpha_1\lambda\mu) + 3l^2\alpha_0^2 - (\omega s + 3l^2 + 3k^2)\alpha_0 - c &= 0 \\ s^4(6\alpha_2\lambda\mu + 2\alpha_1\mu + \alpha_1\lambda^2) + 6l^2\alpha_1\alpha_0 - (\omega s + 3l^2 + 3k^2)\alpha_1 &= 0 \\ s^4(8\alpha_2 + 3\alpha_1\lambda + 4\alpha_2\lambda^2) + 3l^2(\alpha_1^2 + 2\alpha_2\alpha_0) - (\omega s + 3l^2 + 3k^2)\alpha_2 &= 0 \\ s^4(2\alpha_1 + 10\alpha_2\lambda) + 6l^2\alpha_2\alpha_1 &= 0 \end{aligned}$$

$$6\alpha^2 s^4 + 3l^2 \alpha^2 = 0$$

On solving the above algebraic Eq. above by using the Maple, we get

$$\alpha_1 = \frac{-2s^4 \lambda}{l^2}, \alpha_2 = \frac{-2s^4}{l^2} \quad (3.8)$$

$$\omega = -\frac{-8s^4 - s^4 \lambda^2 - 6l^2 \alpha_0 + 3l^2 + 3k^2}{s}$$

$$c = -\frac{4s^8 \mu^2 + 2s^8 \mu \lambda^2 + 3l^4 \alpha_0^2 + 8s^4 l^2 \alpha_0 + l^2 \alpha_0 s^4 \lambda^2}{l^2}$$

λ, μ and α_0 are arbitrary constants.

By using (3.8), expression (3.5) can be written as

$$u(\xi) = \frac{-2s^4}{l^2} \left(\frac{G'}{G} \right)^2 - \frac{2s^4 \lambda}{l^2} \left(\frac{G'}{G} \right) + \alpha_0 \quad (3.9)$$

Where $\xi = sx + ly + kz + \frac{3l^2 + 3k^2 - 8s^4 - s^4 \lambda^2 - 6l^2 \alpha_0}{s} t$.

On solving the Eq. (3.4), we deduce after some reduction that

$$\frac{G'}{G} = \frac{1}{2} \sqrt{\lambda^2 - 4\mu} \times \left(\frac{C_1 \sinh \frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi + C_2 \cosh \frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi}{C_1 \cosh \frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi + C_2 \sinh \frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi} \right) - \frac{\lambda}{2}$$

where C_1 and C_2 are arbitrary constants. Substituting the general solutions of Eq. (3.4) into (3.5) we have three types of travelling wave solutions of the (3+1)-Dimensional Kadomtsev-Petviashvili Equation (3.1) as follows:

When $\lambda^2 - 4\mu > 0$

$$u(\xi) = \frac{-2s^4}{l^2} (\lambda^2 - 4\mu) \times \left(\frac{C_1 \sinh \frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi + C_2 \cosh \frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi}{C_1 \cosh \frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi + C_2 \sinh \frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi} \right)^2 - \frac{2s^4 \lambda}{l^2} - \frac{\lambda}{2} + \alpha_0$$

Where $\xi = sx + ly + kz + \frac{3l^2 + 3k^2 - 8s^4 - s^4 \lambda^2 - 6l^2 \alpha_0}{s} t$, C_1 and C_2 are arbitrary constants.

If C_1 and C_2 are taken as special values, the various known results in the literature can be rediscovered, for instance, if $C_1 > 0$, $C_1^2 > C_2^2$, then $u = u(\xi)$ can be written as

$$u(\xi) = \frac{-2s^4}{l^2} (\lambda^2 - 4\mu) \times \operatorname{sech}^2 \left(\frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi + \xi_0 \right) - \frac{2s^4 \lambda}{l^2} - \frac{\lambda}{2} + \alpha_0$$

When $\lambda^2 - 4\mu < 0$

$$u(\xi) = \frac{-2s^4}{l^2} (4\mu - \lambda^2) \times \left(\frac{-C_1 \sin \frac{1}{2} \sqrt{4\mu - \lambda^2} \xi + C_2 \cos \frac{1}{2} \sqrt{4\mu - \lambda^2} \xi}{C_1 \cos \frac{1}{2} \sqrt{4\mu - \lambda^2} \xi + C_2 \sin \frac{1}{2} \sqrt{4\mu - \lambda^2} \xi} \right)^2 - \frac{2s^4 \lambda}{l^2} - \frac{\lambda}{2} + \alpha_0$$

When $\lambda^2 - 4\mu = 0$

$$u(\xi) = \frac{-2s^4 C_2^2}{l^2 (C_1 + C_2 \xi)^2} - \frac{2s^4 \lambda}{l^2} - \frac{\lambda}{2} + \alpha_0 \quad (3.10)$$

Which are the solutions of (3+1)-Dimensional Kadomtsev-Petviashvili Equation.

4 conclusion

In this paper we have seen that three types of travelling solutions of the (3+1)-Dimensional Kadomtsev-Petviashvili equation are successfully found out by using the $\frac{G'}{G}$ -expansion method. The solutions of these non-linear evolution equations have many potential applications in physics. These equations are very difficult to be solved by traditional methods. The performance of this method is reliable, simple and gives many new exact solutions.

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